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RESEARCH ARTICLE

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Absorption of electromagnetic wave by inhomogeneous, unmagnetized plasma

Abstract In this article, a novel and normalized *Z*-transform finite-difference time-domain (ZTFDTD) method is presented. This method uses a more general form of Maxwell's equations using the ***E***, ***H***, ***D*** fields. The iterative model of ***D-E-H-D*** can be obtained by using the *Z*-transform resulted frequency-dependent formula between ***D*** and ***E***. The advantages of the ZTFDTD consist in that the discrete equations are simple, the results are precise, easy to program and capable of dealing with the key technologies of finite-difference time-domain (FDTD), such as absorbing boundary conditions (uniaxial anisotropic perfectly matched layer, UPML) and near-to-far-field transformation. The ZTFDTD method is then used to simulate the interaction of electromagnetic wave with plasma. Using a simplified two-dimensional model, the stealth effect of inhomogeneous, unmagnetized plasma is studied both in different electron densities of plasma, different electromagnetic wave frequencies and different plasma collision frequencies. The numerical results indicate that plasma stealth is effective in theory and a reasonable selection with the plasma parameters that can greatly enhance the effectiveness of plasma stealth.

Keywords finite-difference time-domain (FDTD), *Z*-transform, anisotropic perfectly matched layer, plasma stealth, radar cross section (RCS)

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1 Introduction

Plasma stealth is a novel technology in theory. The study of targeting plasma stealth has attracted much attention around the world [1–3]. Considerable work has been done in plasma stealth and modeling plasma with the finite-difference time-domain (FDTD) technique.

In this article a novel and normalized *Z*-transform FDTD (ZTFDTD) method is presented to simulate the interaction of electromagnetic (EM) wave with unmagnetized plasma. This method uses a more general form of Maxwell's equations using the ***E***, ***H***, ***D*** fields, by which we can obtain the iterative model of ***D-E-H-D*** using the *Z*-transform resulting frequency-dependent formula between ***D*** and ***E***. This FDTD method was originally suggested by Sullivan [4,5] and has been applied to dispersive media. The advantages of the ZTFDTD are that the discrete equations are simple, the results are precise, easy to program and capable of dealing with the key technologies of FDTD, uniaxial anisotropic perfectly matched layer (UPML), and near-to-far-field transformation. The main purpose of this article is to introduce the ZTFDTD method into plasma stealth technology. The two-dimensional (2-D) ZTFDTD formulations for unmagnetized plasma are derived. Using a simplified 2-D model, the stealth effect of inhomogeneous, unmagnetized plasma (IUP) is studied both in different electron densities of plasma, different electromagnetic wave frequencies and different plasma collision frequencies.

2 *Z*-transform-based finite-difference time-domain method

Using the Wentzel Kramers Brillouin (WKB) approximation [6], the permittivity of unmagnetized plasma is described as follows:

$$\varepsilon_r^*(\omega) = 1 + \frac{\omega_{\mathrm{p}}^2}{\omega(\mathrm{j}\nu_{\mathrm{c}} - \omega)}, \tag{1}$$

$$\omega_p = 2\pi f_p = \left(\frac{N_e e^2}{m_e \varepsilon_0}\right)^{1/2}, \tag{2}$$

where f_p is the plasma frequency, v_c is the electron collision frequency, N_e is the electron density, e is the electron charge, and m_e is the mass of the electron.

A more general form of Maxwell's equations coupling with plasma movement equation which uses the $\boldsymbol{E}$, $\boldsymbol{H}$, $\boldsymbol{D}$ fields is

$$\frac{\partial \boldsymbol{D}}{\partial t} = \nabla \times \boldsymbol{H}, \tag{3}$$

$$\boldsymbol{D}(\omega) = \varepsilon_0 \varepsilon_r^*(\omega)\boldsymbol{E}(\omega), \tag{4}$$

$$\frac{\partial \boldsymbol{H}}{\partial t} = -\frac{1}{\mu_0}\nabla \times \boldsymbol{E}, \tag{5}$$

where $\boldsymbol{D}$ is the electric flux density, $\boldsymbol{E}$ is the electric field, and $\boldsymbol{H}$ is the magnetic intensity. Because μ and ε differ by several orders of magnitude, the $\boldsymbol{E}$ field and $\boldsymbol{H}$ field will differ by several orders of magnitude. This can be avoided by making the following change of variables:

$$\tilde{\boldsymbol{E}} = \sqrt{\frac{\varepsilon_0}{\mu_0}}\boldsymbol{E}, \quad \tilde{\boldsymbol{D}} = \sqrt{\frac{1}{\varepsilon_0 \mu_0}}\boldsymbol{D}. \tag{6}$$

Inserting Eq. (6) into Eqs. (3)–(5) yields

$$\frac{\partial \tilde{\boldsymbol{D}}}{\partial t} = \sqrt{\frac{1}{\varepsilon_0 \mu_0}}\nabla \times \boldsymbol{H}, \tag{7}$$

$$\tilde{\boldsymbol{D}}(\omega) = \varepsilon_r^*(\omega)\tilde{\boldsymbol{E}}(\omega), \tag{8}$$

$$\frac{\partial \boldsymbol{H}}{\partial t} = -\sqrt{\frac{1}{\varepsilon_0 \mu_0}}\nabla \times \tilde{\boldsymbol{E}}. \tag{9}$$

Equations (7) and (9) will lead to very simple finite difference equations. The only change is to use variable $\boldsymbol{D}$ instead of $\boldsymbol{E}$. Equation (8) has to be formulated into a time domain difference equation for the implementation of FDTD. The first task is to get it from the frequency domain to the time domain.

In frequency-dependent media, we can avoid dealing with troublesome convolution integrals in the time domain using Z-transforms for the FDTD formulation. By using partial fraction expansion and Z-transform, Eq. (1) can be written as

$$\varepsilon_r^*(z) = \frac{1}{\Delta t} + \frac{\omega_p^2/v_c}{1-z^{-1}} - \frac{\omega_p^2/v_c}{1-e^{-v_c\Delta t}z^{-1}}. \tag{10}$$

By the convolution theorem, the Z-transform of Eq. (8) is

$$\tilde{\boldsymbol{D}}(z) = \Delta t \varepsilon_r^*(z)\tilde{\boldsymbol{E}}(z). \tag{11}$$

By inserting Eq. (10) into Eq. (11), we obtain

$$\tilde{\boldsymbol{D}}(z) = \tilde{\boldsymbol{E}}(z) + \frac{\omega_p^2 \Delta t}{v_c}\left[\frac{(1-e^{-v_c\Delta t})z^{-1}}{1-(1+e^{-v_c\Delta t})z^{-1}+e^{-v_c\Delta t}z^{-2}}\right]\tilde{\boldsymbol{E}}(z). \tag{12}$$

An auxiliary term will be defined as

$$\boldsymbol{M}(z) = \frac{\omega_p^2 \Delta t}{v_c}\left[\frac{(1-e^{-v_c\Delta t})}{1-(1+e^{-v_c\Delta t})z^{-1}+e^{-v_c\Delta t}z^{-2}}\right]\tilde{\boldsymbol{E}}(z). \tag{13}$$

$\tilde{\boldsymbol{E}}(z)$ can be solved for by

$$\tilde{\boldsymbol{E}}(z) = \tilde{\boldsymbol{D}}(z) - z^{-1}\boldsymbol{M}(z). \tag{14}$$

Therefore, the FDTD simulation becomes

$$\begin{aligned} \tilde{\boldsymbol{E}}^n &= \tilde{\boldsymbol{D}}^n - \boldsymbol{M}^{n-1}, \\ \boldsymbol{M}^n &= (1+e^{-v_c\Delta t})\boldsymbol{M}^{n-1} - e^{-v_c\Delta t}\boldsymbol{M}^{n-2} \\ &\quad + \frac{\omega_p^2 \Delta t}{v_c}(1-e^{-v_c\Delta t})\tilde{\boldsymbol{E}}^n. \end{aligned} \tag{15}$$

3 Research on absorption of EM wave by IUP

Figure 1 shows three boundaries and zoning of the ZTFDTD grid.

An ideal plasma boundary condition is

$$\begin{cases} N_e(b) = 0, \ n(b) = 1, \\ N_e(a) = N_{e0}, \ n(a) = 0, \end{cases} \tag{16}$$

where N_e is the electron density, n is the refractive index, and N_{e0} is the peak value electron density. According to Eq. (16), the electron density of plasma is taken as

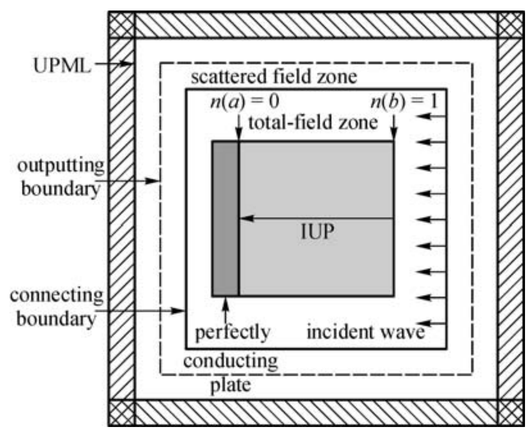

Fig. 1 Three boundaries and zoning of ZTFDTD grid

$$N_e(x) = N_{e0}\frac{(b-x)^{2m}}{(b-a)^{2m}}. \tag{17}$$

Inserting Eq. (17) into Eq. (2):

$$\omega_p(r) = \omega_{pmax}\frac{(b-x)^m}{(b-a)^m}. \tag{18}$$

The definition of radar cross section (RCS) is [7]

$$\begin{cases} R(f) = 10\lg\left(\dfrac{2\pi r}{\lambda}\left|\dfrac{\boldsymbol{E}_s(f)}{\boldsymbol{E}_i(f)}\right|^2\right), \\ \boldsymbol{E}_s(f) = \displaystyle\int_{+\infty}^{-\infty}\boldsymbol{E}_s(t)e^{-j2\pi ft}dt, \\ \boldsymbol{E}_i(f) = \displaystyle\int_{+\infty}^{-\infty}\boldsymbol{E}_i(t)e^{-j2\pi ft}dt, \end{cases} \tag{19}$$

where $\boldsymbol{E}_i(f)$ is the electric field of the incident EM wave, $\boldsymbol{E}_s(f)$ is the scatter field of the far-field of the EM wave, and λ is the wavelength of the vacuum.

To demonstrate the aforementioned ZTFDTD formulation for unmagnetized plasma, we compute the RCS of the EM wave through a perfectly conducting plate covered with unmagnetized plasma. The incident wave is introduced to Eqs. (7)–(9) by connecting boundaries, and the reflected wave of the EM wave is absorbed by the UPML. Using a simplified 2-D model, a perfectly conducting plate covered with unmagnetized plasma (the length and width are both 10 cm), the stealth effect of unmagnetized plasma is studied in different thicknesses of plasma, electron densities of plasma, EM wave frequencies and plasma collision frequencies. The incident EM wave is a sinusoidal plane wave (E_{in}= sin(2πft)).

Figure 2 shows the amplitude and the RCS of the perfectly conducting plate covered with nothing and IUP. The value of m, maximum electron density, plasma frequency, and plasma collision frequency of IUP are 1, $2.80\times10^{18}/m^3$, 15 GHz and 40 GHz, respectively. The EM wave frequency is 15 GHz. The RCS of the IUP is better, mainly because the IUP has absorbed the incident EM wave.

Figure 3 shows the RCS of the perfectly conducting plate covered with the IUP in different electron densities ($m = 0.5$, 1, 1.5, 2). The EM wave frequency is 15 GHz. The plasma collision frequency, maximum electron density and corresponding plasma frequency of the IUP are 10 GHz, $2.80 \times 10^{18}/m^3$ and 15 GHz, respectively. In this condition the plasma stealth is effective. The smaller the value of m is, the more remarkable the plasma stealth effect is.

Figure 4 shows the RCS of the perfectly conducting

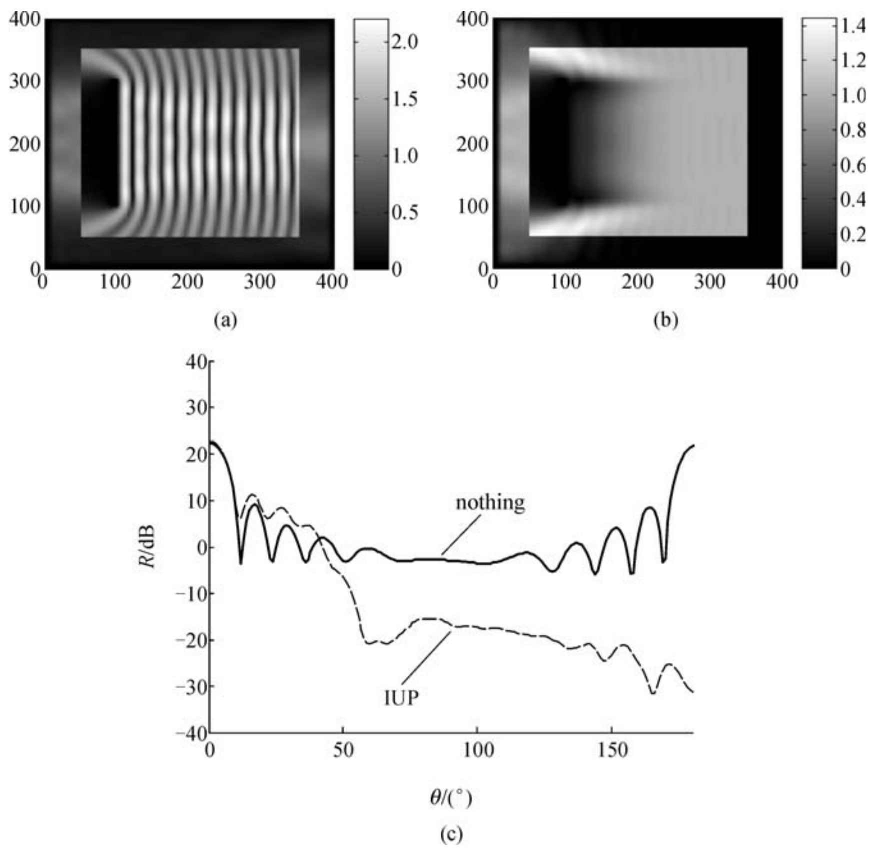

Fig. 2 Amplitude and RCS of perfectly conducting plate covered with nothing and IUP. (a) Amplitude of perfectly conducting plate covered with nothing; (b) amplitude of perfectly conducting plate covered with IUP; (c) RCS of perfectly conducting plate covered with nothing and IUP

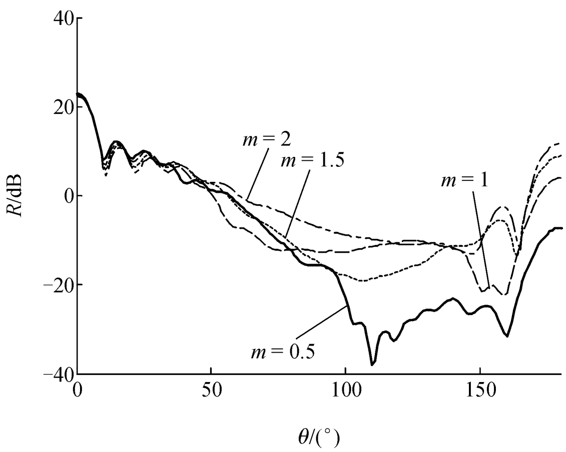

Fig. 3 RCS of perfectly conducting plate covered with IUP in different electron densities

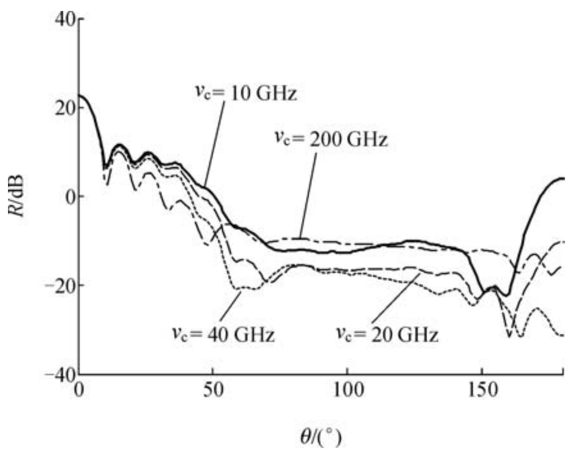

Fig. 5 RCS of perfectly conducting plate covered with IUP in different plasma collision frequencies

plate covered with the IUP in different EM wave frequencies (f = 15 GHz, 20 GHz, 30 GHz). The value of m, plasma collision frequency, maximum electron density and corresponding plasma frequency of IUP are 1, 20 GHz, $2.80 \times 10^{18}/\mathrm{m}^3$ and 15 GHz, respectively. The ability of IUP absorption of EM waves of different frequencies is greatly different. When the EM wave frequency approaches to the lower limit of the plasma frequency, the attenuation of the EM wave is increased because of the plasma resonance absorption of EM waves.

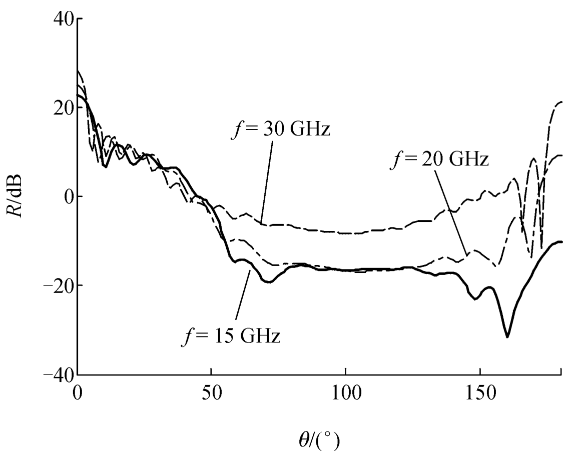

Fig. 4 RCS of perfectly conducting plate covered with IUP in different EM wave frequencies

Figure 5 shows the RCS of the perfectly conducting plate covered with the IUP in different plasma collision frequencies (ν_c = 10 GHz, 20 GHz, 40 GHz, 200 GHz). The EM wave frequency is 15 GHz. The value of m, maximum electron density and corresponding plasma frequency of IUP are 1, $2.80 \times 10^{18}/\mathrm{m}^3$ and 15 GHz, respectively. The plasma collision frequency has achieved its optimal value. When the plasma has this collision frequency, the attenuation of the EM wave reaches its maximum.

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