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RESEARCH ARTICLE

Haijun WANG, Xin SU, Yi XU, Shidong ZHOU, Jing WANG

Optimal cooperative energy spectrum sensing in cognitive radio network

Abstract Cooperative energy spectrum sensing has been proved effective to detect the spectrum holes in cognitive radio (CR). In this paper, we present the optimal energy sensing algorithm in CR network and prove its optimality through the duality theorem of Neyman-Pearson theorem. Then, the optimal energy sensing algorithm is expanded to a cooperative sensing algorithm based on channel covariance matrix. We compare the proposed algorithms with traditional cooperative sensing algorithms in terms of complexity and required transmission bits. Simulation results validate the optimality of the proposed cooperative sensing algorithm. Furthermore, it is intuitively reasonable for the sensing station to choose the sensing nodes with better channel condition to cooperate, which is verified by our analysis and simulation.

Keywords cognitive radio (CR), cooperative sensing, likelihood-ratio test

1 Introduction

Nowadays, spectrum resources have become increasingly scarce. However, statistics show that the utilization of spectrum resources is very low in terms of time and space. Cognitive radio (CR) emerges as a potential technique to solve this unreasonable spectrum allocation problem by detecting and utilizing spectrum holes. Generally, CR networks can be categorized into three classes: underlay, overlay, and interweave [1]. Only interweave cognitive system is considered in this paper, wherein the cognitive user needs to accurately detect whether the current band is occupied by a licensed user to ensure the licensed user's use of specific bands. Spectrum sensing algorithms in CR can be mainly divided into three types: energy detection, matched filter detection, and cyclostationary detection [2]. Among them, energy detection has been widely applied since its algorithm is simple, and it does not require transcendental knowledge of the licensed user's signals.

Due to the interference factors, such as multipath and shadow effect of wireless channels, energy sensing conducted by single cognitive sensing node that has low signal-to-noise ratio (SNR) of the received signal may be unreliable. However, this problem can be eased by cooperative sensing strategies, which are extensively studied and can be found in many references. Reference [3] studied the AND/OR cooperative spectrum sensing algorithms and discussed how to achieve the optimal performance by choosing different cooperative users. Reference [4] proposed a half-voting cooperative sensing algorithm and a fast spectrum sensing algorithm for a large CR network, which requires less than the total number of cognitive nodes. Since local observations of each sensing nodes radios are quantized to one bit to the sensing station [5], these two cooperative sensing algorithms belong to hard (binary) cooperative sensing algorithm. In soft cooperative sensing algorithms, the sensing nodes send soft information, e.g., quantized likelihood ratio or received signals (usually more than one bit), to the sensing station. For instance, several liner cooperative spectrum sensing algorithms have been proposed in Refs. [6–8]. However, these cooperative spectrum sensing algorithms are not optimal. In this paper, we propose the optimal energy sensing algorithm and prove it with Neyman-Pearson duality theorem. Then, we expand this optimal algorithm to cooperative energy spectrum sensing. Analytical results show that these liner cooperative spectrum sensing algorithms in Refs. [6–8] can be simply derived from the optimal cooperative energy spectrum sensing algorithm under some given condition.

The rest of this paper is organized as follows. In Sect. 2, the system model is briefly introduced, and the optimal energy sensing algorithm with Neyman-Pearson duality theorem is proved. In Sect. 3, we propose the optimal

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Haijun WANG (✉), Xin SU, Yi XU, Shidong ZHOU, Jing WANG
Department of Electronic Engineering, Tsinghua University, Beijing 100084, China
E-mail: wanghj02@mails.tsinghua.edu.cn

cooperative energy spectrum sensing algorithm based on channel covariance matrix and compare it with several existing cooperative energy spectrum sensing algorithms. Simulation results and analysis are given in Sect. 4. Conclusions are drawn in Sect. 5.

2 System model and optimal energy spectrum sensing algorithm

2.1 System model and notation

A cognitive sensing node is supposed to detect M consecutive sampling points in the licensed user's band each time:

$$y[i] = \begin{cases} n[i], & H_0, \\ hx[i] + n[i], & H_1, \end{cases} \tag{1}$$

where $n[i]$ is the noise of the ith sampling point (here, it is assumed that the noise is independent and identically distributed Gaussian white noise and $n[i] \sim N(0, \sigma^2)$); $x[i]$ is the licensed user's signal at the ith sampling point; $y[i]$ is the signal detected by the cognitive sensing node; and h denotes the complex channel gain. As energy sensing requires very short detection period, h is supposed to remain unchanged during each detection period. Binary hypothesis is adopted here: H_0 means that there is no licensed user's signal, i.e., the band is idle; while H_1 indicates that the band is currently occupied by the licensed user. The key problem is to decide which hypothesis is true for the sequence $Y = [y[1]\ y[2]\ \cdots\ y[M]]$.

There are two important parameters in this binary hypothesis-testing problem: probability of false alarm P_f and the probability of detection P_d.

P_f: the probability of decision result is H_1 when H_0 is true.

P_d: the probability of decision result is H_1 when H_1 is true.

2.2 Optimal energy spectrum sensing algorithm in CR

The binary hypothesis-testing problem has been widely discussed in signal detection. Especially in radar signal detection, the studies often try to construct the decision algorithm to achieve maximum P_d while keeping the P_f below a certain threshold α. Moreover, the Neyman-Pearson lemma [9] has proved that the likelihood-ratio test

$$L(Y) = \frac{p(Y|H_1)}{p(Y|H_0)} > \eta \tag{2}$$

is the optimal test. Here, η must satisfy

$$P_\mathrm{f} = \int_{\{Y:L(Y)>\eta\}} p(Y|H_0)\mathrm{d}Y = \alpha.$$

In CR, in order to guarantee the licensed user's use of specific spectrum, the collision probability $1-P_\mathrm{d}$ should never exceed a certain threshold α, e.g., 1%. This means that in CR, the target of decision algorithm has been changed to achieve the minimum P_f (which means the highest utilization of the free spectrum) while ensuring the P_d above a certain threshold $1-\alpha$. This problem can be also solved by the likelihood-ratio test.

Lemma 1 (duality theorem of the Neyman-Pearson theorem) The likelihood-ratio test defined by Eq. (2) is also the optimal test to achieve minimum P_f while ensuring the P_d above a certain threshold η.

Proof Similar with the proof of Neyman-Pearson theorem [9], we use the Lagrange multiplier,

$$\begin{aligned} F &= P_\mathrm{f} + \lambda(P_\mathrm{d} - \eta) \\ &= \int_R p(Y|H_0)\mathrm{d}Y + \lambda\left(\int_R p(Y|H_1)\mathrm{d}Y - \eta\right) \\ &= \int_R (p(Y|H_0) + \lambda p(Y|H_1))\mathrm{d}Y - \lambda\eta, \end{aligned} \tag{3}$$

where R is the aggregation of Y that makes the decision of H_1. The minimum F can be obtained if and only if all the Ys that make $p(Y|H_0) + \lambda p(Y|H_1) < 0$ belong to R and all the Ys that make $p(Y|H_0) + \lambda p(Y|H_1) > 0$ do not belong to R. Here, $\lambda < 0$. In fact, if $\lambda > 0, p(Y|H_0) + \lambda p(Y|H_1) > 0$, and the decision result will be always H_0, which means $P_\mathrm{d} = 0$. Therefore, when

$$L(Y) = \frac{p(Y|H_1)}{p(Y|H_0)} > -\frac{1}{\lambda}, \tag{4}$$

we can decide that the H_1 is true. Moreover, denoting $-1/\lambda = \eta$, the likelihood-ratio test Eq. (2) can be obtained. Assuming that the mean and covariance matrix of sequence $X = [x[1]\ x[2]\ \cdots\ x[M]]$ are 0 and Cs, and $X \sim N(0, Cs)$. The traditional NP detection algorithm is the optimal energy spectrum sensing algorithm in CR:

$$T(Y) = YCs\left(|h|^2 Cs + \sigma^2 I\right)^{-1} Y^\mathrm{H} > \eta, \tag{5}$$

where

$$Y \sim \begin{cases} N\left(0, \sigma^2 I\right), & H_0, \\ N\left(0, Cs + \sigma^2 I\right), & H_1. \end{cases}$$

However, in practical systems, it is hard to obtain the covariance matrix Cs without transcendental knowledge of the licensed user's signals. Therefore, each $x[i]$ is usually assumed to be independent and identically distributed (i.d.d.). Gaussian random process with mean 0 and variance $\mathrm{Var}(x[i]) = p$, then Eq. (5) can be simply rewritten as

$$T(Y) = \sum_{i=1}^{M} |y[i]|^2 > \eta'. \tag{6}$$

Equation (6) is the common energy spectrum sensing algorithm in CR. It is worthy to note that in the remaining parts of this paper, we also take the assumption that the primary signal $x[i]$ is an i.d.d. Gaussian random process.

3 Optimal cooperative energy spectrum sensing algorithm in CR network

In cooperative sensing, the sensing node can send the individual judgments or detail information (e.g., likelihood ratio or received signals) to the sensing station, and then, the sensing station makes an overall judgment based on these data. Here, we first assume that all the sensing nodes send received signal sequence Y_k to the sensing station, and there are M cooperative users; then, the kth user's signal can be rewritten as

$$y_k[i] = \begin{cases} n[i], & H_0, \\ h_k x[i] + n[i], & H_1. \end{cases} \tag{7}$$

We can rewrite all the signal sequence Y_k as a horizontal vector

$$\begin{aligned} Y &= [\, Y_1 \quad Y_2 \quad \cdots \quad Y_M \,] \\ &= [\, y_1[1] \quad y_1[2] \quad \cdots \quad y_1[N] \quad y_2[1] \quad y_2[2] \quad \cdots \quad y_2[N] \quad \cdots \quad y_M[1] \quad y_M[2] \quad \cdots \quad y_M[N] \,], \end{aligned}$$

and denote covariance matrix of sequence $[\, h_1X \quad h_2X \quad \cdots \quad h_MX \,]$ as

$$\begin{aligned} C &= E\big([\, h_1X \quad h_2X \quad \cdots \quad h_MX \,]^{\mathrm{H}} [\, h_1X \quad h_2X \quad \cdots \quad h_MX \,]\big) \\ &= \begin{bmatrix} E\big((h_1X)^{\mathrm{H}} h_1X\big) & E\big((h_1X)^{\mathrm{H}} h_2X\big) & \cdots & E\big((h_1X)^{\mathrm{H}} h_MX\big) \\ E\big((h_2X)^{\mathrm{H}} h_1X\big) & E\big((h_2X)^{\mathrm{H}} h_2X\big) & \cdots & E\big((h_2X)^{\mathrm{H}} h_MX\big) \\ \vdots & \vdots & \ddots & \vdots \\ E\big((h_MX)^{\mathrm{H}} h_1X\big) & E\big((h_MX)^{\mathrm{H}} h_2X\big) & \cdots & E\big((h_MX)^{\mathrm{H}} h_MX\big) \end{bmatrix}. \end{aligned} \tag{8}$$

Since the primary signal $X = [\, x[1] \quad x[2] \quad \cdots \quad x[N] \,]$ is i.d.d. Gaussian random process and $E(x[j]) = 0$, $\mathrm{Var}(x[j]) = P$, the covariance matrix C can be simply rewritten as

$$C = \begin{bmatrix} (P|h_1|^2)I & (Ph_1^{\mathrm{H}} h_2)I & \cdots & (Ph_1^{\mathrm{H}} h_M)I \\ (Ph_2^{\mathrm{H}} h_1)I & (P|h_2|^2)I & \cdots & (Ph_2^{\mathrm{H}} h_M)I \\ \vdots & \vdots & \ddots & \vdots \\ (Ph_M^{\mathrm{H}} h_1)I & (Ph_M^{\mathrm{H}} h_2)I & \cdots & (P|h_M|^2)I \end{bmatrix}, \tag{9}$$

where I is an $N\times N$ unit matrix.

Equation (5) offers the optimal spectrum sensing algorithm for sequence Y:

$$T(Y) = YC(C + \sigma^2 I)^{-1} Y^{\mathrm{H}} > \eta. \tag{10}$$

Equation (10) is the optimal cooperative energy spectrum sensing algorithm based on channel covariance matrix in CR network. In this algorithm, the sensing station needs to know all the channel gain between the primary user and sensing nodes, and all the signals received by each sensing node will be sent to the sensing station, which requires satisfactory communication links between sensing nodes and the sensing station.

In practical systems, the channel gain h_i may be difficult to obtain, and the communication links between sensing nodes and the sensing station are restricted, i.e., these wireless communication links only offer 2 or 3 bit for transmitting detail information. Therefore, we derive the suboptimal cooperative energy spectrum sensing algorithm from Eq. (10). Now, suppose that $E(h_i x[j]) = 0$ for ith sensing node, and the received signal power is $\mathrm{Var}(h_i x[j]) = P_i$. Since it is impossible to know the elements $C_{i,j} = Ph_i^{\mathrm{H}} h_j$, $i \neq j$, of the covariance matrix C, we simply rewrite Eq. (9) as

$$C' = \begin{bmatrix} P_1 I & 0 & \cdots & 0 \\ 0 & P_2 I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & P_M I \end{bmatrix}. \tag{11}$$

Now, the suboptimal cooperative energy spectrum sensing algorithm derived from Eq. (10) is as follows:

$$\begin{aligned} &T(Y) \\ &= YC'(C' + \sigma^2 I)^{-1} Y^{\mathrm{H}} \\ &= Y \begin{bmatrix} P_1 I & 0 & \cdots & 0 \\ 0 & P_2 I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & P_M I \end{bmatrix} \end{aligned}$$

$$\times \begin{bmatrix} (P_1+\sigma^2)I & 0 & \cdots & 0 \\ 0 & (P_2+\sigma^2)I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & (P_M+\sigma^2)I \end{bmatrix}^{-1} Y^{\mathrm{H}}$$
$$= \sum_{i=1}^{M} \frac{P_i}{P_i+\sigma^2} Y_i Y_i^{\mathrm{H}}$$
$$= \sum_{i=1}^{M} \frac{P_i}{P_i+\sigma^2} T_i, \tag{12}$$

where $T_i = Y_i Y_i^{\mathrm{H}}$ is the energy of received signal sequence in the ith sensing node.

Actually, some traditional cooperative sensing algorithms can be derived from Eq. (12). Therefore, some expanded discussion about Eq. (12) is made as follows.

When each P_i is equal, Eq. (12) can be rewritten as

$$T(Y) = \sum_{i=1}^{M} T_i. \tag{13}$$

Equation (13) has been proposed in Ref. [8] as the average weight liner cooperative spectrum sensing algorithm.

Usually, in cognitive radio systems, the primary signal power received by cognitive sensing nodes is quite low [2], which means $P_i \ll \sigma^2$. Under this assumption, $P_i/(P_i+\sigma^2)$ can be approximated as P_i/σ^2. Denoting

$$\omega_i = \frac{\frac{P_i}{\sigma^2}}{\sqrt{\left(\frac{P_1}{\sigma^2}\right)^2 + \left(\frac{P_2}{\sigma^2}\right)^2 + \cdots + \left(\frac{P_M}{\sigma^2}\right)^2}},$$

$$T(Y)' = \frac{T(Y)}{\sqrt{\left(\frac{P_1}{\sigma^2}\right)^2 + \left(\frac{P_2}{\sigma^2}\right)^2 + \cdots + \left(\frac{P_M}{\sigma^2}\right)^2}},$$

Eq. (12) can be rewritten as

$$T(Y)' = \sum_{i=1}^{M} \omega_i T_i. \tag{14}$$

Equation (14) is another liner cooperative spectrum sensing algorithm proposed in Ref. [7], which is called soft combination cooperative spectrum sensing algorithm.

Table 1 compares the complexity and the required transmission bits of several cooperative energy spectrum sensing algorithms in CR network.

Here, M is the cooperative sensing node number, N is the signal length (sampling point's number), and K is the quantification bit number for each transmission data. The complexity issue focuses on the multiplication of the optimal cooperative algorithm Eq. (10). The complexity of matrix inversion $(C+\sigma^2 I)^{-1}$ is $2M^2$, and it can be proved that the matrix $(C+\sigma^2 I)^{-1}$ has the similar structure with matrix C. Therefore, the multiplication number of C multiplied by $(C+\sigma^2 I)^{-1}$ is M^3. Since we suppose that the sensing channel is time-invariant, the required transmission bits in optimal cooperative algorithm are mainly used to transmit received signal sequence. In soft combination cooperative algorithm Eq. (14), since the sensing station needs to know the SNR of each sensing node, it requires more transmission bits than suboptimal and average weight liner cooperative energy spectrum sensing algorithm. It is worth noting that the optimal and suboptimal cooperative sensing algorithms also belong to the soft cooperative sensing algorithm.

Table 1 Complexity and required transmission bits of each cooperative algorithm

algorithms	complexity	required transmission bits
optimal cooperative	$2M^2+M^3+2MN$	MNK
suboptimal cooperative	$MN+2M$	MK
average weight liner cooperative	MN	MK
soft combination cooperative	$MN+2M$	$2MK$~MK
AND rule	MN	M
half-voting rule	MN	M

4 Simulation results and analysis

To evaluate the performance of the optimal and suboptimal cooperative spectrum sensing algorithms in Sect. 3, a couple of numerical simulations are carried out, and the results are shown in Figs. 1–3. The signal length (sampling points number) N is 32, and noise variance σ^2 is 1. There are 10 sensing nodes, and the channel conditions between the primary use and the sensing nodes are given in Table 2.

Figure 1 depicts the relationship between the P_{d} and P_{f} of different sensing algorithms. In this simulation, the SNR of the primary user's signal is -6 dB, and the sensing node i ($i=1,2,\ldots,8$) takes part in cooperation. Figure 1 shows that the optimal cooperative sensing algorithm based on channel covariance matrix provides the best performance, and in fact, this curve can be regarded as the upper bound for the performance of cooperative sensing. Also, Fig. 1 illustrates that soft cooperative sensing algorithms, which transmits more information to sensing station (such as suboptimal cooperative, soft combination cooperative, and average weight liner cooperative) can achieve significant performance gain.

Figure 2 illustrates the decrease in P_{f} as the SNR of

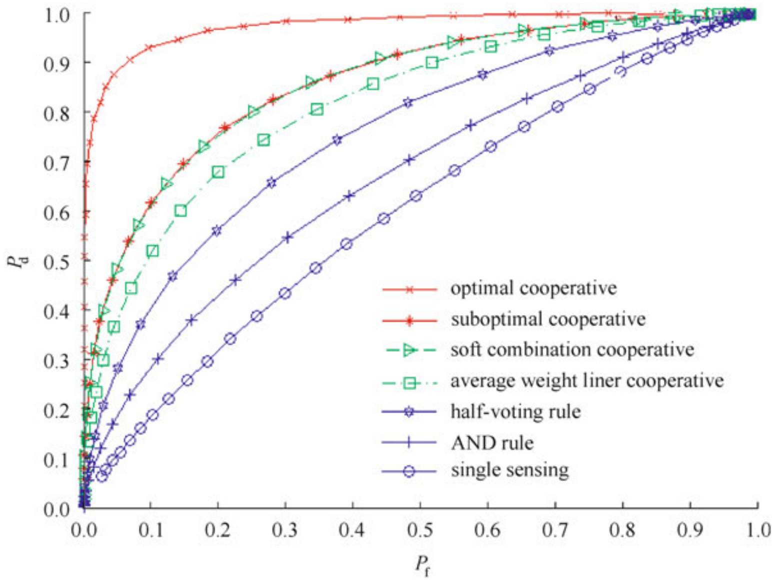

Fig. 1 P_d versus P_f with different sensing algorithms

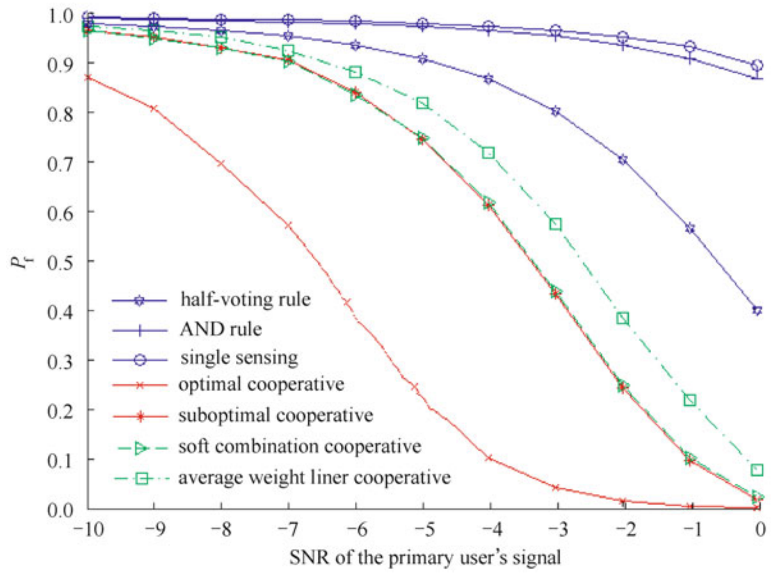

Fig. 2 P_f versus SNR with different sensing algorithms

the primary user's signal increases. Here, P_d is always above the threshold 99%, and also, the sensing node i ($i=1,2,\ldots,8$) participates in the cooperation. Apparently, when the SNR of the primary user's signal is low, P_f of optimal cooperative sensing algorithm declines quickly, but the other curves change tardily. As the SNR of the primary user's signal mounts up, e.g., the SNR is higher than −7 dB, the performance of other soft cooperative sensing algorithms obtain remarkable improvements. Especially, when the SNR is quite high, e.g., the SNR is higher than 0 dB, the hard cooperative sensing algorithms, e.g., half-voting rule, may be satisfactory since they have low P_f and require less transmission bits.

Figure 3 portrays the relationship between P_f and the number of cooperative sensing nodes. In this simulation, the P_d is also above the threshold 99%, and the SNR of the primary user's signal is −5 dB. Here, the sensing node participates in the cooperation sequentially with its index in Table 2. In Fig. 3, most P_fs decrease as the number of cooperative sensing nodes increase, which demonstrate that the cooperative sensing can improve the sensing performance. Moreover, Fig. 3 illustrates that the optimal

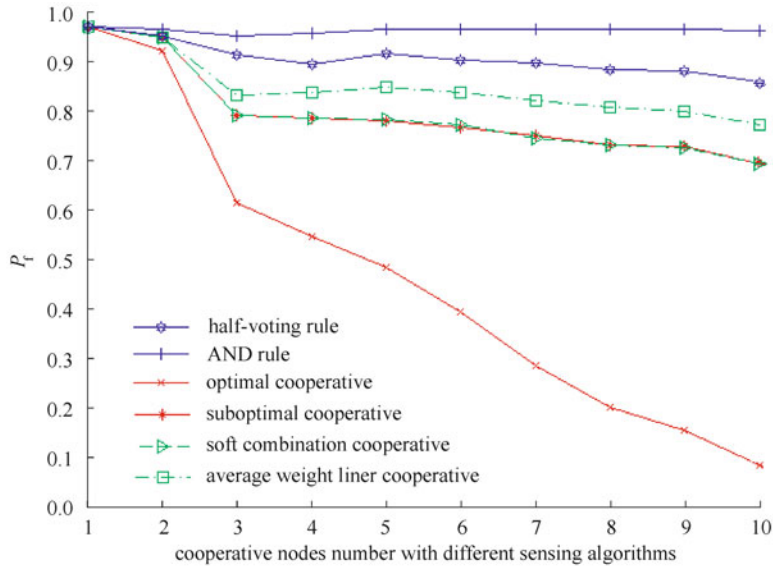

Fig. 3 P_{f} versus cooperative nodes number with different sensing algorithms

Table 2 Channel state information between the primary use and sensing nodes

node number	channel state h	$\lvert h\rvert^2$
1	$-0.0299+0.4991\mathrm{i}$	0.25
2	$-0.3351-0.4978\mathrm{i}$	0.36
3	$0.8544+0.5196\mathrm{i}$	1.00
4	$-0.4224+0.1399\mathrm{i}$	0.20
5	$0.3066-0.1349\mathrm{i}$	0.11
6	$-0.2166-0.4507\mathrm{i}$	0.25
7	$0.0276-0.5694\mathrm{i}$	0.32
8	$-0.3740-0.3008\mathrm{i}$	0.23
9	$0.2371+0.4391\mathrm{i}$	0.25
10	$0.2068-0.5633\mathrm{i}$	0.36

cooperative sensing algorithm present the best performance since it makes good use of the correlation of different channels, and the soft cooperative sensing algorithms is better than hard cooperative sensing algorithms. Furthermore, Fig. 3 indicates that different sensing nodes with different channel gain may bring distinct effect on cooperative sensing performance. On one hand, in Table 2, we can find that sensing node 3 has the best channel condition, which is also obviously represented in Fig. 3: the P_{f} decreases markedly when the sensing node 3 attends the cooperation. On the other hand, the P_{f} contrarily increases when sensing node 5, which has the worst channel state, takes part in the average weight liner, half-voting, or AND rule cooperative sensing. Besides, considering the complexity and required transmission bits for cooperative sensing algorithms, it is reasonable to adopt these sensing nodes with better channel condition into the cooperative sensing.

5 Conclusions

In this paper, we studied the optimal energy sensing algorithm in cognitive radio and prove its optimality through the duality theorem of Neyman-Pearson theorem. Then, we proposed the optimal cooperative sensing algorithm based on channel covariance matrix and simplified it to the suboptimal cooperative sensing algorithm when it is hard to obtain the channel state. Then, the comparison between the proposed cooperative sensing algorithms and the traditional cooperative sensing algorithms in the aspects of complexity and required transmission bits was made. A couple of numerical simulations were carried out to evaluate the performance of different cooperative sensing algorithms. Simulation results demonstrated that the optimal cooperative sensing algorithm provided the best performance with any given cooperative nodes number and the primary user's signal

SNR. Furthermore, the sensing nodes with better channel states should be encouraged to participate in the cooperation from the perspective of the CR network.

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