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RESEARCH ARTICLE

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Location-aware downlink and uplink power control and throughput optimization in cellular cognitive radio networks

Abstract In this paper, we consider throughput maximization in cognitive radio systems with proper power control. In particular, we incorporate location-awareness into the power control design and maximize the average throughput of the cognitive system. As we shall show, the proposed approach effectively utilizes the "spatial opportunity" to maximize the system throughput, which clearly outperforms traditional power control methods. Further, the proposed approach still exhibits significant throughput gain even considering imperfect position information, with appropriate robust design modifications.

Keywords cognitive radios, spectrum hole, power control, localization, optimization

1 Introduction

In cognitive radio [1] networks, secondary users (SUs) are considered to be intelligently agile to perceive the spectrum usage of the licensed users and utilize the unoccupied frequency resource opportunistically to accomplish information conveying [2]. To ensure continuous protection to primary user (PU) system, periodic spectrum sensing has been considered as an effective method tracing and understanding the PU system's activities. By such sensing procedure, temporal spectrum holes can be recognized for opportunistic spectrum access [3,4].

More recently, some works have proposed several ways that exploit the sensing results of large numbers of distributed SUs together with their location information to estimate the coordinates of PU transmitters (PU-Tx), which are referred to as PU localization algorithms [5]. For instance, Ref. [6] proposed a high order geometric range-free algorithm, while Ref. [7] resorted to a range-based algorithm assuming precise knowledge of the received signal strength (RSS) information from PUs. Recently, in our work [8], we proposed a semi-range based iterative localization algorithm that improves the accuracy of pure range-free algorithms and relaxes the stringent requirement on the precision of physical layer measurements in conventional range-based algorithms.

The secondary system can take advantage of PU location information to avoid harmful interference to the primary communications. For instance, a geographical location database technique is proposed in Ref. [9] to avoid interference with Television (TV) band PUs. More importantly, we have found that the knowledge of PU locations can further facilitate the design of link layer control schemes, such as power optimization and channel selection in the secondary access, which in turn, boosts the transmission efficiency of the secondary system.

Intuitively, the benefits of knowing PU locations are two-fold [10]. On one hand, when some active PUs are present in the detection range of SUs, with PU location information, SU transmitters (SU-Tx) may still manage to transmit at a power level that is controlled low enough to avoid harmful interference to even the nearest PU, which is otherwise impossible. On the other hand, if all the PUs are outside the detection range of SUs, instead of assuming the primary users are in the closest possible positions and applying conservative worst-case power control, with PU location information, SU-Tx may act more aggressively by employing larger transmit power. In short, the knowledge of PU locations creates spatial opportunities for secondary spectrum access, which can indeed be significantly useful in enhancing the secondary system's throughput by location-aware power control.

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In this work, we consider a multiuser cellular cognitive radio networks sharing multiple licensed channels with a set of PUs, whose locations are available at the secondary system via PU localization algorithm we proposed in Ref. [8]. For a secondary system that can only transmit on one channel at any given time, the knowledge of PU locations simplifies the channel selection procedure such that always selecting the channel where PU receiver (PU-Rx) is farthest from SU-Tx serves as a rational choice. Therefore, we focus on the location-aware transmit power optimization to maximize the average system throughput for both downlink and uplink scenarios. In addition, we also take localization errors into consideration to make the power control design robust.

The rest of the paper is organized as follows. Section 2 describes the system model and formulates the transmit power optimization problem. The solutions to the downlink and uplink scenarios are given in Sects. 3 and 4. Section 5 shows some numerical results that confirm the benefit of PU location information and the robustness of our design. Section 6 concludes the whole paper.

2 System model and problem formulation

We consider a cellular cognitive radio system with N SUs and one centralized controller, i.e., cognitive base station (BS). N SUs are assumed to be moving around in a disk with radius r_{S}, while the cognitive BS is assumed fixed at the center of the disk. The cognitive system opportunistically shares the licensed spectrum with M randomly scattered PUs. Without loss of generality, we assume that M PUs are uniformly distributed on the disk centered by the cognitive BS but with a larger radius r_{P}. The locations of all PUs are assumed to be known (estimated) to the cognitive system via some localization algorithms [8]. Moreover, each PU is assumed to transmit on one licensed band while receiving at another licensed band. The cellular cognitive system works in time division multiple access (TDMA) fashion and the cognitive BS schedules N SUs in a round-robin (RR) manner: In each time slot, one of the SUs is scheduled to receive (downlink) or transmit (uplink) according to a predetermined order. Driven by practical concerns, we assume that the secondary system has limited adaptation and reconfiguration capability, in particular, both the BS (downlink) and SUs (uplink) can only transmit on one frequency band in any given slot, and the transmit power values are not adjustable but can be designed (optimized) in advance. Only macroscopic fading (path-loss) is considered, i.e., the average channel power gain between node i and j is represented by

$$G_{ij} = \kappa_0 d_{ij}^{-\alpha},$$

where κ_0 and α are path-loss coefficient and exponent, respectively, while d_{ij} is the distance between the two nodes. By taking path-loss into account, the proposed method may maximize the spatial spectrum reuse while avoiding the harmful interference to PUs in an average sense. In practice, multipath fading can also affect the instantaneous channel gain. However, this is out of the scope of this paper.

The cognitive BS shall decide for each SU which channel (band) to access and how much power to use on that channel, for both downlink and uplink scenarios, respectively. Based on PU location information, channel selection becomes extremely simple: selecting the channel on which PU-Rx is farthest from SU-Tx. Consequently, the resource allocation task left for the cognitive BS is to set the optimal transmit power. In this paper, we consider the objective of the transmit power optimization as to maximize the system's total average (over time) throughput of all SUs, i.e.,

$$\Lambda_1 : \quad \max_{0 \leqslant P_t^s \leqslant P_{\max}} \frac{1}{T} \lim_{T\to\infty} \sum_{t=1}^{T}\sum_{i=1}^{N} R_i(P_t^s, t), \tag{1}$$

where P_t^s is the transmit power to be optimized (superscript "$s = \mathrm{d}$" for downlink and "$s = \mathrm{u}$" for uplink), and $R_i(P_t^s, t)$ is the achievable rate for the ith SU in the tth scheduling round given transmit power P_t^s. Since each SU is randomly and independently located in each time slot it is scheduled, we may assume uniform possibility for an SU to be at any coordinate on the disk of radius r_{S}. As a result, each SU contributes equally to the total system throughput in a long run, and the optimization objective can thus be replaced by the following:

$$\Lambda_2 : \quad \max_{0 \leqslant P_t^s \leqslant P_{\max}} \mathbb{E}_{\Phi_{\mathrm{P}},\Phi_{\mathrm{S}}}[R_i(P_t^s, \phi_{\mathrm{P}}, \phi_{\mathrm{S}})], \tag{2}$$

where Φ_{S} and Φ_{P} denote the sets of SU's and PU's all possible coordinates, while ϕ_{S} and ϕ_{P} stand for topology realizations that belongs to Φ_{S} and Φ_{P} , respectively[1)]. To better understand the objective Λ_2, we further introduce the following definitions.

Definition 1 (Interference range of the secondary system) Given the transmit power P_t^s, the corresponding interference region is approximated by a disk centered by the transmitter with radius r such that the average interference received by the nodes outside the disk is less than or equal to a threshold I_0, i.e.,

$$\kappa_0 P_t^s \tilde{r}^{-\alpha} \leqslant I_0, \quad \forall \tilde{r} \geqslant r,$$

which implies that r is a function of P_t^s, i.e.,

$$r(P_t^s) = \left(\frac{\kappa_0 P_t^s}{I_0}\right)^{1/\alpha}. \tag{3}$$

1) Without introducing ambiguity, we may omit the subscripts of user index i and expectation space Φ_{S} and Φ_{P} for convenience in the rest of this paper.

Definition 2 (Transmission outage of the secondary system) A transmission outage is defined as the event that all M PU-Rx are inside the interference range r of SU-Tx for a given transmit power P_t^s, denoted by $\varepsilon_{\text{out}}(r)$, which refers to the situation where SU-Tx cannot find any idle channel to access. Denoted by $p_{\text{out}}(r)$ (or $p_{\text{out}}(P_t^s)$), the probability of transmission outage given interference range of SU-Tx as r is

$$p_{\text{out}}(r) = \Pr[\varepsilon_{\text{out}}(r)].$$

Definition 3 (Achievable rate of the secondary system) Given the transmit power of SU-Tx and PU-Tx as P_t^s and P_{P}, respectively, the instantaneous achievable rate of SU transmission can be expressed as the product of two parts, namely,

$$R(P_t^s,\phi_{\text{P}},\phi_{\text{S}}) = \log_2\left(1+\frac{P_t^s G_{\text{S}}(\phi_{\text{S}})}{\sigma^2 + P_{\text{P}} G_{\text{P}}(\phi_{\text{P}},\phi_{\text{S}})}\right)\left(1-1_{\{\varepsilon_{\text{out}}(P_t^s,\phi_{\text{P}},\phi_{\text{S}})\}}\right). \tag{4}$$

The first part is Shannon formula, while the second part indicates the feasibility of transmission. $G_{\text{S}}(\phi_{\text{S}})$ and $G_{\text{P}}(\phi_{\text{P}},\phi_{\text{S}})$ are channel power gain given topology realization ϕ_{S} and ϕ_{P}, σ^2 is the noise power at SU-Rx which is normalized to 1 for simplicity, and $1_{\{\mathcal{A}\}}$ is an indicator function that equals 1 when event $\mathcal{A}$ is true and 0 otherwise. Taking expectation over all possible topology of PUs and SUs, we obtain the average achievable rate as

$$\overline{R}(P_t^s) = \mathbb{E}_{\Phi_{\text{P}},\Phi_{\text{S}}}[R(P_t^s,\phi_{\text{P}},\phi_{\text{S}})].$$

We shall point out that finding the optimal transmit power to satisfy Λ_2 is not trivial. On one hand, increasing the transmit power P_t^s enlarges the interference range r, which in turn increases the potential probability of outage $p_{\text{out}}(r)$. On the other hand, the achievable information rate of SU is monotonic increasing function of P_t^s. Therefore, there is an interesting tradeoff between smaller interference range and higher possible information rate. We shall find the optimal solutions to the downlink and uplink scenarios, respectively, in the following two sections. In addition, to take into consideration the estimation errors in PU localization results, we further define the concept of localization error radius as follows, which will be used in later calculation.

Definition 4 (Localization error radius) The localization error radius e is defined as the radius of a small circle such that PU under estimation in the localization algorithm lies almost surely inside the circle.

As have been proved in Ref. [8], under our PU localization algorithm, the probability that the actual position of PU is in the circle centered by the estimated coordinates with radius $3\sigma_{\text{e}}$ is about 98.89%, where σ_{e} is the mean square error of localization, so the localization error radius is set as $e = 3\sigma_{\text{e}}$ in the rest of the paper.

3 Downlink transmit power optimization

Take Fig. 1 as an example (BS denotes the secondary base station that will transmit data to an SU station; r denotes the interference radius of BS), in the downlink transmission, the cognitive BS transmits to SU_n, while PU_1 is the farthest PU-Rx away from BS. For the current time slot, PU_1's receiving channel is selected for secondary access, so the interference from PU_4, PU-Tx, must be counted in the calculation of the achievable rate R. The objective of downlink transmit power optimization is to find the optimal P_t^{d} that satisfy Λ_2. It is not hard to observe from Eq. (3) that the interference range r has a one-one correspondence with P_t^{d}. Therefore, instead of directly optimizing P_t^{d}, we focus on finding the optimal interference range r^* in what follows. Taking the transmit power P_t^0 that corresponds to interference range equaling r_{S} as a reference, the optimal P_t^{d} can be derived from the equation below:

$$\frac{P_t^{\text{d}}}{P_t^0} = \left(\frac{r^*}{r_{\text{S}}}\right)^{\alpha}, \tag{5}$$

where

$$P_t^0 = I_0 \kappa_0^{-1} r_{\text{S}}^{\alpha}.$$

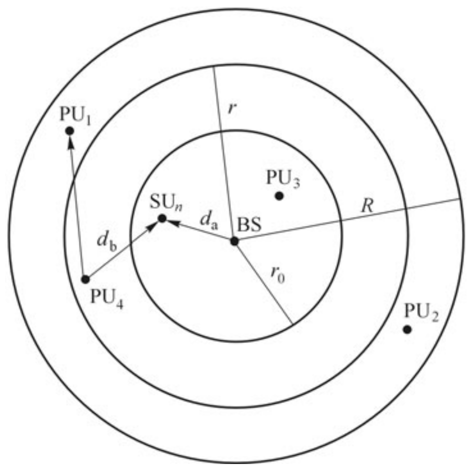

Fig. 1 Downlink system structure

In the following, we shall first introduce some random variables as parameters to describe topology realizations and then calculate the average achievable rate of SU_n according to Definition 3. Denote by d_{a} the distance between the SU-Tx (the cognitive BS in this case) and SU_n (represents SU-Rx), d_{b} the distance between the PU-Tx (on the selected channel) and SU-Rx, and d_{P} the distance between PU-Rx (on the selected channel) and SU-Tx. Note that each triple of (d_{a}, d_{b}, d_{P}) represents one realization of the network topology, i.e., a pair of ϕ_{P} and ϕ_{S}. Let $f_{d_{\text{a}},d_{\text{b}},d_{\text{P}}}^{\text{d}}(\rho_{\text{a}},\rho_{\text{b}},\rho_{\text{P}})$ denote the joint probability density function (PDF) of d_a and d_b and d_P in downlink case. Due to the independence between d_P and d_a, d_b, we have

$$f^{d}_{d_a,d_b,d_P}(\rho_a,\rho_b,\rho_P) = f^{d}_{d_b|d_a}(\rho_b|\rho_a) f^{d}_{d_a}(\rho_a) f^{d}_{d_P}(\rho_P),$$

where $f^{d}_{d_a}(\rho_a)$ and $f^{d}_{d_P}(\rho_P)$ are the marginal PDF of d_a and d_P, respectively. According to the uniform distribution assumption of both SUs and PUs (but with different disk radius), it is easy to write

$$f^{d}_{d_a}(\rho_a) = \frac{2\rho_a}{r_S^2}, \tag{6}$$

$$f^{d}_{d_P}(\rho_P) = \frac{2\rho_P}{r_P^2}, \tag{7}$$

while given $d_a = l$, the conditional PDF of d_b is

$$f^{d}_{d_b|d_a}(d_b = \rho_b|d_a = l)$$

$$= \begin{cases} \dfrac{2\rho_b}{r_P^2}, & \rho_b \leqslant r_P - l, \\ \dfrac{1}{\pi}\dfrac{2\rho_b}{r_P^2}\arccos\dfrac{\rho_b^2 - r_P^2}{2\rho_b}l^{-1} + \dfrac{1}{2\rho_b}l, & r_P - l < \rho_b \leqslant r_P + l, \\ 0, & \rho_b > r_P + l. \end{cases} \tag{8}$$

Given a set of d_a, d_b, and d_P, and assuming the interference range is r, according to Eq. (4), then the instantaneous information rate for SU_n is

$$R(r,d_a,d_b,d_P) = \log_2\left[1 + \frac{\left(\dfrac{r}{r_S}\right)^{\alpha} P_t^0 \kappa_0 d_a^{-\alpha}}{\sigma^2 + P_P\kappa_0 d_b^{-\alpha}}\right](1 - 1_{\{d_P<r\}}). \tag{9}$$

Taking expectation over (d_a, d_b, d_P), the average achievable data rate for SU_n as a function of r is given by

$$\overline{R}(r) = \mathbb{E}_{d_a,d_b,d_P}[R(r,d_a,d_b,d_P)]$$

$$= \mathbb{E}_{d_a,d_b}\left\{\log_2\left[1 + \frac{\left(\dfrac{r}{r_S}\right)^{\alpha} P_t^0 \kappa_0 d_a^{-\alpha}}{\sigma^2 + P_P\kappa_0 d_b^{-\alpha}}\right]\right\}$$

$$\times (1 - \mathbb{E}_{d_P}[1_{\{d_P<r\}}])$$

$$= \left\{\int_0^{r_S}\int_0^{d_a+R} \log_2\left[1 + \frac{\left(\dfrac{r}{r_S}\right)^{\alpha} P_t^0 \kappa_0 d_a^{-\alpha}}{\sigma^2 + P_P\kappa_0 d_b^{-\alpha}}\right]\right.$$

$$\left. \times f^{d}_{d_a,d_b}(d_a,d_b)\mathrm{d}d_b\mathrm{d}d_a\right\}[1 - p_{\text{out}}(r)]. \tag{10}$$

Since the selection channel is the one with PU that is farthest from the cognitive BS, the outage probability $p_{\text{out}}(r)$ for the downlink case equals the probability that all M PUs are inside the interference range, i.e.,

$$p_{\text{out}}(r) = \left[\frac{\pi(r+e)^2}{\pi r_P^2}\right]^M = \left(\frac{r+e}{r_P}\right)^{2M}. \tag{11}$$

Substituting Eqs. (6), (8), and (11) into Eq. (10), $\overline{R}(r)$ can finally be expressed as

$$\overline{R}(r) = \left[1 - \left(\frac{r+e}{r_P}\right)^{2M}\right]g(r), \tag{12}$$

where $g(r)$ is given as follows (in which we denote $\gamma_0 = P_P/\sigma^2$):

$$g(r) = \int_0^{r_S}\frac{2d_a}{r_S^2}\left\{\int_0^{r_P-d_a}\log_2\left[1 + \frac{\kappa_0\dfrac{\gamma_0}{d_a^{\alpha}}\left(\dfrac{r}{r_S}\right)^{\alpha}}{1 + k\dfrac{\gamma_0}{d_b^{\alpha}}}\right]\frac{2d_b}{r_P^2}\mathrm{d}d_b\right.$$

$$+\int_{r_P-d_a}^{r_P+d_a}\log_2\left[1 + \frac{\kappa_0\dfrac{\gamma_0}{d_a^{\alpha}}\left(\dfrac{r}{r_S}\right)^{\alpha}}{1 + k\dfrac{\gamma_0}{d_b^{\alpha}}}\right]\frac{1}{\pi}\frac{2d_b}{r_P^2}$$

$$\left.\times\arccos\left(\frac{d_b^2 - r_P^2}{2d_b}d_a^{-1} + \frac{1}{2d_b}d_a\right)\mathrm{d}d_b\right\}\mathrm{d}d_a. \tag{13}$$

As can be seen that Eq. (13) is too complicated for us to find the optimal r. We now make the following approximation based on the assumption that the interference from PU-Tx is much smaller than that from the cognitive BS, i.e., $\kappa\gamma_0 d_a^{-\alpha} \gg 1$ and $\kappa\gamma_0 d_b^{-\alpha} \ll 1$. Accordingly, $g(r)$ can be approximated as

$$g(r) \approx \frac{1}{\ln 2}\left[\frac{\alpha}{2} + \ln(\kappa_0\gamma_0) - 2\alpha\ln r_S + \alpha\ln r\right]. \tag{14}$$

Let r^* be the optimal r, by setting the derivative of $\overline{R}(r)$ to be 0, we have

$$\frac{\mathrm{d}\overline{R}(r^*)}{\mathrm{d}r^*} = \left[1 - \left(\frac{r^*+e}{r_P}\right)^{2M}\right]\frac{g(r^*)}{r^*}$$

$$-2M\frac{(r^*+e)^{2M-1}}{r_P^{2M}}g(r^*) = 0, \tag{15}$$

which implies the following transcendental equation of r^* as

$$r^* = -e + r_{\mathrm{P}}\left\{1 + M\frac{r^*}{r^*+e}\left[1+\frac{2}{\alpha}\ln(\kappa_0\gamma_0)\right.\right.$$
$$\left.\left.-4\ln r_{\mathrm{S}}\right] + 2M\ln r^*\right\}^{-\frac{1}{2M}}. \quad (16)$$

When $M \gg 1$, we can approximate $\ln r^*$ as $-\frac{1}{2M}\ln(r_{\mathrm{P}}+M)$. Substituting such approximation into Eq. (16), we get a better approximation of r^* as

$$r^* \approx -e + \left(1+\frac{e}{M}\right)r_{\mathrm{P}}\left\{1 + M\left[1+\frac{2}{\alpha}\ln(\kappa_0\gamma_0)\right.\right.$$
$$\left.\left.-4\ln r_{\mathrm{S}}\right] - \ln(M+r_{\mathrm{P}})\right\}^{-\frac{1}{2M}}. \quad (17)$$

Finally, according to Eq. (5), the optimal downlink transmit power P_t^{d} is given by

$$P_t^{\mathrm{d}*} \approx P_t^0\left\{-\frac{e}{r_{\mathrm{S}}} + \frac{r_{\mathrm{P}}}{r_{\mathrm{S}}}\left(1+\frac{e}{M}\right)\left[1 + M\left(1+\frac{2}{\alpha}\ln(\kappa_0\gamma_0)\right.\right.\right.$$
$$\left.\left.\left.-4\ln r_{\mathrm{S}}\right) - \ln(M+r_{\mathrm{P}})\right]^{-\frac{1}{2M}}\right\}^{\alpha}. \quad (18)$$

4 Uplink transmit power optimization

The uplink analysis is similar to the downlink case. Take Fig. 2 as an example (SU_n is SU-Tx that will transmit data to the BS; r denotes the interference radius of SU-Tx), for the current time slot, SU_n is the next transmitter of the cognitive system (i.e., SU-Tx) the SU-Tx transmits to BS, while PU_1 is the farthest PU-Rx away from the SU-Tx. PU_1's receiving channel is selected for secondary access, so the interference from PU_4, the PU-Tx, must be counted in the calculation of the achievable rate R. The objective of uplink transmit power optimization is to find the optimal P_t^{u} that satisfy Λ_2. Instead of directly optimizing P_t^{u}, we also focus on finding the optimal interference range r^* just as the downlink case does.

Denote by d_{a} the distance between SU-Tx and the cognitive BS, d_{b} the distance between PU-Tx (on the selected channel) and BS, and d_{P} the distance between PU-Rx (on the selected channel) and SU-Tx. Similar to the downlink case, we have the joint PDF

$$f^{\mathrm{u}}_{d_{\mathrm{a}},d_{\mathrm{b}},d_{\mathrm{P}}}(\rho_{\mathrm{a}},\rho_{\mathrm{b}},\rho_{\mathrm{P}}) = f^{\mathrm{u}}_{d_{\mathrm{a}}}(\rho_{\mathrm{a}})f^{\mathrm{u}}_{d_{\mathrm{b}}}(\rho_{\mathrm{b}})f^{\mathrm{u}}_{d_{\mathrm{P}}|d_{\mathrm{a}}}(\rho_{\mathrm{P}}|\rho_{\mathrm{a}}).$$

We can simply get that

$$f^{\mathrm{u}}_{d_{\mathrm{a}}}(\rho_{\mathrm{a}}) = \frac{2\rho_{\mathrm{a}}}{r_{\mathrm{S}}^2}, \quad (19a)$$

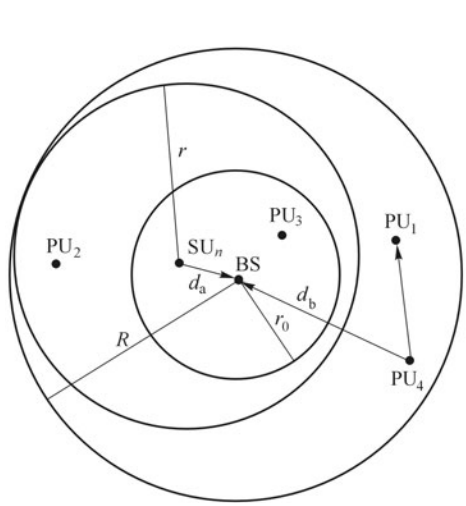

Fig. 2 Uplink system structure

$$f^{\mathrm{u}}_{d_{\mathrm{b}}}(\rho_{\mathrm{b}}) = \frac{2\rho_{\mathrm{b}}}{r_{\mathrm{P}}^2}. \quad (19b)$$

The outage probability of uplink under the condition of given d_{a} is deduced as follows:

$$p_{\mathrm{out}}(r|d_a = l) = \mathbb{E}_{d_{\mathrm{P}}}[1_{\{d_{\mathrm{P}}<r|d_{\mathrm{a}}=l\}}]$$
$$= \left[\frac{S(r|d_a = l)}{\pi R^2}\right]^M, \quad (20)$$

where

$$S(r|d_{\mathrm{a}} = l)$$
$$= \begin{cases} \pi(r+e)^2, & 0 \leqslant l \leqslant r_{\mathrm{P}} - r - e, \\ \pi r_{\mathrm{P}}^2, & 0 \leqslant l \leqslant r + e - r_{\mathrm{P}}, \\ (r+e)^2(\pi - \phi + \sin\phi\cos\phi) + r_{\mathrm{P}}^2(\theta - \sin\theta\cos\theta), & \\ & |r_{\mathrm{P}} - r - e| < l \leqslant r_{\mathrm{S}}, \end{cases} \quad (21)$$

and

$$\theta = \arccos\frac{r_{\mathrm{P}}^2 + l^2 - (r+e)^2}{2lr_{\mathrm{P}}}, \quad (22)$$

$$\phi = \pi - \arccos\frac{l^2 + (r+e)^2 - r_{\mathrm{P}}^2}{2l(r+e)}. \quad (23)$$

According to Eq. (9), the average achievable data rate is given by

$$\overline{R}(r) = \mathbb{E}_{d_{\mathrm{a}},d_{\mathrm{b}},d_{\mathrm{P}}}[R(r,d_{\mathrm{a}},d_{\mathrm{b}},d_{\mathrm{P}})]$$
$$= \int_0^{r_{\mathrm{S}}}\left[\int_{r_{\mathrm{S}}}^{R} R(r,d_{\mathrm{a}},d_{\mathrm{b}},d_{\mathrm{P}})f^{\mathrm{u}}_{d_{\mathrm{b}}}(d_{\mathrm{b}})\mathrm{d}d_{\mathrm{b}}\right]$$

$$\times[1-p_{\mathrm{out}}(r|d_{\mathrm{a}})]f_{d_{\mathrm{a}}}^{\mathrm{u}}(d_{\mathrm{a}})\mathrm{d}d_{\mathrm{a}}$$

$$=\int_0^{r_{\mathrm{S}}}\frac{2d_{\mathrm{a}}}{r_{\mathrm{S}}^2}\log_2\left[\kappa_0\frac{\gamma_0}{d_{\mathrm{a}}^{\alpha}}\left(\frac{r}{r_{\mathrm{S}}}\right)^{\alpha}\right]\mathrm{d}d_{\mathrm{a}}$$

$$-\int_0^{r_{\mathrm{S}}}\frac{2d_{\mathrm{a}}}{r_{\mathrm{S}}^2}\log_2\left[\kappa_0\frac{\gamma_0}{d_{\mathrm{a}}^{\alpha}}\left(\frac{r}{r_{\mathrm{S}}}\right)^{\alpha}\right]\left[\frac{S(r|d_{\mathrm{a}})}{\pi R^2}\right]^M\mathrm{d}d_{\mathrm{a}}$$

$$\triangleq \overline{R}_1(r)-\overline{R}_2(r). \quad (24)$$

We take the similar approximation as Eq. (14), we can have that

$$\overline{R}_1(r)=\frac{1}{\ln 2}\left[\frac{\alpha}{2}+\ln(\kappa_0\gamma_0)-2\alpha\ln r_{\mathrm{S}}+\alpha\ln r\right]. \quad (25)$$

The approximation of $S(r|d_{\mathrm{a}}=l)$ can be obtained geometrically by replacing the circles with radius r_{S} and r_{P} with squares with side length $2r_{\mathrm{S}}$ and $2r_{\mathrm{P}}$, respectively:

$$S(r|d_{\mathrm{a}}=l)\approx\pi(r+e)^2\left(1-\frac{l}{2r_{\mathrm{P}}}\right). \quad (26)$$

Moreover, $\overline{R}_2(r)$ can then be given by

$$\overline{R}_2(r)=\frac{1}{\ln 2}\int_0^{r_{\mathrm{S}}}\frac{2d_{\mathrm{a}}}{r_{\mathrm{S}}^2}[\ln(\kappa_0\gamma_0)-\alpha\ln d_{\mathrm{a}}+\alpha\ln r$$

$$-\alpha\ln r_{\mathrm{S}}]\left(\frac{r+e}{r_{\mathrm{P}}}\right)^{2M}\left(1-\frac{d_{\mathrm{a}}}{2r_{\mathrm{P}}}\right)^M\mathrm{d}d_{\mathrm{a}}. \quad (27)$$

According to

$$\frac{\mathrm{d}\overline{R}(r)}{\mathrm{d}r}=\frac{\mathrm{d}\overline{R}_1(r)}{\mathrm{d}r}-\frac{\mathrm{d}\overline{R}_2(r)}{\mathrm{d}r}=\frac{\alpha}{r\ln 2}-\frac{\mathrm{d}\overline{R}_2(r)}{\mathrm{d}r}=0. \quad (28)$$

We get that

$$\left[2M\frac{r}{r+e}\left(\ln(\kappa_0\gamma_0)+\alpha\ln r-\alpha\ln r_{\mathrm{S}}\right)\right]$$

$$\times\left(\frac{r+e}{r_{\mathrm{P}}}\right)^{2M}\frac{2}{r_{\mathrm{S}}^2}\int_0^{r_{\mathrm{S}}}d_{\mathrm{a}}\left(1-\frac{d_{\mathrm{a}}}{2r_{\mathrm{P}}}\right)^M\mathrm{d}d_{\mathrm{a}}=\alpha. \quad (29)$$

Similar to the approximation of Eq. (17), r^* can be expressed as

$$r^*\approx -e+\left(1+\frac{e}{M}\right)r_{\mathrm{P}}\left\{4M\left[\frac{1}{\alpha}\ln(\kappa_0\gamma_0)-\ln r_{\mathrm{S}}\right]\right.$$

$$-2\ln(1+M)-\frac{4}{M+2}\left(\frac{r_{\mathrm{P}}}{r_{\mathrm{S}}}\right)^2\left[\left(1-\frac{r_{\mathrm{S}}}{2r_{\mathrm{P}}}\right)^{M+2}-1\right]$$

$$\left.-\frac{4}{M+1}\left(\frac{r_{\mathrm{P}}}{r_{\mathrm{S}}}\right)^2\left[\left(1-\frac{r_{\mathrm{S}}}{2r_{\mathrm{P}}}\right)^{M+1}-1\right]\right\}^{-\frac{1}{2M}}. \quad (30)$$

Finally, the optimal uplink transmit power of SU-Tx is given by

$$P_t^{\mathrm{u}*}\approx P_{t0}\left\{-\frac{e}{r_{\mathrm{S}}}+\left(1+\frac{e}{M}\right)\frac{r_{\mathrm{P}}}{r_{\mathrm{S}}}\left[4M\left(\frac{1}{\alpha}\ln(\kappa_0\gamma_0)-\ln r_{\mathrm{S}}\right)\right.\right.$$

$$-2\ln(1+M)-\frac{4}{M+2}\left(\frac{r_{\mathrm{P}}}{r_{\mathrm{S}}}\right)^2\left[\left(1-\frac{r_{\mathrm{S}}}{2r_{\mathrm{P}}}\right)^{M+2}-1\right]$$

$$\left.\left.-\frac{4}{M+1}\left(\frac{r_{\mathrm{P}}}{r_{\mathrm{S}}}\right)^2\left[\left(1-\frac{r_{\mathrm{S}}}{2r_{\mathrm{P}}}\right)^{M+1}-1\right]\right]^{-\frac{1}{2M}}\right\}^{\alpha}. \quad (31)$$

5 Simulation results

In this section, we shall verify our analytical results using numerical simulations. Each point on the curves is averaged over many thousands times of randomly generated topology realizations (for both SUs and PUs). SU-Tx (cognitive BS for downlink and each SU for uplink) calculates the interference range according to the optimized transmit power value. At the beginning of each scheduling round, SU-Tx first checks whether the farthest PU-Rx is inside the interference range. If yes, an outage event is detected, and SU-Tx remains silent in this round in order to avoid causing harmful interference to the PUs. Otherwise, it uses the calculated optimal transmit power in Eqs. (18) and (31) for downlink and uplink transmission, respectively. The system parameters are set as follows: $\kappa_0=0.03$, $\alpha=2$, $r_{\mathrm{S}}=0.5$, $r_{\mathrm{P}}=1$, and $\gamma_0=10$ dB. Moreover, we also take the localization error into consideration and will investigate how the average achievable rate varies with the mean square error (MSE) of the estimation error.

Figures 3 and 4 show the average achievable spectrum efficiency in terms of $\overline{R}(r)$ in downlink and uplink cases as function of the radius of the interference range r (which has one-to-one correspondence with the transmit power). In Figs. 3 and 4, the root of localization mean square error (RMSE) denotes the root of normalized localization MSE. Dots denote simulation results, and lines represent analytical results. Since there is a one-to-one correspondence between the interference radius r and the transmit power, for the convenience of illustration, we present results in terms of the normalized optimal interference radius. As explained in Sect. 2, the reason for the non-monotonicity of the curves is due to the tradeoff between smaller outage probability and higher possible information rate. Besides, we also observe, the average achievable rate of cognitive system decreases as RMSE increases. Finally, for the scenarios that the position of PU is not available, the transmit power of SU can only cover a radius of r_{S}. The data rate in this case corresponds to the point $r=r_{\mathrm{S}}$ in the

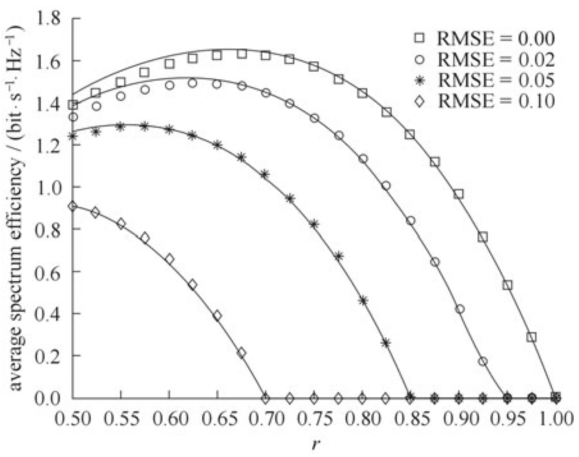

Fig. 3 Average spectrum efficiency versus r in downlink case corresponding to $M = 2$

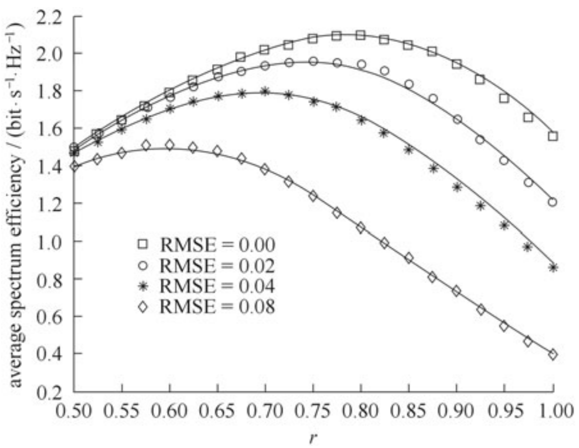

Fig. 4 Average spectrum efficiency versus r in uplink case corresponding to $M = 4$

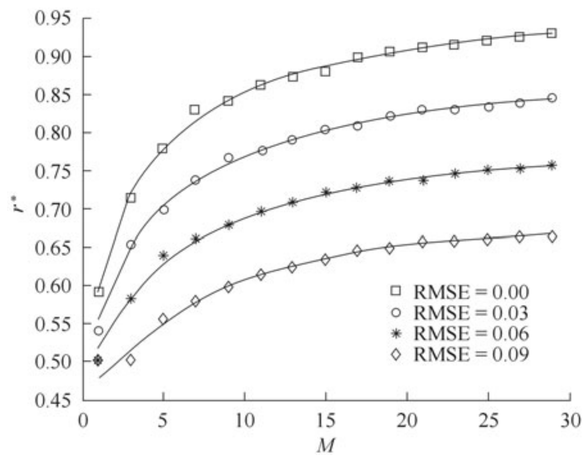

Fig. 5 Maximum interference radius r^* versus number of channels M in downlink case

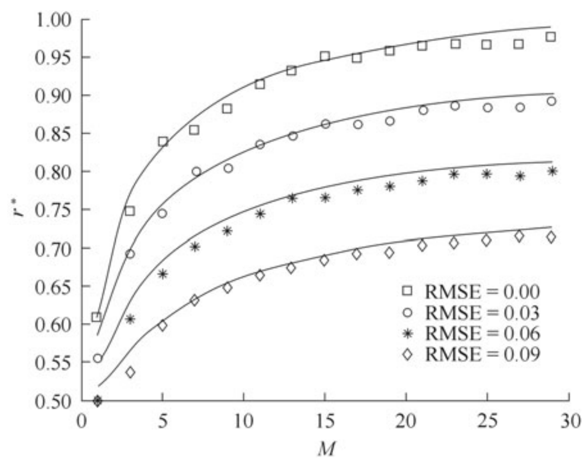

Fig. 6 Maximum interference radius r^* versus number of channels M in uplink case

figures, which is lower than the maximum, especially when the RMSE is small. Therefore, we can see that the utilization of the spatial opportunity brings additional performance enhancement compared with conventional periodic sensing methods.

Figures 5 and 6 depict the optimal radius of the interference range versus the number of PUs (namely, the number of potential channels), i.e., M, for both uplink and downlink case. In Figs. 5 and 6, RMSE denotes the root of normalized localization MSE. Dots denote simulation results, and lines represent analytical results. As can be seen from the figures, simulated points match well with the analytical results obtained via approximation, which verifies the correctness of our approximation. Moreover, when M increases, the secondary system gets more choices, and the outage probability reduces as well, which makes the optimal r^* become larger. In addition, given a fixed M, the optimal r decreases as RMSE of localization increases.

6 Conclusion

A multiuser cellular cognitive radio networks sharing multiple licensed channels with a set of PUs was considered in this paper. A combined PU location aware channel selection and transmit power optimization problem was formulated and solved for both downlink and uplink scenarios, respectively. In addition, localization errors were also taken into consideration to make the power control design robust. Mathematical approximations were further used to help derive the close-form expressions for the optimization results. Numerical simulations confirmed the benefits of exploiting PU location information in the access design for the cognitive system as well as the robustness of our design.

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