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RESEARCH ARTICLE

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Concatenated Alamouti codes using multi-level modulation and symbol mapping diversity technique

Abstract A family of space-time block codes (STBCs) for systems with even transmit antennas and any number of receive antennas is proposed. The new codeword matrix is constructed by concatenating Alamouti space-time codes to form a block diagonal matrix, and its dimension is equal to the number of transmit antennas. All Alamouti codes in the same codeword matrix have the same information; thus, full transmit diversity can be achieved over fading channels. To improve the spectral efficiency, multi-level modulations such as multi-quadrature amplitude modulation (M-QAM) are employed. The symbol mapping diversity is then exploited between transmissions of the same information from different antennas to improve the bit error rate (BER) performance. The proposed codes outperform the diagonal algebraic space-time (DAST) codes presented by Damen [Damen et al. IEEE Transactions on Information Theory, 2002, 48(3): 628–636] when they have the same spectral efficiency. Also, they outperform the 1/2-rate codes from complex orthogonal design. Moreover, compared to DAST codes, the proposed codes have a low decoding complexity because we only need to perform linear processing to achieve single-symbol maximum-likelihood (ML) decoding.

Keywords space-time block codes (STBCs), symbol mapping, spectral efficiency, decoding complexity, pairwise error probability (PEP)

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1 Introduction

It is well known that multiple-input multiple-output (MIMO) communication systems can elegantly provide spatial diversity to mitigate the effect of fading [1,2]. Using space-time codes, transmit diversity can be exploited for systems with multiple transmit antennas. Thus, it is advantageous to a cellular base station. Alamouti proposes a simple transmit diversity scheme using two transmit antennas [3]. This scheme can provide a diversity order of $2n$ with full rate (i.e., one symbol per transmission) for n receive antennas and its computation complexity is similar to that of maximal-ratio receiver combining (MRRC). Recently, orthogonal space-time block codes (OSTBCs) were designed to achieve full diversity for more than two transmit antennas [4]. Moreover, the maximum-likelihood (ML) decoding algorithm for OSTBC is based only on linear processing at the receiver. Though the codes presented in Ref. [4] can offer full rate for real signal constellations, the complex OSTBC which provides full rate does not exist for more than two transmit antennas [4]. Furthermore, it is proved in Ref. [5] that the symbol rates of complex OSTBC are upper bounded by 3/4 for more than two transmit antennas, and a tight upper bound approaching 1/2 is conjectured when the number of transmit antennas is large [5]. To improve symbol rates for systems with more than two transmit antennas, some designs of space-time codes are proposed in Refs. [6–10]. Those proposed code designs in Refs. [6–10] can guarantee full or near full rate. In Ref. [6], the quasi-orthogonal space-time block codes are presented, but they cannot offer full diversity and the decoders work with pairs of transmitted symbols. The symbol rate of the codes presented in Refs. [7,8] can approach one as the block size goes to infinity, and full diversity can be achieved by employing linear zero-forcing (ZF) or minimum mean square error (MMSE) receiver. However, for the codes designed in Refs. [7,8], the block size increases and the performance degrades when the symbol rate increases. In Refs. [9,10], space-time block code (STBC) with full rate and full diversity are designed based on constellation rotation, which is presented in Ref. [11]. In Ref. [9], an L-dimensional constellation is rotated, and then the component of each dimension is transmitted through a different antenna. In Ref. [10], using constellation rotation and Hadamard transform, diagonal algebraic space-time (DAST) block codes are designed. When the number of transmit antennas is large, the codes presented in Refs. [9,10] are better than those from orthogonal design [4]. However, the decoding complexity of these codes presented in Refs. [9,10] is very high. In fact, the same information is transmitted in different symbol intervals in Alamouti scheme and those schemes based on constellation rotation that are presented in Refs. [9,10]. That is to say, repetition coding is employed to offer diversity.

Multi-level modulation techniques, such as multi-pulse amplitude modulation (M-PAM), multi-phase shift keying (M-PSK), and multi-quadrature amplitude modulation (M-QAM) are bandwidth efficient; thus, they are usually employed to provide high data rates. To improve the robustness of multi-level modulations in noise channels, some enhanced schemes [12,13] extracting additional diversity, called symbol mapping diversity [13] by using constellation rearrangement, have been proposed in automatic repeat request (ARQ) systems. Using the symbol mapping diversity technique, the bit error rate (BER) performance can be improved greatly. On the other hand, Alamouti codes [3] have the best performance and the lowest decoding complexity for two antennas. Motivated by these results, in this paper, we construct a family of STBCs by concatenating Alamouti codes for systems with even transmit antennas and any number of receive antennas. We also employ repetition coding. Several Alamouti codes with the same information are transmitted from different pairs of antennas in different pairs of symbol intervals. Hence, full diversity is obtained with ML decoding that is based only on linear processing. Moreover, multi-level modulations are employed for high spectral efficiency, and the symbol mapping diversity is exploited between transmissions from different pairs of antennas. The proposed codes outperform the DAST codes in terms of the BER performance, and especially in the decoding complexity.

The rest of this paper is organized as follows. In Sect. 2, the system model is introduced, and a family of concatenated Alamouti codes is designed. The pairwise error probability (PEP) and the decoding complexity are analyzed in Sect. 3, and some simulation results are provided to compare the performance of different codes in Sect. 4. Finally, conclusions are contained in Sect. 5.

2 System model and concatenated Alamouti codes

In each symbol interval, only two antennas transmit and they transmit simultaneously using Alamouti's scheme. Without loss of generality, we consider a multiple-input single-output (MISO) system with $2m$ transmit antennas and one receive antenna. Assume the channel is quasi-static Rayleigh flat fading, which remains constant over at least two symbol intervals, and denoted as

$$\boldsymbol{H} = [h_1, h_2, \ldots, h_{2m}], \tag{1}$$

where the coefficient h_k $(k = 1,2,\ldots,2m)$ is the path gain from transmit antenna k to the receive antenna. All coefficients are modeled as independent complex zero-mean Gaussian random variables with variance 0.5 per real dimension.

Assume that $2\log_2 M$ information bits are encoded into a space-time block codeword and an M-ary modulation, which has an alphabet $\{s_0, s_1, \ldots, s_{M-1}\}$, is employed. The space-time block codeword matrix $\boldsymbol{C}$ can be expressed as

$$\boldsymbol{C} = \mathrm{diag}(\boldsymbol{C}_1, \boldsymbol{C}_2, \ldots, \boldsymbol{C}_m), \tag{2}$$

where $\mathrm{diag}(\boldsymbol{C}_1, \boldsymbol{C}_2, \ldots, \boldsymbol{C}_m)$ is a block diagonal matrix, $\boldsymbol{C}_k$ $(k = 1,2,\ldots,m)$ is the Alamouti code, and

$$\boldsymbol{C}_k = \begin{bmatrix} c_{k1} & -c_{k2}^* \\ c_{k2} & c_{k1}^* \end{bmatrix}, \tag{3}$$

where $c_{k1}, c_{k2} \in \{s_0, s_1, \ldots, s_{M-1}\}$, and $(\cdot)^*$ denotes the complex conjugate operation. Hence, the codeword matrix $\boldsymbol{C}$ is orthogonal. Codes $\boldsymbol{C}_1, \boldsymbol{C}_2, \ldots, \boldsymbol{C}_m$ have the same information, but they have different bit-to-symbol mapping to exploit symbol mapping diversity.

The code $\boldsymbol{C}$ is transmitted from $2m$ antennas in $2m$ symbol intervals, and the total power of the transmitter is constrained, then the received baseband signal $\boldsymbol{r} = [r_1, r_2, \ldots, r_{2m}]$ can be expressed as

$$\boldsymbol{r} = \boldsymbol{H}\frac{\boldsymbol{C}}{\sqrt{2}} + \boldsymbol{n}, \tag{4}$$

where $\boldsymbol{n} = [n_1, n_2, \ldots, n_{2m}]$. Each noise term n_k $(k = 1,2,\ldots,2m)$ is an independent complex zero-mean Gaussian random variable and $\varepsilon\{|n_k|^2\} = N_0$. Note that ε denotes the expectation operator. When the receiver knows $\boldsymbol{H}$, the decoder can decode by employing ML decoding algorithm and it only works with single symbols.

3 PEP and decoding complexity

We consider a 4ISO system which means there are four transmit antennas and one receive antenna. The modulations are 4-QAM and 16-QAM for DAST codes and proposed concatenated codes, respectively. Thus, the spectral efficiency is 2 bit/(s·Hz). Since the rectangular signal constellation is equivalent to two PAM signals on quadrature carriers, we only consider one-dimensional PAM modulations. Hence, 4-PAM signal is transmitted in each transmit antenna for the scheme employing concatenated codes, and the received signal in each symbol interval is the 16-PAM signal. That is to say, the same information is transmitted for four times in four symbol intervals.

Assume that the receiver has perfect channel state information (CSI) and employs coherent detection. The metric $P(x \to y)$ is defined as the PEP when the transmitted symbol x is detected as symbol $y\,(x \neq y)$. We derive the PEP for concatenated Alamouti codes when constellation rearrangement is and is not used, respectively.

3.1 Concatenated Alamouti codes without constellation rearrangement

Codeword matrix $\boldsymbol{C}$ can be expressed as

$$\boldsymbol{C} = \mathrm{diag}(\boldsymbol{C}_1, \boldsymbol{C}_1). \tag{5}$$

Gray code is used to map bits to symbols, and the mapping for 4-PAM is

$$\{00,01,10,11\} \to \frac{1}{\sqrt{5}}\{-3,-1,3,1\}. \tag{6}$$

Assume that s_0 and s_1 are transmitted, using the Alamouti's combining scheme, we obtain

$$\begin{aligned}\tilde{s}_0^{(1)} &= \frac{h_1^* r_1 + h_2 r_2^*}{\sqrt{|h_1|^2 + |h_2|^2}} \\ &= \sqrt{|h_1|^2 + |h_2|^2}\cdot\frac{s_0}{\sqrt{2}} + \frac{h_1^* n_1 + h_2 n_2^*}{\sqrt{|h_1|^2 + |h_2|^2}}\end{aligned} \tag{7}$$

and

$$\begin{aligned}\tilde{s}_0^{(2)} &= \frac{h_3^* r_3 + h_4 r_4^*}{\sqrt{|h_3|^2 + |h_4|^2}} \\ &= \sqrt{|h_3|^2 + |h_4|^2}\cdot\frac{s_0}{\sqrt{2}} + \frac{h_3^* n_3 + h_4 n_4^*}{\sqrt{|h_3|^2 + |h_4|^2}}.\end{aligned} \tag{8}$$

Then we can estimate s_0 from Eqs. (7) and (8) using MRRC. Similar processes are executed for estimating s_1. Without loss of generality, we derive the PEP $P(0000 \to 0001)$ from Eqs. (7) and (8), and the result is

$$\begin{aligned}&P(0000 \to 0001) \\ &\approx \varepsilon\left\{Q\left(\sqrt{\frac{|h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2}{5N_0}}\right)\right\},\end{aligned} \tag{9}$$

where $Q(\cdot)$ is the Gaussian Q-function. Using the Chernoff bounding technique, Eq. (9) is upper-bounded as

$$P(0000 \to 0001) \leqslant \frac{1}{2}\left(1 + \frac{1}{10N_0}\right)^{-4}. \tag{10}$$

Let $\gamma = 1/(10N_0)$, Eq. (10) can be written as

$$P(0000 \to 0001) \leqslant \frac{1}{2}(\gamma^4 + 4\gamma^3 + 6\gamma^2 + 4\gamma + 1)^{-1}. \tag{11}$$

3.2 Concatenated Alamouti codes with constellation rearrangement

Codeword matrix $\boldsymbol{C}$ can be expressed as

$$\boldsymbol{C} = \mathrm{diag}(\boldsymbol{C}_1, \boldsymbol{C}_2). \tag{12}$$

Codes $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$ have the same information, but they have different symbol mapping. Symbol mapping for code $\boldsymbol{C}_1$ is

$$\{00,01,10,11\} \to \frac{1}{\sqrt{5}}\{-3,-1,1,3\}, \tag{13}$$

and symbol mapping for code $\boldsymbol{C}_2$ is

$$\{00,01,10,11\} \to \frac{1}{\sqrt{5}}\{1,-3,3,-1\}. \tag{14}$$

The single-symbol ML decoding algorithm is similar to that described in Sect. 3.1, and we can obtain the PEP $P(0000 \to 0001)$ as

$$\begin{aligned}&P(0000 \to 0001) \\ &\approx \varepsilon\left\{Q\left(\sqrt{\frac{|h_1|^2 + |h_2|^2 + 4|h_3|^2 + 4|h_4|^2}{5N_0}}\right)\right\}.\end{aligned} \tag{15}$$

Then Eq. (15) can be upper-bounded as

$$\begin{aligned}&P(0000 \to 0001) \\ &\leqslant \frac{1}{2}\left(1 + \frac{1}{10N_0}\right)^{-2}\left(1 + \frac{4}{10N_0}\right)^{-2} \\ &= \frac{1}{2}(16\gamma^4 + 40\gamma^3 + 33\gamma^2 + 10\gamma + 1)^{-1}.\end{aligned} \tag{16}$$

Here, we use the symbol mapping that can achieve the optimal performance. If another non-optimal symbol mapping is used, then the value of the PEP is larger, and the performance is worse.

3.3 DAST codes

To construct DAST codes for dimension 4 proposed in Ref. [10], a rotation matrix $\boldsymbol{M}_4$ is applied to a 4-dimensional 2-PAM signal, which is constructed from four 2-PAM signals. Here the rotation matrix $\boldsymbol{M}_4$ given in Ref. [10] is used, and the first row of $\boldsymbol{M}_4$ is (0.2012, 0.3255, −0.4857, −0.7859). Hence, the transmitted and received signals in each symbol interval are also 16-PAM signals, and the same information is transmitted four times in four symbol intervals. Thus, DAST codes also transmit multi-level modulation signals several times to offer diversity, which is the same as that in the proposed concatenated codes.

The PEP $P(0000 \rightarrow 0001)$ can be written as [11]

$$P(0000 \rightarrow 0001) = \varepsilon\left\{Q\left(\sqrt{\frac{2.47|h_1|^2 + 0.94|h_2|^2 + 0.42|h_3|^2 + 0.16|h_4|^2}{2N_0}}\right)\right\}, \tag{17}$$

and it can be upper-bounded as

$$\begin{aligned} &P(0000 \rightarrow 0001) \\ &\leqslant \frac{1}{2}\left(1+\frac{2.47}{4N_0}\right)^{-1}\left(1+\frac{0.94}{4N_0}\right)^{-1} \\ &\quad \cdot \left(1+\frac{0.42}{4N_0}\right)^{-1}\left(1+\frac{0.16}{4N_0}\right)^{-1} \\ &\approx \frac{1}{2}(6.1\gamma^4 + 24.6\gamma^3 + 27.3\gamma^2 + 10\gamma + 1)^{-1}. \end{aligned} \tag{18}$$

We observe from Eqs. (11), (16), and (18) that the PEP performance of concatenated Alamouti codes without constellation rearrangement is worse than that of the DAST codes. However, if the symbol mapping diversity technique is used, then the concatenated Alamouti codes outperform the DAST codes. Therefore, the symbol mapping is important for the proposed scheme employing concatenated codes. Moreover, these values of PEP expressed in Eqs. (11), (16), and (18) are mainly determined by the value of γ^{-4} when $\gamma \gg 1$; thus, all of these schemes can provide a diversity order of four.

Furthermore, the concatenated Alamouti codes have single-symbol ML decoding algorithm that is based only on linear processing due to the orthogonality. Each 4-PAM signal can be decoded independently. However, for DAST codes, four 2-PAM signals cannot be decoded separately. Hence, the decoding complexity of concatenated Alamouti codes is much lower than that of the DAST codes.

4 Simulation results

In the simulation, the channel model described in Sect. 2 is used. We constrain the total power of the transmitter to 1 for one-dimensional modulations, and define signal-to-noise ratio (SNR) in the receiver as

$$\mathrm{SNR} = \frac{1}{N_0}.$$

Figure 1 shows the simulation results of the BER performance of 4ISO system for DAST codes and concatenated Alamouti codes. DAST codes with 2-PAM modulation and concatenated codes with 4-PAM modulation, and DAST codes with 4-PAM modulation and concatenated codes with 16-PAM modulation are simulated and they have the same spectral efficiency, respectively. For 16-PAM modulation, the results of

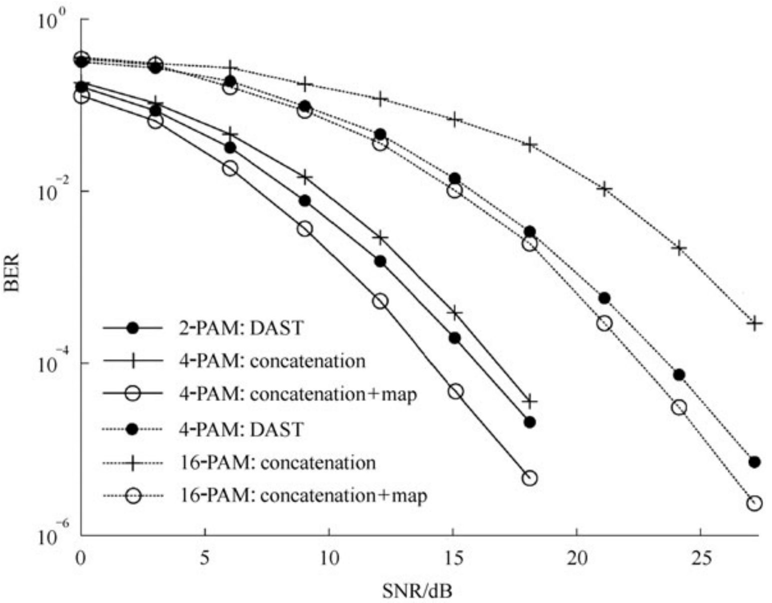

Fig. 1 BER performance of DAST codes and concatenated Alamouti codes in quasi-static Rayleigh flat fading channel (four transmit antennas and one receive antenna)

mapping rearrangement, which are designed here using the rule proposed in Ref. [13], are that the first mapping is

$$\{0000,0001,0010,0011,0100,0101,0110,0111,1000,1001,1010,1011,1100,1101,1110,1111\}$$

$$\rightarrow \frac{1}{\sqrt{85}}\{-15,-11,-3,-7,-1,-5,-13,-9,1,5,13,9,3,7,15,11\}, \tag{19}$$

and the second mapping is

$$\{0000,0001,0010,0011,0100,0101,0110,0111,1000,1001,1010,1011,1100,1101,1110,1111\}$$

$$\rightarrow \frac{1}{\sqrt{85}}\{1,5,13,9,-1,-5,-13,-9,-15,-11,-3,-7,3,7,15,11\}. \tag{20}$$

The BER performance of the proposed concatenated codes is the worst if the symbol mapping diversity is not exploited. However, when using the symbol mapping diversity technique, the performance of the proposed concatenated codes can be improved greatly, and it is better than that of the DAST codes. Hence, it is important to employ good symbol mapping that can provide good BER performance. Moreover, for DAST codes with 4-PAM modulation, the decoder must jointly decode four 4-PAM signals. While the corresponding proposed scheme only needs to decode each 16-PAM signal independently.

Figure 2 shows the simulation results of the BER performance of the 6ISO system. The performance of DAST codes, OSTBC, and concatenated Alamouti codes are compared. The rotation matrix $\boldsymbol{M}_6$ given in Ref. [11] is used for DAST codes, and the first row of $\boldsymbol{M}_6$ is $(-0.3199, 0.7189, 0.5765, -0.0590, 0.1326, 0.1654)$. Since 1/2-rate complex OSTBC and full-rate real OSTBC have the same spectral efficiency, we use real OSTBC proposed in Ref. [4] for six transmit antennas. 4-QAM, 4-PAM, and 64-QAM modulations are employed for DAST codes, OSTBC, and concatenated codes, respectively. Hence, these codes have the same spectral efficiency of 2 bit/(s·Hz). Also, for simplicity of simulation, we only consider one-dimensional PAM modulations with half energy, that is to say, we use 2-PAM and 8-PAM modulations for DAST codes and concatenated codes, respectively. Note that the total power of the transmitter is constrained to 2 for real OSTBC. For 8-PAM modulation, the results of mapping rearrangement designed here are that the first mapping is

$$\{000,001,010,011,100,101,110,111\}$$

$$\rightarrow \frac{1}{\sqrt{21}}\{-7,-5,-1,-3,3,5,1,7\}, \tag{21}$$

the second mapping is

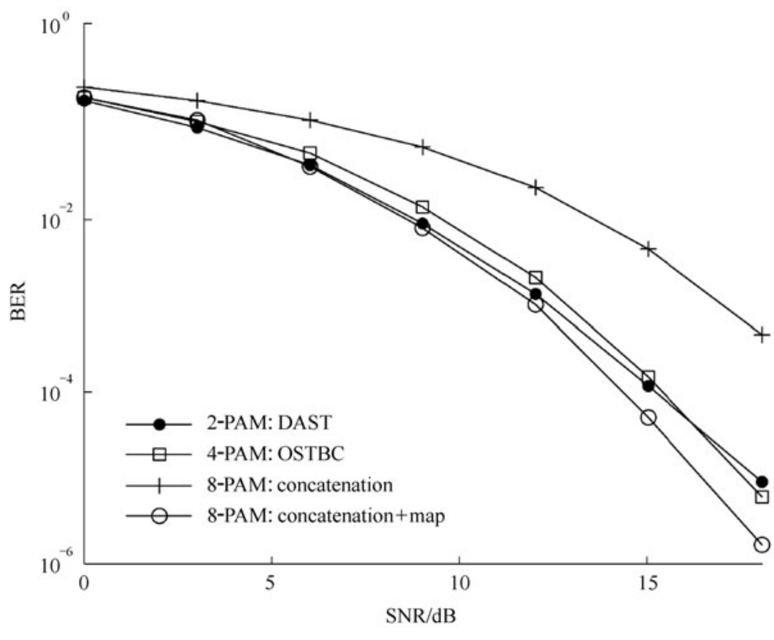

Fig. 2 BER performance of DAST codes, OSTBC, and concatenated Alamouti codes in quasi-static Rayleigh flat fading channel (six transmit antennas and one receive antenna)

$$\{000,001,010,011,100,101,110,111\} \rightarrow \frac{1}{\sqrt{21}}\{1,-5,-1,5,3,-3,-7,7\}, \quad (22)$$

and the third mapping is

$$\{000,001,010,011,100,101,110,111\} \rightarrow \frac{1}{\sqrt{21}}\{-3,1,-7,5,-1,3,7,-5\}. \quad (23)$$

We observe that the BER performance of OSTBC is slightly worse at moderate SNR and slightly better at high SNR than that of DAST codes. The BER performance of the proposed concatenated codes without constellation rearrangement is the worst. Using the symbol mapping diversity technique, the proposed codes have better performance than DAST codes and OSTBC. The decoding complexity of OSTBC is the lowest. It only needs to decode each 4-PAM signal independently. The proposed scheme needs to decode each 8-PAM signal independently. Hence, the decoding complexity is slightly higher than that of OSTBC. However, six 2-PAM signals must be jointly decoded for DAST codes, which results in a high decoding complexity.

5 Conclusions

By concatenating Alamouti codes, we design a new family of STBC. The new codes can provide full diversity because the same information is transmitted from all transmit antennas, which is similar to that in the DAST codes, and the codeword matrix is orthogonal. Simultaneously, high spectral efficiency is achieved by using multi-level modulations, and the BER performance is improved by using mapping rearrangement to extract symbol mapping diversity. Because of the orthogonality of the codeword matrix, the decoding complexity is low. Furthermore, the BER performance of the proposed codes is better than that of the DAST codes and the 1/2-rate codes from complex orthogonal design.

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