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RESEARCH ARTICLE

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Distributed fusion white noise deconvolution estimators

Abstract The white noise deconvolution or input white noise estimation problem has important applications in oil seismic exploration, communication and signal processing. By combining the Kalman filtering method with the modern time series analysis method, based on the autoregressive moving average (ARMA) innovation model, new distributed fusion white noise deconvolution estimators are presented by weighting local input white noise estimators for general multisensor systems with different local dynamic models and correlated noises. The new estimators can handle input white noise fused filtering, prediction and smoothing problems, and are applicable to systems with colored measurement noise. Their accuracy is higher than that of local white noise deconvolution estimators. To compute the optimal weights, the new formula for local estimation error cross-covariances is given. A Monte Carlo simulation for the system with Bernoulli-Gaussian input white noise shows their effectiveness and performance.

Keywords multisensor information fusion, deconvolution, white noise estimator, seismology, modern time series analysis method, Kalman filtering method

1 Introduction

The white noise deconvolution or input white noise estimation problem is important in oil seismic exploration [1–4] as well as many other fields, including communication, signal processing and state estimation. Mendel and Kormylo [1–4] presented an optimal input white noise estimator with application in oil seismic exploration based on the Kalman filter. Deng et al. [5] presented a unified white noise estimation theory based on the modern time series analysis method, which includes input noise estimator and measurement noise estimator.

To improve estimation accuracy based on a single sensor, multisensor information fusion has received much attention in recent years. For Kalman filtering-based fusion, the two basic fusion methods are the centralized fusion method and distributed fusion method. The former offers globally optimal state estimation by directly combining local measurement data, but it is disadvantageous in that it may require a larger computational burden. Meanwhile, the distributed fusion method offers globally suboptimal state estimation by weighting the local state estimators, but can reduce computational burden and facilitate fault detection and isolation more conveniently. The optimal fusion rules weighted by matrices, diagonal matrices, and scalars have been presented in the linear minimum variance sense in Ref. [6], which gives three globally suboptimal Kalman fusers.

Recently, some results about white noise estimation have been presented using the Kalman filtering method. For multisensor linear discrete time-varying systems with correlated noises, Sun [7] presented the optimal weighted fusion white noise filter, which did not solve the smoothing problem of the white noise. In further research, Sun [8] presented an optimal weighted fusion white noise smoother. For multisensor linear discrete time-varying systems with colored measurement noises, Sun et al. [9] presented three optimal information fusion white noise deconvolution smoothers weighted by matrices, diagonal matrices and scalars, whereas the results in Refs. [7–9] are only suitable for the systems with the same local dynamic models. For a multisensor linear discrete time-varying system with different local dynamic models, Sun et al. [10,11] presented three information fusion white noise deconvolution estimators weighted by matrices, diagonal matrices and scalars. However, it is well known that classical Kalman filtering requires solving the Riccati equation making the computation of Kalman smoothers very complex. Using the modern time series analysis method based on the autoregressive moving average (ARMA) innovation model, Deng et al. [5] presented an optimal white noise filter, although it is only applicable to a

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single sensor system. For a multisensor linear discrete system with white and colored measurement noises, Deng et al. [12] presented the information fusion white noise deconvolution filter weighted by scalars, whereas it is only suitable for single channel systems. For multisensor linear discrete systems with different local dynamic models, Sun et al. [13] presented information fusion white noise deconvolution estimators using the modern time series analysis method, which have a disadvantage in that the infinite series is required with a larger computational burden.

To overcome the above drawbacks and limitations, the Kalman filtering method is combined with the modern time series analysis method for new distributed fusion white noise deconvolution estimators for linear discrete time-invariant stochastic systems with different local models. They are completely different from the estimators in Refs. [10,11,13]. They can handle white noise fused filtering, prediction and smoothing problems, and are applicable to systems with colored measurement noise. To compute the optimal weights, the new formula for local estimation error covariances is also presented, which is different from that in Ref. [13].

2 Problem formulation

Consider the linear discrete time-invariant stochastic system with different local dynamic models:

$$\boldsymbol{x}_i(t+1) = \boldsymbol{\Phi}_i\boldsymbol{x}_i(t) + \boldsymbol{\Gamma}_i\boldsymbol{w}_i(t), \tag{1}$$

$$\boldsymbol{y}_i(t) = \boldsymbol{H}_i\boldsymbol{x}_i(t) + \boldsymbol{v}_i(t),\ i = 1,2,...,L, \tag{2}$$

$$\boldsymbol{w}_{\mathrm{c}}(t) = \boldsymbol{C}_i\boldsymbol{w}_i(t), \tag{3}$$

where t is the discrete time, $\boldsymbol{x}_i(t) \in \mathbb{R}^{n_i}$ is the state, $\boldsymbol{y}_i(t) \in \mathbb{R}^{m_i}$ is the measurement, $\boldsymbol{w}_i(t) \in \mathbb{R}^{r_i}$ and $\boldsymbol{v}_i(t) \in \mathbb{R}^{m_i}$ are the input white noise and measurement white noise of the ith sensor subsystem, respectively. $\boldsymbol{w}_{\mathrm{c}}(t) \in \mathbb{R}^{r}$ is the common input white noise of all subsystems. $\boldsymbol{\Phi}_i$, $\boldsymbol{\Gamma}_i$, $\boldsymbol{H}_i$, $\boldsymbol{C}_i$ are known matrices with compatible dimensions.

Assumption 1 $\boldsymbol{w}_i(t)$ and $\boldsymbol{v}_i(t)$ are correlated white noises with zero mean and

$$\mathrm{E}\left\{\begin{bmatrix}\boldsymbol{w}_i(t)\\ \boldsymbol{v}_i(t)\end{bmatrix}\begin{bmatrix}\boldsymbol{w}_j^{\mathrm{T}}(k) & \boldsymbol{v}_j^{\mathrm{T}}(k)\end{bmatrix}\right\} = \begin{bmatrix}\boldsymbol{Q}_{ij} & \boldsymbol{S}_{ij}\\ \boldsymbol{S}_{ji}^{\mathrm{T}} & \boldsymbol{R}_{ij}\end{bmatrix}\delta_{tk}, \tag{4}$$

where E denotes the mathematical expectation, the superscript T denotes the transpose, and δ_{tk} is the Kronecker delta function, $\delta_{tt} = 1$, $\delta_{tk} = 0\ (t \neq k)$.

Assumption 2 $(\boldsymbol{\Phi}_i, \boldsymbol{H}_i)$ is a completely observable pair with the observability index β_i, and the ith subsystem is completely stabilizable.

Assumption 3 The initial time $t_0 = -\infty$.

The objectives are to find the local steady-state common white noise deconvolution estimators $\hat{\boldsymbol{w}}_{\mathrm{c}i}(t|t+N)$, $i = 1,2,...,L$, for the ith sensor subsystem, and to find the optimal weighted fusion white noise deconvolution estimators $\hat{\boldsymbol{w}}_{\mathrm{c}0}(t|t+N)$ of $\boldsymbol{w}_{\mathrm{c}}(t)$, weighted by matrices, diagonal matrices and scalars respectively. When $N = 0$, $N > 0$, $N < 0$, they are called filters, smoothers and predictors, respectively.

Remark 1 For the multisensor system with the local dynamic model Eq. (1), $i = 1,2,...,L$, if for different i, $\boldsymbol{\Phi}_i$, $\boldsymbol{\Gamma}_i$, $\boldsymbol{x}_i(t)$ and $\boldsymbol{w}_i(t)$ are different, then it is called a multisensor system with different local dynamic models [6,13]. If $\boldsymbol{\Phi}_i = \boldsymbol{\Phi}$, $\boldsymbol{\Gamma}_i = \boldsymbol{\Gamma}$, $\boldsymbol{x}_i(t) = \boldsymbol{x}(t)$, $\boldsymbol{w}_i(t) = \boldsymbol{w}(t)$, $i = 1,2,...,L$, then it is called a multisensor system with the same local dynamic models.

3 ARMA innovation model

For the ith subsystem, from Eqs. (1) and (2) we have

$$\boldsymbol{y}_i(t) = \boldsymbol{H}_i(\boldsymbol{I}_{n_i} - q^{-1}\boldsymbol{\Phi}_i)^{-1}\boldsymbol{\Gamma}_i q^{-1}\boldsymbol{w}_i(t) + \boldsymbol{v}_i(t), \tag{5}$$

where q^{-1} is the backward shift operator, $q^{-1}\boldsymbol{x}_i(t) = \boldsymbol{x}_i(t-1)$. The left-coprime factorization

$$\boldsymbol{H}_i(\boldsymbol{I}_{n_i} - q^{-1}\boldsymbol{\Phi}_i)^{-1}\boldsymbol{\Gamma}_i q^{-1} = \boldsymbol{A}_i^{-1}(q^{-1})\boldsymbol{B}_i(q^{-1}), \tag{6}$$

where $\boldsymbol{A}_i(q^{-1})$ and $\boldsymbol{B}_i(q^{-1})$ are polynomial matrices having the form $\boldsymbol{X}_i(q^{-1}) = \boldsymbol{X}_{i0} + \boldsymbol{X}_{i1}q^{-1} + \cdots + \boldsymbol{X}_{in_{xi}}q^{-n_{xi}}$ with coefficient matrices $\boldsymbol{X}_{ij}$ and degree n_{xj}, and $\boldsymbol{A}_{i0} = \boldsymbol{I}_{m_i}$.

Inserting Eq. (6) into Eq. (5) yields the ARMA innovation model of $\boldsymbol{y}_i(t)$:

$$\boldsymbol{A}_i(q^{-1})\boldsymbol{y}_i(t) = \boldsymbol{D}_i(q^{-1})\boldsymbol{\varepsilon}_i(t), \tag{7}$$

where $\boldsymbol{D}_i(q^{-1}) = \boldsymbol{D}_{i0} + \boldsymbol{D}_{i1}q^{-1} + \cdots + \boldsymbol{D}_{in_{di}}q^{-n_{di}}$ is stable (i.e., all zeros of $\det\boldsymbol{D}_i(x)$ are excluded from the unit circle), $\boldsymbol{D}_{i0} = \boldsymbol{I}_{m_i}$, the innovation process $\boldsymbol{\varepsilon}_i(t) \in \mathbb{R}^{m_i}$ is white noise with zero mean and variance matrix $\boldsymbol{Q}_{\boldsymbol{\varepsilon}_i}$, and

$$\boldsymbol{D}_i(q^{-1})\boldsymbol{\varepsilon}_i(t) = \boldsymbol{B}_i(q^{-1})\boldsymbol{w}_i(t) + \boldsymbol{A}_i(q^{-1})\boldsymbol{v}_i(t), \tag{8}$$

where $\boldsymbol{D}_i(q^{-1})$ and $\boldsymbol{Q}_{\boldsymbol{\varepsilon}_i}$ can be obtained by using the Gevers and Wouters algorithm [14].

4 Distributed fusion white noise deconvolution estimators

Lemma 1 [6] For the multisensor time-invariant systems Eqs. (1)–(3) with Assumptions 1–3, the ith sensor subsystem has the local steady-state optimal Kalman predictor:

$$\hat{\boldsymbol{x}}_i(t+1|t) = \boldsymbol{\Psi}_{pi}\hat{\boldsymbol{x}}_i(t|t-1) + \boldsymbol{K}_{pi}\boldsymbol{y}_i(t), \tag{9}$$

$$\boldsymbol{\varepsilon}_i(t) = \boldsymbol{y}_i(t) - \boldsymbol{H}_i\hat{\boldsymbol{x}}_i(t|t-1), \tag{10}$$

$$\boldsymbol{\Psi}_{pi} = \boldsymbol{\Phi}_i - \boldsymbol{K}_{pi}\boldsymbol{H}_i, \tag{11}$$

where $\boldsymbol{\Psi}_{pi}$ is a stable matrix. The prediction gain $\boldsymbol{K}_{pi}$ is given as

$$\boldsymbol{K}_{pi}=\begin{bmatrix}\boldsymbol{H}_i\\ \boldsymbol{H}_i\boldsymbol{\Phi}_i\\ \vdots\\ \boldsymbol{H}_i\boldsymbol{\Phi}_i^{\beta_i-1}\end{bmatrix}^{+}\begin{bmatrix}\boldsymbol{M}_1^{(i)}\\ \boldsymbol{M}_2^{(i)}\\ \vdots\\ \boldsymbol{M}_{\beta_i}^{(i)}\end{bmatrix},\tag{12}$$

where the pseudo-inverse of matrix $\boldsymbol{X}$ is defined as $\boldsymbol{X}^{+}=(\boldsymbol{X}^{\mathrm{T}}\boldsymbol{X})^{-1}\boldsymbol{X}^{\mathrm{T}}$. The matrices $\boldsymbol{M}_k^{(i)}$ can be recursively computed as

$$\boldsymbol{M}_k^{(i)}=-\boldsymbol{A}_{i1}\boldsymbol{M}_{k-1}^{(i)}-\cdots-\boldsymbol{A}_{in_{ai}}\boldsymbol{M}_{k-n_{ai}}^{(i)}+\boldsymbol{D}_{ik},\tag{13}$$

where $k=1,2,\ldots,\beta_i$, and we define that $\boldsymbol{M}_0^{(i)}=\boldsymbol{I}_{m_i}$, $\boldsymbol{M}_k^{(i)}=\boldsymbol{0}\ (k<0)$, $\boldsymbol{D}_{ik}=\boldsymbol{0}\ (k>n_{di})$.

The prediction error cross-covariances $\Sigma_{ij}=E[\tilde{\boldsymbol{x}}_i(t+1|t)\tilde{\boldsymbol{x}}_j^{\mathrm{T}}(t+1|t)]$ are given as

$$\Sigma_{ij}=\boldsymbol{\Psi}_{pi}\Sigma_{ij}\boldsymbol{\Psi}_{pj}^{\mathrm{T}}+[\boldsymbol{\Gamma}_i\quad-\boldsymbol{K}_{pi}]\begin{bmatrix}\boldsymbol{Q}_{ij}&\boldsymbol{S}_{ij}\\ \boldsymbol{S}_{ji}^{\mathrm{T}}&\boldsymbol{R}_{ij}\end{bmatrix}\begin{bmatrix}\boldsymbol{\Gamma}_j^{\mathrm{T}}\\ -\boldsymbol{K}_{pj}^{\mathrm{T}}\end{bmatrix}\tag{14}$$

or

$$\Sigma_{ij}=\boldsymbol{\Psi}_{pi}\Sigma_{ij}\boldsymbol{\Psi}_{pj}^{\mathrm{T}}+\boldsymbol{\Gamma}_i\boldsymbol{Q}_{ij}\boldsymbol{\Gamma}_j^{\mathrm{T}}-\boldsymbol{K}_{pi}\boldsymbol{S}_{ji}^{\mathrm{T}}\boldsymbol{\Gamma}_j^{\mathrm{T}}-\boldsymbol{\Gamma}_i\boldsymbol{S}_{ij}\boldsymbol{K}_{pj}^{\mathrm{T}}+\boldsymbol{K}_{pi}\boldsymbol{R}_{ij}\boldsymbol{K}_{pj}^{\mathrm{T}}.\tag{15}$$

Proof The proof is given in Ref. [6] and is omitted here.

Lemma 2 [15] For the multisensor time-invariant systems Eqs. (1)–(3) with Assumptions 1–3, the ith sensor subsystem has the local steady-state optimal white noise deconvolution estimators:

$$\hat{\boldsymbol{w}}_i(t|t+N)=\sum_{k=0}^{N}\boldsymbol{M}_{ii}(k)\boldsymbol{\varepsilon}_i(t+k),\quad N\geqslant 0,\ i=1,2,\ldots,L,\tag{16}$$

$$\hat{\boldsymbol{w}}_i(t|t+N)=\boldsymbol{0},\ N<0,\ i=1,2,\ldots,L,\tag{17}$$

where we define

$$\boldsymbol{M}_{ii}(0)=\boldsymbol{S}_{ii}\boldsymbol{Q}_{\boldsymbol{\varepsilon}_i}^{-1},\ i=1,2,\ldots,L,\tag{18}$$

$$\boldsymbol{M}_{ii}(1)=\boldsymbol{D}_{ii}\boldsymbol{H}_i^{\mathrm{T}}\boldsymbol{Q}_{\boldsymbol{\varepsilon}_i}^{-1},\ i=1,2,\ldots,L,\tag{19}$$

$$\boldsymbol{M}_{ii}(k)=\boldsymbol{D}_{ii}\boldsymbol{\Psi}_{pi}^{\mathrm{T}(k-1)}\boldsymbol{H}_i^{\mathrm{T}}\boldsymbol{Q}_{\boldsymbol{\varepsilon}_i}^{-1},\ k>1,\ i=1,2,\ldots,L,\tag{20}$$

with the definition

$$\boldsymbol{D}_{ii}=\boldsymbol{Q}_{ii}\boldsymbol{\Gamma}_i^{\mathrm{T}}-\boldsymbol{S}_{ii}\boldsymbol{K}_{pi}^{\mathrm{T}}.\tag{21}$$

The corresponding steady-state estimation error variances $\boldsymbol{P}_{ii}^{w}(N)$ are given as

$$\boldsymbol{P}_{ii}^{w}(N)=\boldsymbol{Q}_{ii}-\sum_{k=0}^{N}\boldsymbol{M}_k^{iw}\boldsymbol{Q}_{\boldsymbol{\varepsilon}_i}\boldsymbol{M}_k^{iw\mathrm{T}},\quad N\geqslant 0,\ i=1,2,\ldots,L,\tag{22}$$

$$\boldsymbol{P}_{ii}^{w}(N)=\boldsymbol{Q}_{ii},\ N<0,\ i=1,2,\ldots,L.\tag{23}$$

Proof The proof is given in Ref. [15] and is omitted here.

Theorem 1 For the multisensor time-invariant system Eqs. (1) –(3) with Assumptions 1–3, the cross-covariances among local estimation errors are given as

$$\boldsymbol{P}_{ij}^{w}(N)=\boldsymbol{\Psi}_{iN}\Sigma_{ij}\boldsymbol{\Psi}_{jN}^{\mathrm{T}}+\sum_{\rho=0}^{N}[\boldsymbol{K}_{i\rho}^{w}\quad\boldsymbol{K}_{i\rho}^{v}]\begin{bmatrix}\boldsymbol{Q}_{ij}&\boldsymbol{S}_{ij}\\ \boldsymbol{S}_{ji}^{\mathrm{T}}&\boldsymbol{R}_{ij}\end{bmatrix}\begin{bmatrix}\boldsymbol{K}_{j\rho}^{w\mathrm{T}}\\ \boldsymbol{K}_{j\rho}^{v\mathrm{T}}\end{bmatrix},\tag{24}$$

where $i,j=1,2,\ldots,L$, $N>0$, and we define

$$\boldsymbol{\Psi}_{iN}=-\sum_{k=0}^{N}\boldsymbol{M}_{ii}(k)\boldsymbol{H}_i\boldsymbol{\Psi}_{pi}^{k},\tag{25}$$

$$\boldsymbol{K}_{i\rho}^{w}=\delta_{\rho 0}\boldsymbol{I}_r-\sum_{k=\rho+1}^{N}\boldsymbol{M}_{ii}(k)\boldsymbol{H}_i\boldsymbol{\Psi}_{pi}^{k-\rho-1}\boldsymbol{\Gamma}_i,$$
$$\rho=0,1,\ldots,N-1,\ \delta_{00}=1,\ \delta_{\rho 0}=0\ (\rho\neq 0),$$
$$\boldsymbol{K}_{iN}^{w}=\boldsymbol{0}\ (N>0),\tag{26}$$

$$\boldsymbol{K}_{i\rho}^{v}=\sum_{k=\rho+1}^{N}\boldsymbol{M}_{ii}(k)\boldsymbol{H}_i\boldsymbol{\Psi}_{pi}^{k-\rho-1}\boldsymbol{K}_{pi}-\boldsymbol{M}_{ii}(\rho),$$
$$\rho=0,1,\ldots,N-1,\ \boldsymbol{K}_{iN}^{v}=-\boldsymbol{M}_{ii}(N)\ (N>0),\tag{27}$$

$$\boldsymbol{\Psi}_{pi}^{k-1}=\underbrace{\boldsymbol{\Psi}_{pi}\cdots\boldsymbol{\Psi}_{pi}}_{k-1},\quad\boldsymbol{\Psi}_{pi}^{0}=\boldsymbol{I}_n.\tag{28}$$

In particular, for $i=j$, we have

$$\boldsymbol{P}_{ii}^{w}(N)=\boldsymbol{\Psi}_{iN}\Sigma_{ii}\boldsymbol{\Psi}_{iN}^{\mathrm{T}}+\sum_{\rho=0}^{N}[\boldsymbol{K}_{i\rho}^{w}\ \boldsymbol{K}_{i\rho}^{v}]\begin{bmatrix}\boldsymbol{Q}_{ii}&\boldsymbol{S}_{ii}\\ \boldsymbol{S}_{ii}^{\mathrm{T}}&\boldsymbol{R}_{ii}\end{bmatrix}\begin{bmatrix}\boldsymbol{K}_{i\rho}^{w\mathrm{T}}\\ \boldsymbol{K}_{i\rho}^{v\mathrm{T}}\end{bmatrix},\tag{29}$$

which is different from Eq. (22).

Proof See Appendix A.

Corollary 1 When $N=0$, we have the cross-covariances among local filtering errors as

$$\boldsymbol{P}_{ij}^{w}(0)=\boldsymbol{Q}_{ij}+\boldsymbol{S}_{ii}\boldsymbol{Q}_{\boldsymbol{\varepsilon}_i}^{-1}\boldsymbol{H}_i\Sigma_{ij}\boldsymbol{H}_i^{\mathrm{T}}(\boldsymbol{Q}_{\boldsymbol{\varepsilon}_i}^{-1})^{\mathrm{T}}\boldsymbol{S}_{ii}^{\mathrm{T}}-\boldsymbol{S}_{ii}\boldsymbol{Q}_{\boldsymbol{\varepsilon}_i}^{\mathrm{T}}\boldsymbol{S}_{ji}^{\mathrm{T}}-\boldsymbol{S}_{ij}(\boldsymbol{Q}_{\boldsymbol{\varepsilon}_i}^{-1})^{\mathrm{T}}\boldsymbol{S}_{ii}^{\mathrm{T}}.\tag{30}$$

Proof From Eq. (16), we have

$$\tilde{\boldsymbol{w}}_i(t|t)=\boldsymbol{w}_i(t)-\boldsymbol{S}_{ii}\boldsymbol{Q}_{\boldsymbol{\varepsilon}_i}^{-1}\boldsymbol{\varepsilon}_i(t).\tag{31}$$

From Eqs. (2) and (10), we have

$$\boldsymbol{\varepsilon}_i(t) = \boldsymbol{H}_i\tilde{\boldsymbol{x}}_i(t|t-1) + \boldsymbol{v}_i(t). \tag{32}$$

Inserting Eq. (32) into Eq. (31) yields

$$\tilde{\boldsymbol{w}}_i(t|t) = \boldsymbol{w}_i(t) - \boldsymbol{S}_{ii}\boldsymbol{Q}_{\boldsymbol{\varepsilon}_i}^{-1}[\boldsymbol{H}_i\tilde{\boldsymbol{x}}_i(t|t-1) + \boldsymbol{v}_i(t)]. \tag{33}$$

Note that $\tilde{\boldsymbol{x}}_i(t|t-1)$ is uncorrelated with $\boldsymbol{w}_i(t)$ and $\boldsymbol{v}_j(t)$. Using Eqs. (4) and (33), we can obtain Eq. (30).

Theorem 2 For the multisensor time-invariant system Eqs. (1)–(3) with Assumptions 1–3, the local optimal common white noise deconvolution estimators $\hat{\boldsymbol{w}}_{ci}(t|t+N)$ for the common white noise $\boldsymbol{w}_c(t)$ are given as

$$\hat{\boldsymbol{w}}_{ci}(t|t+N) = \boldsymbol{C}_i\hat{\boldsymbol{w}}_i(t|t+N),\ i = 1,2,\ldots,L, \tag{34}$$

and the cross-covariances of the local estimation errors $\tilde{\boldsymbol{w}}_{ci}(t|t+N) = \boldsymbol{w}_c(t) - \hat{\boldsymbol{w}}_{ci}(t|t+N)$ are given as

$$\boldsymbol{P}_{cij}(N) = \boldsymbol{C}_i\boldsymbol{P}_{ij}^{w}(N)\boldsymbol{C}_j^{\mathrm{T}},\ i = 1,2,\ldots,L. \tag{35}$$

The distributed (weighted) optimal information fusion common white noise deconvolution estimators are given as

$$\hat{\boldsymbol{w}}_{c0}(t|t+N) = \sum_{i=1}^{L}\boldsymbol{\Omega}_i^{(N)}\hat{\boldsymbol{w}}_{ci}(t|t+N),\ N \geqslant 0, \tag{36}$$

$$\hat{\boldsymbol{w}}_{c0}(t|t+N) = 0,\ N < 0. \tag{37}$$

For the fusers with matrix weights, we have

$$[\boldsymbol{\Omega}_1^{(N)}\ \boldsymbol{\Omega}_2^{(N)}\ \cdots\ \boldsymbol{\Omega}_L^{(N)}] = (\boldsymbol{e}^{\mathrm{T}}P_c^{-1}(N)\boldsymbol{e})^{-1}\boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}_c^{-1}(N), \tag{38}$$

$$\boldsymbol{P}_c(N) = (\boldsymbol{P}_{cij}(N))_{nL\times nL},\ i,j = 1,2,\ldots,L, \tag{39}$$

where $\boldsymbol{e}^{\mathrm{T}} = [\boldsymbol{I}_n\ \cdots\ \boldsymbol{I}_n]$, and the fusion error covariances $\boldsymbol{P}_0^{wcm}(N)$ are given as

$$\boldsymbol{P}_0^{wcm}(N) = \left(\boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}_c^{-1}(N)\boldsymbol{e}\right)^{-1}. \tag{40}$$

For the fusers with scalar weights $\boldsymbol{\Omega}_i^{(N)} = \boldsymbol{\omega}_i^{(N)}$, we have

$$[\boldsymbol{\omega}_1^{(N)}\ \boldsymbol{\omega}_2^{(N)}\ \cdots\ \boldsymbol{\omega}_L^{(N)}] = (\boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}_{\mathrm{trc}}(N)\boldsymbol{e})^{-1}\boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}_{\mathrm{trc}}^{-1}(N), \tag{41}$$

$$\boldsymbol{P}_{\mathrm{trc}}(N) = (\mathrm{tr}\boldsymbol{P}_{cij}(N))_{L\times L},\ i,j = 1,2,\ldots,L, \tag{42}$$

where tr denotes the trace of matrix $\boldsymbol{e}^{\mathrm{T}} = [1\ 1\ \cdots\ 1]$ and the fusion error covariances $\boldsymbol{P}_0^{wcs}(N)$ are given as

$$\boldsymbol{P}_0^{wcs}(N) = \sum_{i,j=1}^{L}\boldsymbol{\omega}_i^{(N)}\boldsymbol{\omega}_j^{(N)}\boldsymbol{P}_{cij}(N). \tag{43}$$

For the fusers with diagonal matrix weights, we have

$$\boldsymbol{\Omega}_i^{(N)} = \mathrm{diag}(\boldsymbol{\omega}_{i1}^{(N)}\ \cdots\ \boldsymbol{\omega}_{ir}^{(N)}), \tag{44}$$

$$[\boldsymbol{\omega}_{i1}^{(N)}\ \boldsymbol{\omega}_{i2}^{(N)}\ \cdots\ \boldsymbol{\omega}_{iL}^{(N)}] = (\boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}_{cii}^{-1}(N)\boldsymbol{e})^{-1}\boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}_{cii}^{-1}(N), \tag{45}$$

$$\boldsymbol{P}_{cii}(N) = (\boldsymbol{P}_{ckj}^{ii}(N))_{L\times L},\ k,j = 1,2,\ldots,L, \tag{46}$$

where $\boldsymbol{e}^{\mathrm{T}} = [1\ 1\ \cdots\ 1]$, and $\boldsymbol{P}_{ckj}^{ii}(N)$ are the (i,i)th diagonal elements of $\boldsymbol{P}_{ckj}(N)\boldsymbol{P}_{ckj}(N)k,\ j = 1,2,\ldots,L,\ i = 1,2,\ldots,r.$

The trace of the fusion error covariances $\boldsymbol{P}_0^{wcd}(N)$ are given as

$$\mathrm{tr}\boldsymbol{P}_0^{wcd}(N) = \sum_{i=1}^{r}\left(\boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}_{cii}^{-1}(N)\boldsymbol{e}\right)^{-1}. \tag{47}$$

Denoting the centralized fusion error variance as $\boldsymbol{P}_0^{wcc}(N)$, we have the accuracy relation as

$$\mathrm{tr}\boldsymbol{P}_0^{wcm}(N) \leqslant \mathrm{tr}\boldsymbol{P}_0^{wcd}(N) \leqslant \mathrm{tr}\boldsymbol{P}_0^{wcs}(N) \leqslant \mathrm{tr}\boldsymbol{P}_{ii}^{wcc}(N). \tag{48}$$

Proof From Eq. (3) and the projection property we have Eq. (34). From Eqs. (3) and (34) we have $\tilde{\boldsymbol{w}}_{ci}(t|t+N) = \boldsymbol{C}_i\tilde{\boldsymbol{w}}_i(t|t+N)$, which yields Eq. (35). Applying the three optimal fusion formulas weighted by matrices, diagonal matrices and scalars in Ref. [6], we directly obtain Theorem 2.

Equation (48) shows that the three weighted fusers obtained are locally optimal and globally suboptimal. The accuracy of the fusers is higher than that of every local estimator. The accuracy of the fuser with matrix weights is higher than that of the fuser with scalar weights, and the accuracy of the fuser with diagonal matrix weights is between both. The accuracy of each fuser is lower than that of the centralized fuser.

5 Simulation example

In oil seismic exploration, the random reflectivity sequence is described by the Bernoulli-Gaussian white noise [3,16]. Estimating the reflectivity sequence is important for finding oil fields [1–4].

Consider the Bernoulli-Gaussian white noise information fusion deconvolution smoother for the time-invariant system with 3-sensor and colored measurement noise:

$$\boldsymbol{x}(t+1) = \boldsymbol{\Phi}\boldsymbol{x}(t) + \boldsymbol{\Gamma}\boldsymbol{w}_c(t), \tag{49}$$

$$\boldsymbol{w}_c(t) = \boldsymbol{b}(t)\boldsymbol{g}(t), \tag{50}$$

$$\boldsymbol{y}_i(t) = \boldsymbol{H}_0\boldsymbol{x}(t) + \boldsymbol{\eta}_i(t) + \boldsymbol{e}_i(t),\ i = 1,2,3, \tag{51}$$

$$\boldsymbol{P}_i(q^{-1})\boldsymbol{\eta}_i(t) = \boldsymbol{R}_i(q^{-1})\boldsymbol{\xi}_i(t),\ i = 1,2,3, \tag{52}$$

where $\boldsymbol{g}(t)$, $\boldsymbol{e}_i(t)$ and $\boldsymbol{\xi}_i(t)$ are independent Gaussian white noises with zero means and variances $\sigma_{\boldsymbol{g}}^2$, $\sigma_{\boldsymbol{e}_i}^2$ and $\sigma_{\boldsymbol{\xi}_i}^2$, respectively. The input white noise $\boldsymbol{w}_c(t) = \boldsymbol{b}(t)\boldsymbol{g}(t)$ is Bernoulli-Gaussian white noise, where $\boldsymbol{b}(t)$ is a Bernoulli white noise with probabilities $P(\boldsymbol{b}(t) = 1) = \lambda$, $P(\boldsymbol{b}(t) = 0) = 1 - \lambda$, and is independent of $\boldsymbol{g}(t)$. The problem is to find the local and optimal weighted fusion white noise deconvolution smoothers $\hat{\boldsymbol{w}}_{ci}(t|t+3)$, $i = 0,1,2,3$ weighted by scalars.

In the simulation we assume that $\lambda = 0.2$, $\sigma_{\boldsymbol{g}}^2 = 1$, then

$$\begin{aligned}
&\sigma_{\boldsymbol{w}_c}^2 = 0.2,\ \sigma_{\boldsymbol{v}_1}^2 = 0.11,\ \sigma_{\boldsymbol{v}_2}^2 = 0.22,\ \sigma_{\boldsymbol{v}_3}^2 = 0.3,\\
&\sigma_{\boldsymbol{\xi}_1}^2 = 0.0021,\ \sigma_{\boldsymbol{\xi}_2}^2 = 0.0032,\ \sigma_{\boldsymbol{\xi}_3}^2 = 0.0041,\\
&\boldsymbol{\Phi} = \begin{bmatrix} 1.3 & 1 \\ -0.42 & 0 \end{bmatrix},\ \boldsymbol{\Gamma} = \begin{bmatrix} 0.6 \\ 0 \end{bmatrix},\\
&\boldsymbol{H}_0 = [1 \quad 0],\\
&P_1(q^{-1}) = 1 - 0.3q^{-1} + 0.02q^{-2},\\
&R_1(q^{-1}) = 1 - 0.4q^{-1},\\
&P_2(q^{-1}) = 1 + 0.1q^{-1} - 0.12q^{-2},\\
&R_2(q^{-1}) = 1 + 0.3q^{-1},\\
&P_3(q^{-1}) = 1 - 0.6q^{-1} + 0.05q^{-2},\\
&R_3(q^{-1}) = 1 - 0.2q^{-1}.
\end{aligned} \tag{53}$$

Transform Eq. (52) into the state-space model

$$\boldsymbol{\alpha}_i(t+1) = \boldsymbol{P}_i\boldsymbol{\alpha}_i(t) + \boldsymbol{R}_i\boldsymbol{\xi}_i(t), \tag{54}$$

$$\boldsymbol{\eta}_i(t) = \boldsymbol{H}_{1i}\boldsymbol{\alpha}_i(t) + \boldsymbol{\xi}_i(t),\ i = 1,2,3, \tag{55}$$

where

$$\begin{aligned}
&\boldsymbol{R}_1 = \begin{bmatrix} -0.1 \\ -0.02 \end{bmatrix},\ \boldsymbol{R}_2 = \begin{bmatrix} 0.2 \\ 0.12 \end{bmatrix},\ \boldsymbol{R}_3 = \begin{bmatrix} 0.4 \\ -0.05 \end{bmatrix},\\
&\boldsymbol{P}_1 = \begin{bmatrix} 0.3 & 1 \\ -0.02 & 0 \end{bmatrix},\ \boldsymbol{P}_2 = \begin{bmatrix} -0.1 & 1 \\ 0.12 & 0 \end{bmatrix},\\
&\boldsymbol{P}_3 = \begin{bmatrix} 0.6 & 1 \\ -0.05 & 0 \end{bmatrix},\\
&\boldsymbol{H}_i = [1 \quad 0],\ i = 1,2,3.
\end{aligned} \tag{56}$$

Introducing the augmented state and noise, we have the 3-sensor system with different local dynamic models and the common white noise $\boldsymbol{w}_c(t)$ as

$$\boldsymbol{x}_i(t+1) = \boldsymbol{\Phi}_i\boldsymbol{x}_i(t) + \boldsymbol{\Gamma}_i\boldsymbol{w}_i(t), \tag{57}$$

$$\boldsymbol{y}_i(t) = \boldsymbol{H}_i\boldsymbol{x}_i(t) + \boldsymbol{v}_i(t), \tag{58}$$

$$\boldsymbol{w}_c(t) = \boldsymbol{C}_i\boldsymbol{w}_i(t), \tag{59}$$

$$\begin{aligned}
&\boldsymbol{w}_i(t) = [\boldsymbol{w}_c(t) \quad \xi_i(t)]^{\mathrm{T}},\\
&\boldsymbol{v}_i(t) = \boldsymbol{e}_i(t) + \xi_i(t),\ i = 1,2,3,
\end{aligned} \tag{60}$$

where

$$\begin{aligned}
&\boldsymbol{x}_i(t) = \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{\alpha}_i(t) \end{bmatrix},\ \boldsymbol{\Phi}_i = \begin{bmatrix} \boldsymbol{\Phi} & 0 \\ 0 & \boldsymbol{P}_i \end{bmatrix},\\
&\boldsymbol{\Gamma}_i = \begin{bmatrix} \boldsymbol{\Gamma} & 0 \\ 0 & \boldsymbol{R}_i \end{bmatrix},\ \boldsymbol{H}_i = [\boldsymbol{H}_0 \quad \boldsymbol{H}_{1i}],\\
&\boldsymbol{C}_i = [1 \quad 0].
\end{aligned} \tag{61}$$

Here, the problem is converted to that of finding the local and fused smoother of the common white noise $\boldsymbol{w}_c(t)$ for the systems with different local models Eqs. (57)–(61). The simulation results are shown in Figs. 1–3 and Table 1. Figure 1 shows the comparison between the true values and estimates of common white noise, where the vertical coordinates at the endpoints of solid lines denote the true values, and the vertical coordinates of the dots denote the estimates. Figure 2 shows the comparison of accumulated error squares for the local and fused common white noise smoothers. Figure 3 shows the comparison between the local and the fused mean square error (MSE) values in 300 Monte Carlo runs. All these figures and Table 1 show that the accuracy of the fused white noise smoother is higher than that of the local white noise smoothers. Here, we define the MSE value at time t as

$$\mathrm{MSE}_i(t) = \frac{1}{m}\sum_{j=1}^{m}\left(\boldsymbol{w}_c^{(j)}(t) - \hat{\boldsymbol{w}}_{ci}^{(j)}(t|t+3)\right)^2,\quad i = 0,1,2,3,\ t = 1,2,\ldots,200, \tag{62}$$

where $\boldsymbol{w}_c^{(j)}(t)$ and $\hat{\boldsymbol{w}}_{ci}^{(j)}(t|t+3)$ are the jth realizations of the stochastic process $\boldsymbol{w}_c(t)$ and $\hat{\boldsymbol{w}}_{ci}(t|t+3)$ respectively, $j = 1,\ldots,m$, and $m = 300$ is the run number.

Table 1 Comparison of local and fused common white noise smoothing error variances

$P_{11}^{wc}(3)$	$P_{22}^{wc}(3)$	$P_{33}^{wc}(3)$	$P_0^{wc}(3)$
0.05797	0.07949	0.09010	0.04783

6 Conclusions

For linear discrete time-invariant stochastic systems with different local dynamic models, the new distributed fusion white noise deconvolution estimators have been presented using the Kalman filtering method combined with the modern time series analysis method. The principle of the method is that the formulas of the local steady-state white noise estimators and their cross-covariances are computed via classical Kalman filtering. However, the steady-state

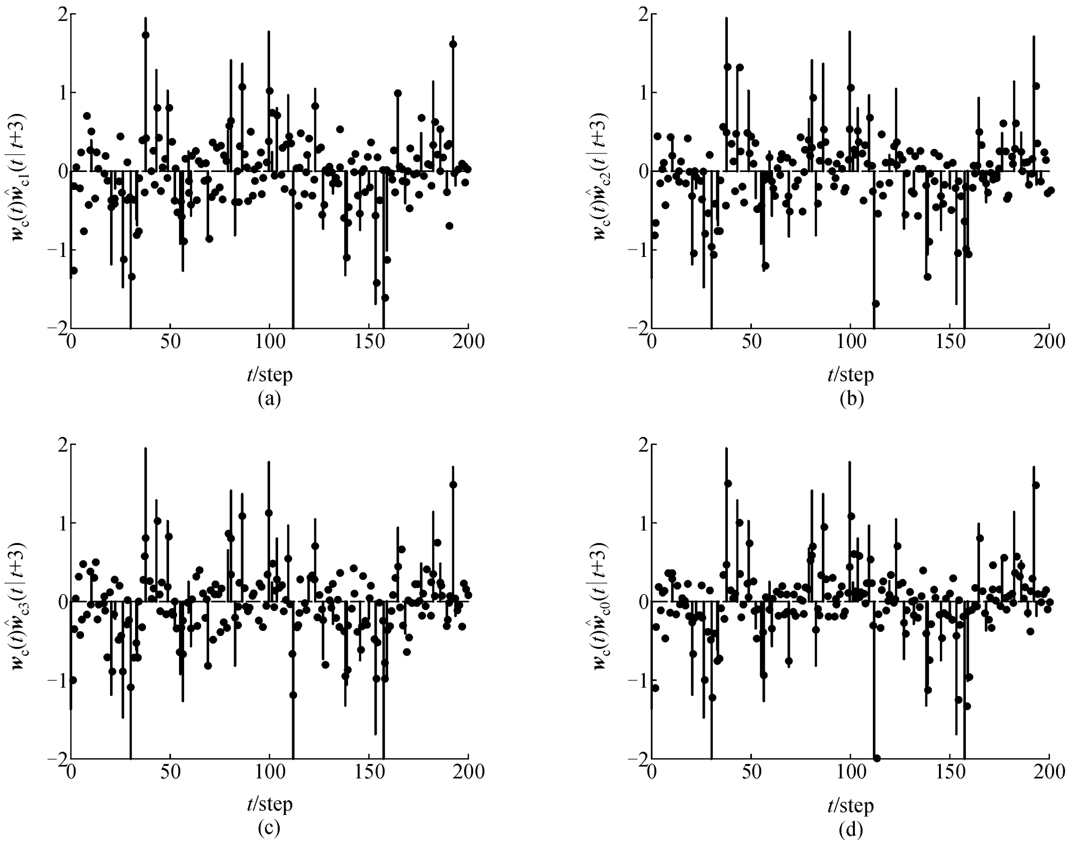

Fig. 1 Common white noise $\boldsymbol{w}_c(t)$ and its local and fused smoothers $\hat{\boldsymbol{w}}_{ci}(t|t+3)$, $i = 0,1,2,3$. (a) $\boldsymbol{w}_c(t)$ and $\hat{\boldsymbol{w}}_{c1}(t|t+3)$; (b) $\boldsymbol{w}_c(t)$ and $\hat{\boldsymbol{w}}_{c2}(t|t+3)$; (c) $\boldsymbol{w}_c(t)$ and $\hat{\boldsymbol{w}}_{c3}(t|t+3)$; (d) $\boldsymbol{w}_c(t)$ and $\hat{\boldsymbol{w}}_{c0}(t|t+3)$

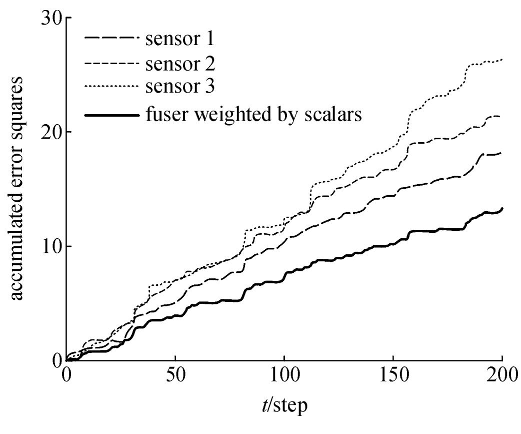

Fig. 2 Comparison of accumulated error squares for local and fused common white noise smoothers $\hat{\boldsymbol{w}}_{ci}(t|t+3)$, $i = 0,1,2,3$

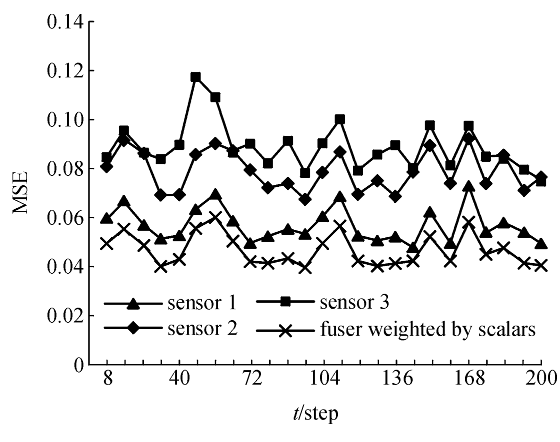

Fig. 3 Comparison of MSE curves for local and fused smoothers

Kalman predictor gains are computed based on the ARMA innovation models, and the Riccati equations are avoided. The fused estimators are locally optimal and globally suboptimal. Their accuracy is higher than that of local white noise estimators and lower than that of centralized fusion estimators. In addition, they can handle systems with colored measurement noise. The new formula for computing cross-covariance among local estimation errors has been presented, which is applied to optimal weights. Computing the infinite series is avoided, and thus the computational burden can be reduced.

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Appendix A Proof of Theorem 1

From Ref. [6], we have the prediction error system

$$\tilde{\boldsymbol{x}}_i(t+1|t) = \boldsymbol{\Psi}_{pi}\tilde{\boldsymbol{x}}_i(t|t-1) + \boldsymbol{\Gamma}_i\boldsymbol{w}_i(t) - \boldsymbol{K}_{pi}\boldsymbol{v}_i(t) \quad \text{(A1)}$$

with $\tilde{\boldsymbol{x}}_i(t|t-1) = \boldsymbol{x}_i(t) - \hat{\boldsymbol{x}}_i(t|t-1)$. Applying the iteration to Eq. (A1) yields

$$\begin{aligned}&\tilde{\boldsymbol{x}}_i(t+k|t+k-1)\\ &= \boldsymbol{\Psi}_{pi}^k\tilde{\boldsymbol{x}}_i(t|t-1) + \sum_{\gamma=1}^{k}\boldsymbol{\Psi}_{pi}^{k-\gamma}[\boldsymbol{\Gamma}_i\boldsymbol{w}_i(t+\gamma-1)\\ &\quad -\boldsymbol{K}_{pi}\boldsymbol{v}_i(t+\gamma-1)]. \quad \text{(A2)}\end{aligned}$$

From Eqs. (2) and (10), we have $\boldsymbol{\varepsilon}_i(t) = \boldsymbol{H}\tilde{\boldsymbol{x}}_i(t|t-1) + \boldsymbol{v}_i(t)$, hence we have

$$\begin{aligned}\boldsymbol{\varepsilon}_i(t+k) &= \boldsymbol{H}_i\boldsymbol{\Psi}_{pi}^k\tilde{\boldsymbol{x}}_i(t|t-1) + \sum_{\gamma=1}^{k}\boldsymbol{H}_i\boldsymbol{\Psi}_{pi}^{k-\gamma}\\ &\quad \times [\boldsymbol{\Gamma}_i \quad -\boldsymbol{K}_{pi}]\begin{bmatrix}\boldsymbol{w}_i(t+\gamma-1)\\ \boldsymbol{v}_i(t+\gamma-1)\end{bmatrix} + \boldsymbol{v}_i(t+k).\end{aligned} \quad \text{(A3)}$$

From Eq. (16), we have

$$\tilde{\boldsymbol{w}}_i(t|t+N) = \boldsymbol{w}_i(t) - \sum_{k=0}^{N}\boldsymbol{M}_i(k)\boldsymbol{\varepsilon}_i(t+k) \quad \text{(A4)}$$

with $\tilde{\boldsymbol{w}}_i(t|t+N) = \boldsymbol{w}_i(t) - \hat{\boldsymbol{w}}_i(t|t+N)$. Inserting Eq. (A3) into Eq. (A4) yields

$$\begin{aligned}\tilde{\boldsymbol{w}}_i(t|t+N) &= \boldsymbol{w}_i(t) - \sum_{k=0}^{N}\boldsymbol{M}_{ii}(k)\boldsymbol{H}_i\boldsymbol{\Psi}_{pi}^k\tilde{\boldsymbol{x}}_i(t|t-1)\\ &\quad -\sum_{k=1}^{N}\sum_{\gamma=1}^{k}\boldsymbol{M}_{ii}(k)\boldsymbol{H}_i\boldsymbol{\Psi}_{pi}^{k-\gamma}\boldsymbol{\Gamma}_i\boldsymbol{w}_i(t+\gamma-1)\\ &\quad +\sum_{k=1}^{N}\sum_{\gamma=1}^{k}\boldsymbol{M}_{ii}(k)\boldsymbol{H}_i\boldsymbol{\Psi}_{pi}^{k-\gamma}\boldsymbol{K}_{pi}\boldsymbol{v}_i(t+\gamma-1)\\ &\quad -\sum_{k=0}^{N}\boldsymbol{M}_{ii}(k)\boldsymbol{v}_i(t+k). \quad \text{(A5)}\end{aligned}$$

Combining the terms in Eq. (A5) for $\tilde{\boldsymbol{x}}_i(t|t-1)$, $\boldsymbol{w}_i(t+\rho)$ and $\boldsymbol{v}_i(t+\rho)$, respectively, we obtain

$$\begin{aligned}\tilde{\boldsymbol{w}}_i(t|t+N) &= \boldsymbol{\Psi}_{iN}\tilde{\boldsymbol{x}}_i(t|t-1) + \sum_{\rho=0}^{N}\boldsymbol{K}_{i\rho}^w\boldsymbol{w}_i(t+\rho)\\ &\quad +\sum_{\rho=0}^{N}\boldsymbol{K}_{i\rho}^v\boldsymbol{v}_i(t+\rho), \quad \text{(A6)}\end{aligned}$$

or

$$\begin{aligned}\tilde{\boldsymbol{x}}_i(t|t+N) &= \boldsymbol{\Psi}_{iN}\tilde{\boldsymbol{x}}_i(t|t-1)\\ &\quad +\sum_{\rho=0}^{N}[\boldsymbol{K}_{i\rho}^w \ \boldsymbol{K}_{i\rho}^v]\begin{bmatrix}\boldsymbol{w}_i(t+\rho)\\ \boldsymbol{v}_i(t+\rho)\end{bmatrix} \quad \text{(A7)}\end{aligned}$$

with $\boldsymbol{\Psi}_{iN}$, $\boldsymbol{K}_{i\rho}^w$ and $\boldsymbol{K}_{i\rho}^v$ defined in Eqs. (25)–(27).

Note that $\tilde{\boldsymbol{x}}_i(t|t-1)$ is uncorrelated with $\boldsymbol{w}_i(t+\rho)$ and $\boldsymbol{v}_j(t+\rho)$, $\rho = 0,1,...,L$. Using Eqs. (4) and (A7), we obtain Eq. (24). The proof is completed.

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