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# Near optimal MIMO detection with reduced search space

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**Abstract** A multi-input multi-output (MIMO) detection scheme that requires considerable low complexity but still achieves the near optimal performance is proposed. The fundamental idea of the proposed MIMO detection scheme consists of two points: 1) the computational complexity is restrained by a complexity limit in low signal-to-noise ratio (SNR) region; 2) while in high SNR region, the complexity is significantly reduced by the proposed search space method. Comparing with existing fixed-complexity techniques of MIMO detection (e.g.,  $K$ -best sphere detector and reduced-search maximum-likelihood (RS ML) detection), the significant benefit of proposed detection scheme is that less computational power will be spent for the given data rate, or the throughput of detector can be increased for high SNR cases. According to the simulation results, the near optimal performance can be obtained while the detection complexity is kept considerable small.

**Keywords** multi-input multi-output (MIMO), search space, computational complexity, posterior probability

## 1 Introduction

Multiple-input multiple-output (MIMO) antenna systems have attracted much attention for more than a decade because of their capabilities of achieving high data rate and spectral efficiency [1]. To exploit the potentials of MIMO, one of the challenges is the very high computing power that is required at the receiver end.

The maximum-likelihood (ML) detection has the optimum performance when a priori information of each bit is unknown, and the signals are transmitted through an additive white Gaussian noise (AWGN) channel [2]. Under

the above assumptions, the ML detector calculates the square of Euclidean distance between a received signal vector and its replica made by a channel response and the candidates of the transmitted signal vector, and it searches for the transmitted signal vector corresponding to the minimum distance from all replica candidates. The ML detector requires such an exhaustive search and it imposes exponentially computational complexity on the receiver.

To simplify the prohibitive complex search problem in ML detection, sphere (lattice) decoders are shown in Refs. [3,4] to be capable of achieving near optimal performance with reasonable complexity. The main idea in sphere decoding is to search over only hypotheses that lie in a certain hypersphere of radius  $r$ , rather than to search over the entire search space [3]. Furthermore,  $K$ -best sphere detector is proposed to guarantee the signal-to-noise ratio (SNR) independent fixed-complexity with performance close to ML [5]. In Ref. [6], another fixed-complexity MIMO detector is proposed, which has the basic idea that the search space is reduced as much as possible under the limit of computational effort before the exhaustive search.

In this paper, we propose a MIMO detection that reduces search space as much as possible based on the posterior probabilities computed from the output of zero-forcing (ZF) equalization. We represent the search space in the Cartesian product; hence, it is convenient for reducing the search space through determining each dimension separately. Comparing with former detection schemes, our proposed scheme owns two benefits, which are given as follows.

1) Most existing sphere detectors are working in the heuristic fashion and lack systematic strategy to connect the complexity and detection performance. The relation between the reduced search space and its corresponding detection performance can be derived in the proposed scheme.

2) In existing fixed-complexity MIMO detectors, the complexity is SNR independent; therefore, it would be a waste of computational power for high SNR. Consider the proposed scheme, less computational power will be spent by the given data rate for high SNR cases (in other words, energy efficiency is increased). Moreover, the throughput

Received April 3, 2009; accepted September 24, 2009

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of detector can be increased in SNR region, which enables the receiver to have the capacity of adaptive throughput control according to the SNR.

**Notation** Throughout this paper, matrix is set in boldface as  $\mathbf{H}$ .  $\mathbf{H}_m$  denotes the  $m$ th column of matrix  $\mathbf{H}$ , and  $\mathbf{H}_{ij}$  is the entry  $(i, j)$  of the matrix.

## 2 System model

We consider a MIMO channel with  $N_t$  transmit and  $N_r$  receive antennas, which can be represented as the matrix  $\mathbf{H}_C$  of dimension  $N_r \times N_t$ . Let  $N_t \times 1$  vector  $\mathbf{x}_C$  be the transmitted symbols that belong to a rectangular  $Q^2$ -quadrature amplitude modulation (QAM) constellation. The received signal is given as

$$\mathbf{y}_C = \mathbf{H}_C \mathbf{x}_C + \mathbf{v}_C, \quad (1)$$

where each entry of the  $N_r \times 1$  noise vector  $\mathbf{v}_C$  is a zero-mean complex Gaussian noise, having a variance of  $\sigma_v^2$ . Moreover, the complex model in Eq. (1) is often described by the equivalent real-valued representation [7], which is

$$\begin{aligned} \mathbf{y} &= \begin{bmatrix} \text{Re}(\mathbf{y}_C) \\ \text{Im}(\mathbf{y}_C) \end{bmatrix} \\ &= \begin{bmatrix} \text{Re}(\mathbf{H}_C) & -\text{Im}(\mathbf{H}_C) \\ \text{Im}(\mathbf{H}_C) & \text{Re}(\mathbf{H}_C) \end{bmatrix} \begin{bmatrix} \text{Re}(\mathbf{x}_C) \\ \text{Im}(\mathbf{x}_C) \end{bmatrix} + \begin{bmatrix} \text{Re}(\mathbf{v}_C) \\ \text{Im}(\mathbf{v}_C) \end{bmatrix} \\ &= \mathbf{H}\mathbf{x} + \mathbf{v}. \end{aligned} \quad (2)$$

This is also referred to as real value decomposition. Here, let  $\mathbf{M}$  denote the covariance matrix of  $\mathbf{v}$ , which fulfills that  $\mathbf{M} = \text{diag}(\sigma_v^2/2, \sigma_v^2/2, \dots, \sigma_v^2/2)$ . Assume the  $m$ th entry of  $\mathbf{x}$  satisfies that  $x_m \in \mathcal{S} = \{s_1, s_2, \dots, s_Q\}$ . We further assume  $N_t = N_r$ . (The case of  $N_r > N_t$  can be studied without much more effort.)

## 3 MIMO detection with predetermined search space

MIMO detection is the problem of estimating the transmitted vector  $\mathbf{x}$ , given  $\mathbf{y}$  and  $\mathbf{H}$ , related as Eq. (1). The ZF estimate of the transmitted vector, denoted  $\mathbf{x}^{\text{ZF}}$ , is written as

$$\mathbf{x}^{\text{ZF}} = \mathbf{H}^\dagger \mathbf{y} = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{y}$$

$$= \mathbf{x} + (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{v} = \mathbf{x} + \mathbf{G}^{\text{ZF}} \mathbf{v}, \quad (3)$$

where  $(\cdot)^\dagger$  is the computation of Moore-Penrose pseudo inverse [8]. Let  $\tilde{\mathbf{v}} = \mathbf{G}^{\text{ZF}} \mathbf{v}$  for the following discussion.

To obtain the optimal performance of MIMO detection, ML estimate can be utilized, which is given by

$$\mathbf{x}^{\text{ML}} = \arg \min_{\mathbf{x} \in \mathcal{D}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2, \quad (4)$$

where  $\mathcal{D}$  denotes the search space of the detection and can be represented in the Cartesian product form that

$$\begin{aligned} \mathcal{D} &= \mathcal{S} \times \mathcal{S} \times \dots \times \mathcal{S} \\ &= \{(x_1, x_2, \dots, x_{2N_t}) | x_m \in \mathcal{S}, m = 1, 2, \dots, 2N_t\}, \end{aligned}$$

and each ordered  $2N_t$ -tuple  $(x_1, x_2, \dots, x_{2N_t})$  corresponds to a possible transmitted vector. The straightforward way to solve ML estimate problem is exhaustive search, in which the value of Eq. (4) is computed for each of the  $Q^{2N_t}$  hypotheses. Since the complexity of exhaustive search method grows exponentially in  $2N_t$  and is not desirable even for a small  $2N_t$ , therefore, such utilization of ML estimate is not available in real-time wireless applications.

Consider the merits of ZF and ML estimates, we propose a novel MIMO detection scheme, which is demonstrated by Fig. 1 and named as reduced search space (RSS) ML detection. In RSS ML detection, the received signal is estimated by ZF equalizer; thus,  $\mathbf{x}^{\text{ZF}}$  can be obtained. With  $\mathbf{x}^{\text{ZF}}$ , further information about the noise and channel conditions can be derived, which can be expressed through sophisticated-constructed metrics, i.e., the posterior probabilities. Then, the required search space is determined, which is always reduced greatly comparing with that of the ideal ML detection before the exhaustive search done in the RSS ML detector.

Let us define the search space for RSS ML detector, which has been reduced and is given as

$$\begin{aligned} \mathcal{D}^{\text{RSS}} &= \mathcal{S}_1 \times \mathcal{S}_2 \times \dots \times \mathcal{S}_{2N_t} \\ &= \{(x_1, x_2, \dots, x_{2N_t}) | x_m \in \mathcal{S}_m, \text{ for all } m\}, \end{aligned}$$

where each  $\mathcal{S}_m \subseteq \mathcal{S}$ . Use  $\mathbf{x}^{(k)}$  to denote the  $k$ th hypothesis within the reduced search space, which corresponds to one and only one  $(x_1, x_2, \dots, x_{2N_t})$  sequence in  $\mathcal{D}^{\text{RSS}}$ , where  $1 \leq k \leq K$  and  $K = |\mathcal{D}^{\text{RSS}}| = \prod_{m=1}^{2N_t} |\mathcal{S}_m|$ ,  $|\cdot|$  denotes the size or cardinality of the set, which is the number of elements in such set. In this paper,  $K$  is called the search space size of the proposed detection scheme.

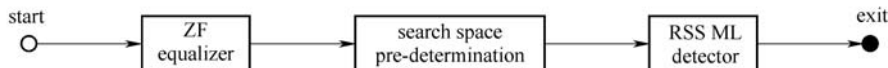


Fig. 1 Block diagram of RSS ML detector

## 4 Search space reduced method

In this section, we introduce a search space reducing method that determines the required search space through computing the posterior probability from ZF equalizer output. Our reducing criterion includes two points: 1) the reduced search space needs to cover all the hypotheses such that the cover probability is large enough; 2) the space size should be reduced as much as possible.

### 4.1 Posterior probability-based reducing

Let  $\tilde{\mathbf{M}}$  denote the covariance matrix of  $\tilde{\mathbf{v}}$ , which can be written as

$$\tilde{\mathbf{M}} = \mathbf{G}^{\text{ZF}} \mathbf{M} (\mathbf{G}^{\text{ZF}})^{\text{T}} = \frac{\sigma_v^2}{2} \mathbf{G}^{\text{ZF}} (\mathbf{G}^{\text{ZF}})^{\text{T}}.$$

During the following discussion, we focus on some detections where the transmitted signal vector is  $\bar{\mathbf{x}}$ .

Consider each entry of  $\tilde{\mathbf{v}}$ , we can get the probability density function (PDF) as

$$p(\tilde{v}_m) = \frac{1}{\sqrt{2\pi\tilde{\mathbf{M}}_{m,m}}} e^{-\frac{\tilde{v}_m^2}{2\tilde{\mathbf{M}}_{m,m}}}, \quad m = 1, 2, \dots, 2N_t, \quad (5)$$

then, given the channel matrix  $\mathbf{H}$  and noise  $\mathbf{v}$ , the conditional PDF of  $x_m^{\text{ZF}}$  can be expressed as

$$p(x_m^{\text{ZF}} | s_i) = \frac{1}{\sqrt{2\pi\tilde{\mathbf{M}}_{m,m}}} e^{-\frac{(x_m^{\text{ZF}} - s_i)^2}{2\tilde{\mathbf{M}}_{m,m}}}, \quad m = 1, 2, \dots, 2N_t. \quad (6)$$

We wish to design a signal identifier that makes the decisions on the search space reducing upon the observation of  $\mathbf{x}^{\text{ZF}}$  such that the probability of each  $\bar{\mathbf{x}}_m$  falling into the search space is large enough. With this goal in mind, we consider a decision rule based on the computation of posterior probabilities expressed as

$$P(s_i | x_m^{\text{ZF}}) = \frac{P(s_i) p(x_m^{\text{ZF}} | s_i)}{\sum_{j=1}^Q P(s_j) p(x_m^{\text{ZF}} | s_j)}, \quad i = 1, 2, \dots, Q. \quad (7)$$

If the a priori probability are equal, i.e., the signals  $\{s_i\}$  are equiprobable, and the above representation can be simplified further.

Therefore, once  $x_m^{\text{ZF}}$  has been known, we are able to define and compute the cover probability as

$$P_c(\mathcal{S}_m | x_m^{\text{ZF}}) = \sum_{s_i \in \mathcal{S}_m} P(s_i | x_m^{\text{ZF}}), \quad m = 1, 2, \dots, 2N_t, \quad (8)$$

that denotes the probability of  $\bar{\mathbf{x}}_m$  being covered (included) in the set  $\mathcal{S}_m$ . Then, we can present our search space reducing method, and the procedure for which can be

described using the pseudocode shown as Algorithm 1 (as shown below), in which the search space is reduced as much as possible if only under the requirement that  $P_c(\mathcal{S}_m | x_m^{\text{ZF}})$  for all  $m = 1, 2, \dots, 2N_t$  are larger than  $P_0$ .

In order to keep the computational complexity under control, we introduce the input  $C_{\text{limit}}$  to Algorithm 1, which is the maximum allowed search space size for the exhaustive search and is called as the complexity limit corresponding to the exhaustive search. Note that the break instruction terminates the execution of a for or while loop.

**Algorithm 1** Search space reducing ( $\mathbf{x}^{\text{ZF}}$ ,  $P_0$ ,  $C_{\text{limit}}$ )

for  $m := 1:2N_t$

$\mathcal{S}_m := \phi$

$\bar{\mathcal{S}}_m := \mathcal{S}$

$P_c(\mathcal{S}_m | x_m^{\text{ZF}}) := 0$

while  $P_c(\mathcal{S}_m | x_m^{\text{ZF}}) < P_0$

$s_j := \arg \max_{s_i \in \bar{\mathcal{S}}_m} P(s_i | x_m^{\text{ZF}})$

$P_c(\mathcal{S}_m | x_m^{\text{ZF}}) := P_c(\mathcal{S}_m | x_m^{\text{ZF}}) + P(s_j | x_m^{\text{ZF}})$

$\mathcal{S}_m := \mathcal{S}_m \cup \{s_j\}$

if  $|\mathcal{D}^{\text{RSS}}| > C_{\text{limit}}$

$\mathcal{S}_m := \mathcal{S}_m - \{s_j\}$

break

else

$\bar{\mathcal{S}}_m := \bar{\mathcal{S}}_m - \{s_j\}$

end

end

end

return  $\mathcal{S}_m$ ,  $m = 1, 2, \dots, 2N_t$

Consider  $\tilde{\mathbf{v}}$  again, the joint PDF can be written as

$$p(\tilde{\mathbf{v}}) = \frac{1}{(2N)^{N_t} (\det \tilde{\mathbf{M}})^{N_t}} e^{-\frac{1}{2} \tilde{\mathbf{v}}^{\text{T}} \tilde{\mathbf{M}}^{-1} \tilde{\mathbf{v}}}. \quad (9)$$

The joint conditional PDF of  $\mathbf{x}^{\text{ZF}}$  is given as

$$p(\mathbf{x}^{\text{ZF}} | \mathbf{x}^{(k)}) = \frac{1}{(2N)^{N_t} (\det \tilde{\mathbf{M}})^{N_t}} e^{-\frac{1}{2} (\mathbf{x}^{\text{ZF}} - \mathbf{x}^{(k)})^{\text{T}} \tilde{\mathbf{M}}^{-1} (\mathbf{x}^{\text{ZF}} - \mathbf{x}^{(k)})}. \quad (10)$$

Hence, the posterior probability can be expressed as

$$P(\mathbf{x}^{(k)} | \mathbf{x}^{\text{ZF}}) = \frac{P(\mathbf{x}^{(k)}) p(\mathbf{x}^{\text{ZF}} | \mathbf{x}^{(k)})}{\sum_{j=1}^K P(\mathbf{x}^{(j)}) p(\mathbf{x}^{\text{ZF}} | \mathbf{x}^{(j)})}, \quad k = 1, 2, \dots, K. \quad (11)$$

Given that  $\mathbf{x}^{\text{ZF}}$  has been known, let us define and compute the cover probability as

$$P_c(\mathcal{D}^{\text{RSS}} | \mathbf{x}^{\text{ZF}}) = \sum_{\mathbf{x}^{(k)} \in \mathcal{D}^{\text{RSS}}} P(\mathbf{x}^{(k)} | \mathbf{x}^{\text{ZF}}), \quad (12)$$

which denotes the probability of the event that the transmitted signal vector  $\bar{\mathbf{x}}$  being covered (included) in the search space  $\mathcal{D}^{\text{RSS}}$ . Since  $\mathcal{D}^{\text{RSS}}$  is defined in the Cartesian product form, Eq. (12) can further be expressed

as

$$P_c(\mathcal{D}^{\text{RSS}}|\mathbf{x}^{\text{ZF}}) = \sum_{\mathbf{x}_1^{(k)} \in \mathcal{S}_1} \sum_{\mathbf{x}_2^{(k)} \in \mathcal{S}_2} \cdots \sum_{\mathbf{x}_{2N_t}^{(k)} \in \mathcal{S}_{2N_t}} P(\mathbf{x}^{(k)}|\mathbf{x}^{\text{ZF}}). \quad (13)$$

The meaning behind  $P_c(\mathcal{D}^{\text{RSS}}|\mathbf{x}^{\text{ZF}})$  can be explained as follows:

**Lemma 1** Since  $\mathbf{H}$  is not always an orthogonal matrix, we obtain the relation as

$$P_c(\mathcal{D}^{\text{RSS}}|\mathbf{x}^{\text{ZF}}) \geq 1 - 2N_t(1 - P_0).$$

Set  $P_0$  large enough, we can make the performance decrease very little; because if only  $P_c(\mathcal{D}^{\text{RSS}}|\mathbf{x}^{\text{ZF}})$  is large enough, the probability of the transmitted signal vector  $\bar{\mathbf{x}}$  falling in the reduced search space is large enough so that the performance decrease can be little.

#### 4.2 Complexity analysis

The computational complexity of RSS ML detection algorithm is going to be investigated. To ignore the difference of hardware implementations, we focus on counting the numerical instructions, including real addition (subtraction) and real multiplication, which massively exist within the detection. For sake of simplicity, we assume each addition or multiplication to be one flop, supposing that one search needs two flops so as to be consistent with the assumption made in Ref. [6].

The resulting computational complexity  $F$  needed by the proposed RSS ML algorithm is the summation of computation complexity required by ZF estimate, by Algorithm 1, and exhaustive search, which fulfills the following relation:

$$\begin{aligned} F &= F_{\text{ZF}} + F_{\text{Algorithm 1}} + F_{\text{searching}} \\ &\leq \underbrace{32N_t^3 + 88N_t^2 + 3N_t}_{\text{ZF}} \\ &\quad + \underbrace{4N_t^2 + (96N_t^2 - 2N_t) + 2N_t}_{\text{worse case of Algorithm 1}} + \underbrace{2C_{\text{limit}}}_{\text{exhaustive search}}, \quad (14) \end{aligned}$$

where  $F_{\text{ZF}}$ ,  $F_{\text{Algorithm 1}}$  and  $F_{\text{searching}}$  are the complexity required by ZF estimate, by Algorithm 1, and exhaustive search. Herein, the complexity counting of ZF estimate and exhaustive search is in the same way as that in Ref. [6]. Hence,

$$F_{\text{searching}} = 2|\mathcal{D}^{\text{RSS}}| \leq 2C_{\text{limit}}.$$

Equation (14) indeed gives the upper bound of resulting computational complexity. When for high SNR, the computational complexity required by exhaustive search would be less than  $2C_{\text{limit}}$  because the proposed search space reducing method will contribute its effect of cutting off the search space. In Algorithm 1, we get the sum of the first five items of Taylor series expansion to approximate  $\exp(\cdot)$ .

## 5 Simulation results

We assume that all elements of the channel matrix  $\mathbf{H}_C$  are independent and identically distributed zero-mean complex Gaussian random variables having a variance of 1. Let

$$\text{SNR} = 10\lg(E_s/\sigma_v^2),$$

where  $E_s$  is the average transmit symbol power. Since in practice  $4 \times 4$  MIMO systems are quite popular; therefore, in this section, we focus on investigating the properties of such case.

Figure 2 shows the symbol error rate (SER) performance of RSS ML detection employing 16-QAM and 64-QAM with the complexity limit  $C_{\text{limit}} = +\infty$ . RSS ML #1 denotes the RSS ML detection with  $P_0 = 1 - 10^{-5}$ , while #2 with  $P_0 = 1 - 10^{-6}$ . It can be seen that, if  $P_0$  is set to  $1 - 10^{-5}$  and  $1 - 10^{-6}$ , the SER performance of RSS ML is approaching to that of the ML detection when SNR is not larger than 25 dB; the performance of RSS ML with  $P_0 = 1 - 10^{-6}$  is tiny better than that with  $P_0 = 1 - 10^{-5}$ .

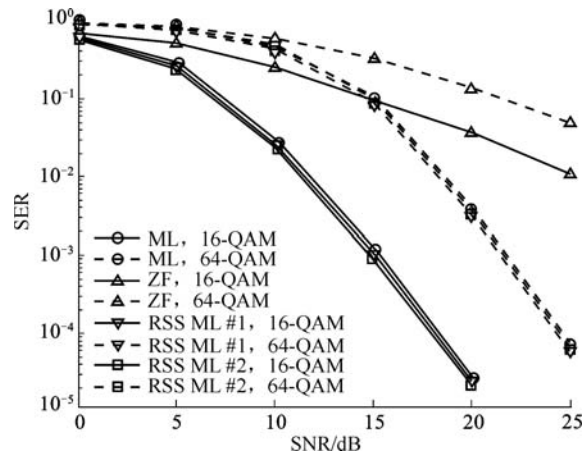


Fig. 2 SER versus SNR in  $4 \times 4$  MIMO system

Figure 3 shows the average search space size  $K$  of RSS ML detection employing 16-QAM and 64-QAM with the complexity limit  $C_{\text{limit}} = +\infty$ . RSS ML #1 denotes the RSS ML detection with  $P_0 = 1 - 10^{-5}$  while #2 with  $P_0 = 1 - 10^{-6}$ . It can be seen that the average search space size decreases greatly compared with ideal ML detection, especially when SNR is large; the space size of RSS ML with  $P_0 = 1 - 10^{-6}$  is tiny larger than that with  $P_0 = 1 - 10^{-5}$ . Since the complexity limit  $C_{\text{limit}} = +\infty$ , results show the efficiency of proposed search space reducing method in high SNR region.

Furthermore, we investigate the performance and average complexity of the proposed detection scheme together with  $K$ -best sphere decoding in Ref. [5] and reduced-search maximum-likelihood (RS ML) detection introduced in Ref. [6]. In this simulation, both RS ML detection and the proposed RSS ML detection use 8000 as

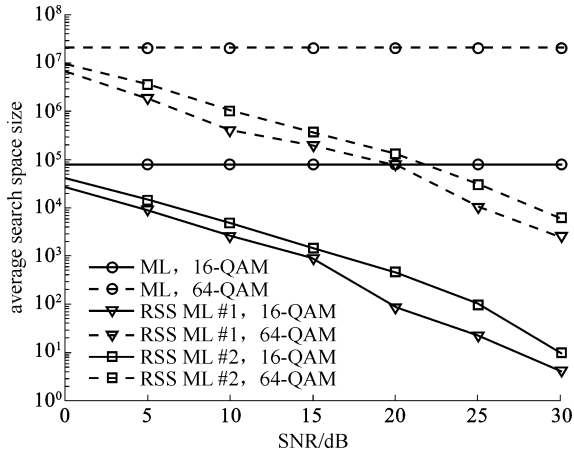


Fig. 3 Average search space size versus SNR in  $4 \times 4$  MIMO system

the complexity limit in the search stage; meanwhile, we set  $K=5$  for 16-QAM and  $K=10$  for 64-QAM considering  $K$ -best sphere decoding. SER is shown in Fig. 4, where the complexity limit is set to be  $C_{\text{limit}}=8000$  for proposed RSS ML detection, RSS ML #2 denotes the RSS ML detection with  $P_0=1-10^{-6}$ . The total number of numerical instructions of one detection considering each above scheme is computed and presented in Fig. 5, where the complexity limit is set as  $C_{\text{limit}}=8000$  for proposed RSS ML detection, RSS ML denotes the RSS ML detection with  $P_0=1-10^{-6}$ . Based on that in Figs. 4 and 5, it can be found that if SNR is larger than 16 dB in 16-QAM (or larger than 20 dB in 64-QAM), the proposed RSS ML detection will have better SER performance and cost less complexity power than  $K$ -best sphere decoding. In low SNR region and if  $Q^2$  and  $K$  values are small,  $K$ -best sphere decoding might cost less complexity power than proposed scheme when they have almost the same SER performance. However, we can also find the constellation size impacts greatly on  $K$ -best sphere decoding because its complexity is determined directly by  $Q^2$  and  $K$ . As shown in Fig. 5, when  $Q^2=64$  and  $K=10$ , the complexity of  $K$ -best sphere decoding becomes more than two times of that of RS ML and RSS ML detections even in a low SNR region. Therefore, we know that taking RSS ML detection and  $K$ -best sphere decoding into consideration, and the former has its advantage in high SNR region or in the case that  $Q^2$  and  $K$  are large and disadvantage only in low SNR region at same time  $Q^2$  and  $K$  are small enough. RSS ML scheme always costs less complexity than RS ML detection when they have almost the same SER performance. The reason herein is clear that RS ML detection always spends the computational power based on the complexity limit, while Algorithm 1 of our proposed scheme can cut off considerable search work even before the search stage once SNR is sufficiently large. Another important observation in Fig. 5 is that the complexity of the proposed

scheme varies according to the constellation size, i.e.,  $Q^2$ , in median SNR region (e.g., 0 to 20 dB) because different constellations have different properties of geometry, which directly affect the search space reducing of Algorithm 1 considering our proposed scheme. While in other two extremes, exactly speaking, in high SNR region, the search stage uses quite low computational power, thus making the item  $F_{\text{searching}}$  in Eq. (14) be far smaller than other two items  $F_{\text{ZF}}$  and  $F_{\text{Algorithm 1}}$ ; in other words, the final complexity is dominated by  $F_{\text{ZF}}$  and  $F_{\text{Algorithm 1}}$ ; while in low SNR region, the reducing efficiency of Algorithm 1 upon search space cannot be guaranteed, and the item  $F_{\text{searching}}$  approaches to  $2C_{\text{limit}}$ . In summary, the final complexity is basically determined by  $F_{\text{ZF}}$ ,  $F_{\text{Algorithm 1}}$ , and  $2C_{\text{limit}}$ . Besides, in high SNR region, the item  $F_{\text{Algorithm 1}}$  of Eq. (14) decays as SNR increases; therefore, the final complexity of proposed scheme approaches to  $F_{\text{ZF}}$  when SNR is high enough, as shown in Fig. 5.

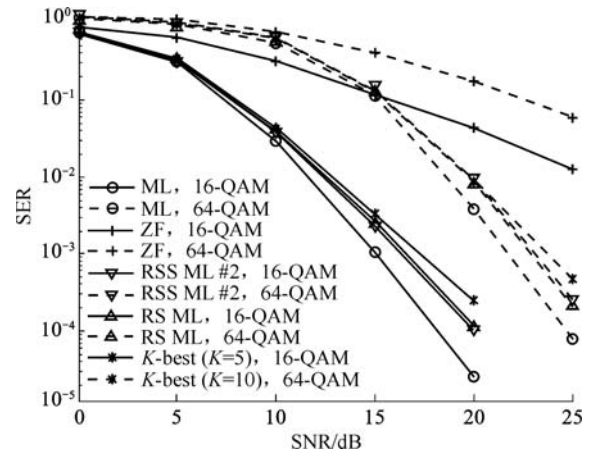


Fig. 4 SER versus SNR in  $4 \times 4$  MIMO system

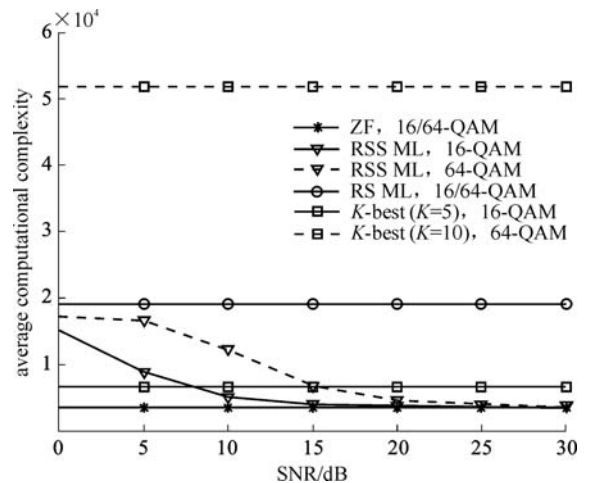


Fig. 5 Average computational complexity versus SNR in  $4 \times 4$  MIMO system

## 6 Conclusions

We present a MIMO detection scheme that reduces the search space based on the metrics calculated in the predetermination stage. The chosen metrics in this paper are the posterior probabilities. The proposed detection scheme can achieve the near optimal performance with the computational complexity decreases greatly.

**Acknowledgements** This work was supported by the National Basic Research Program of China (No. 2007CB310602).

## Appendix A

We give proof to the Lemma 1 given in this paper.

**Proof** Let  $A_m$  denote the event that the transmitted symbol  $\bar{x}_m$  is covered by  $\mathcal{S}_m$  in some detection, and  $\bar{A}_m$  otherwise. Then

$$P_c(\mathcal{D}^{\text{RSS}}|\mathbf{x}^{\text{ZF}}) = P(A_1, A_2, \dots, A_{2N_t})$$

and

$$P_c(\mathcal{S}_m|\mathbf{x}_m^{\text{ZF}}) = P(A_m) = 1 - P(\bar{A}_m).$$

The probability of the joint events fulfils that

$$1 - P(A_1, A_2, \dots, A_{2N_t}) \leq \sum_{m=1}^{2N_t} P(\bar{A}_m), \quad (\text{A1})$$

then

$$P_c(\mathcal{D}^{\text{RSS}}|\mathbf{x}^{\text{ZF}}) \geq 1 - \sum_{m=1}^{2N_t} (1 - P_c(\mathcal{S}_m|\mathbf{x}_m^{\text{ZF}})). \quad (\text{A2})$$

Moreover,

$$P_c(\mathcal{S}_m|\mathbf{x}_m^{\text{ZF}}) \geq P_0.$$

Hence, Lemma 1 is true.

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