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RESEARCH ARTICLE

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Analysis of band gap of non-bravais lattice photonic crystal fiber

Abstract This article designs a novel type of non-bravais lattice photonic crystal fiber. To form the nesting complex-period with positive and negative refractive index materials respectively, a cylinder with the same radius and negative refractive index is introduced into the center of each lattice unit cell in the traditional square lattice air-holes photonic crystal fiber. The photonic band-gap of the photonic crystal fiber is calculated numerically by the plane wave expansion method. The result shows that compared with the traditional square photonic band-gap fiber (PBGF), when R/Λ is 0.35, the refractive index of the substrate, air-hole, and medium-column are 1.30, 1.0, and -1.0, respectively. This new PBGF can transmit signal by the photonic band-gap effect. When the lattice constant Λ varies from 1.5 μm to 3.0 μm, the range of the wavelength ranges from 880 nm to 2300 nm.

Keywords photonic crystal fiber, negative refractive index, non-bravais lattice, photonic band-gap

1 Introduction

Photonic crystal fibers (PCFs) [1] are a two-dimensional photonic crystal with a line-defect lengthwise and periodic structure in the transverse, and the fiber core is the line-defect that damages the periodic structure. The most important characteristic of this photonic crystal is the existence of a photonic band-gap (PBG), which enables light-waves with a specific wavelength range to be limited in the fiber core and to spread along the lengthwise direction. Negative refraction materials are artificial electromagnetic materials with negative permittivity ($\varepsilon < 0$) and permeability ($\mu < 0$) respectively, which have many new features and attract more interest. When the electromagnetic wave transmits in this material, the electric field ***E***, the magnetic field ***H*** and the wave vector ***k*** observe the left-handed rule [2–4].

Research shows that the photonic band-gap is very narrow and the range of the optical wavelength transmitted is very narrow in the traditional square lattice air-holes photonic crystal fiber. Usually, people study the photonic crystal fiber based on the triangular-structure or honeycomb-structure. In this article, by introducing a medium cylinder with the same radius and the refractive index $n = -1$ in the traditional square lattice air-holes photonic crystal fiber, a new square non-bravais lattice photonic crystal fiber is proposed. The numerical results show that the non-bravais lattice photonic crystal fiber can transmit light signal by the photonic band-gap and have excellent properties, compared with the traditional square lattice air-holes photonic crystal fiber. When the air-holes or medium-column filling ratio equals to a certain value, wide ranges of optical wavelength transmitted in the non-bravais lattice photonic crystal fiber can be obtained.

2 Experimental model and theoretical analysis

First, we introduce the experiment model involved in this article briefly. Figure 1(a) gives the X-Y section of a simple square lattice photonic crystal fiber. Generally, the air-holes with a radius R periodically distribute on the cladding substrate, and the lattice constant is Λ. Therefore, the air-hole filling ratio is $f = \pi R^2/\Lambda^2$. Figure 1(b) gives the X-Y section of the square non-bravais lattice photonic crystal fiber, which is a simple square lattice air-hole photonic crystal fiber nesting another simple square lattice medium-column photonic crystal fiber. The cross-section shape of the air-holes or the medium-columns is circular, of which one is the radius of the air-hole whose refractive index is 1, and the other is the radius of the medium-columns whose refractive index is -1. The lattice constant (the centre to

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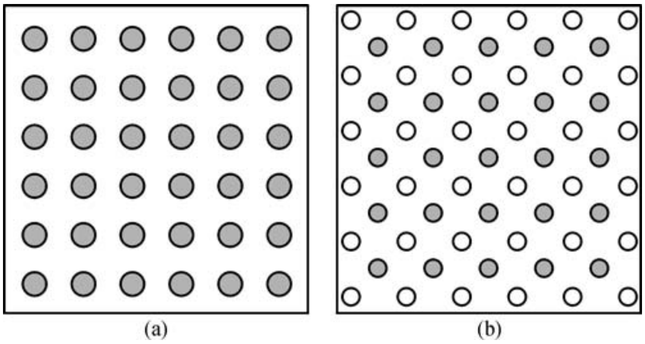


Fig. 1 (a) Structure of traditional square lattice photonic crystal; (b) structure of non-bravais lattice photonic crystal

centre of the adjacent air-hole or medium-column) is Λ; the air-holes filling ratio is $f_1 = \pi R_1^2/\Lambda^2$, while the medium-column filling ratio is $f_2 = \pi R_2^2/\Lambda^2$. By changing the value of R_1/Λ and R_2/Λ or the refractive index of the cladding structure SiO_2, a good photonic band-gap will be obtained.

The plane wave method (PWM) is often used for the numerical simulation of photonic crystals modeling [5]. By the electromagnetic theory, when the distribution of $\varepsilon(\boldsymbol{r})$ is periodic in a medium, Maxwell's equations can be expressed as

$$\nabla \times \frac{1}{\varepsilon(\boldsymbol{r})} \nabla \times \boldsymbol{H}(\boldsymbol{r}) = \frac{\omega^2}{c^2} \boldsymbol{H}(\boldsymbol{r}). \tag{1}$$

Equation (1) has solutions only in a certain frequency ω, that is, the existence of the band-gap. By the waveguide, with implementation of guiding light with the band-gap, the propagation constant β needs to satisfy [6]:

$$\beta\Lambda \leqslant n_{\mathrm{cor}} k_z \Lambda, \tag{2}$$

n_{cor} is the refractive index of the fiber core, k_z is the component of the wave vector in the direction Z, and the implementation of the light guided by the line-defect must simultaneously satisfy the two conditions that the light frequency falls on the photonic band-gap and that Eq. (2) is met.

The PWM is at nature to decompose the electromagnetic wave into sets of plane waves in the reciprocal lattice space, thus, the equivalent form of Eq. (1) can be

$$\sum_{\boldsymbol{G}'} |\boldsymbol{k}+\boldsymbol{G}||\boldsymbol{k}+\boldsymbol{G}'||\varepsilon^{-1}(\boldsymbol{G}-\boldsymbol{G}')| \times \begin{bmatrix} \hat{e}_2\hat{e}_2' & -\hat{e}_2\hat{e}_1' \\ -\hat{e}_1\hat{e}_2' & \hat{e}_1\hat{e}_1' \end{bmatrix} \begin{bmatrix} h_1' \\ h_2' \end{bmatrix} = \frac{\omega^2}{c^2} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}. \tag{3}$$

Equation (3) is a standard eigenvalue equation, and $\boldsymbol{G}$ can be any reciprocal lattice vector. By the numerical method, we obtain a series of eigenvalues at each special wave vector $\boldsymbol{k}$, and form the structure of the photonic band-gap. The key to solve Eq. (3) is to solve $\varepsilon(\boldsymbol{G})$. We defined Fourier coefficient in the reciprocal lattice space as [7]

$$\varepsilon(\boldsymbol{G}) = \frac{1}{S}\int_s \varepsilon(\boldsymbol{r})\, \mathrm{e}^{-\mathrm{i}\boldsymbol{G}\cdot\boldsymbol{r}} \mathrm{d}\boldsymbol{r}, \tag{4}$$

where S is the size of a unit original cell. Under the case of the two-dimensional photonic crystal with the compound lattice structure, Eq. (4) can be simplified as

$$\varepsilon(\boldsymbol{G}) = \begin{cases} \varepsilon_b + f_1(\varepsilon_a - \varepsilon_b) + f_2(\varepsilon_c - \varepsilon_b), & \boldsymbol{G} = \boldsymbol{0}, \\ (\varepsilon_a - \varepsilon_b) I_1(\boldsymbol{G}) + (\varepsilon_c - \varepsilon_b) I_2(\boldsymbol{G}), & \boldsymbol{G} \neq \boldsymbol{0}, \end{cases} \tag{5}$$

where ε_a, ε_b, ε_c represent the air-hole, the cladding substrate materials and the medium-column's dielectric constant, respectively. We define the geometric factor as

$$I(\boldsymbol{G}) = \frac{1}{S}\int_{S_r} \mathrm{e}^{-\mathrm{i}\boldsymbol{G}\cdot\boldsymbol{r}} \mathrm{d}\boldsymbol{r}. \tag{6}$$

When we discuss the simple square lattice air-holes photonic crystal fiber, the geometric factor $I_1(\boldsymbol{G})$ equals $(1/S)2\pi G^{-1} R_1 J_1(GR_1)$. However, when the simple square lattice air-holes photonic crystal fiber nests another medium-column with a dielectric constant ε_c, the geometric factor $I_2(\boldsymbol{G})$ changes in the corner compared with the $I_1(\boldsymbol{G})$ [8–10]. Then, $I_2(\boldsymbol{G}) = (1/S)2\pi \boldsymbol{G}^{-1} R_2 J_1(\boldsymbol{G}R_2) \cdot \cos\alpha$, and $\alpha = (G_x + G_y)\Lambda/2$. In this article, we discuss the case of $R_1 = R_2 = R$, thus, $I_2(\boldsymbol{G}) = I_1(\boldsymbol{G})\cos\alpha$ and $f_1 = f_2 = f$.

For the two-dimensional photonic crystal, we usually assume that the distribution of the dielectric constant is periodic, and is uniform in the direction Z. According to the assumptions above, the dielectric constant in the reciprocal lattice space for Fig. 1(b) structure can be simplified as

$$\begin{cases} \varepsilon_{0,0} = \varepsilon_b + f(\varepsilon_a - \varepsilon_b) + f(\varepsilon_c - \varepsilon_b), & \boldsymbol{G} = \boldsymbol{0}, \\ \varepsilon_{G_x,G_y} = (\varepsilon_a - \varepsilon_b) f \dfrac{J_1(\boldsymbol{G}R)}{\boldsymbol{G}R} + (\varepsilon_c - \varepsilon_b) f \dfrac{J_1(\boldsymbol{G}R)}{\boldsymbol{G}R} \cos\alpha, & \boldsymbol{G} \neq \boldsymbol{0}, \end{cases} \tag{7}$$

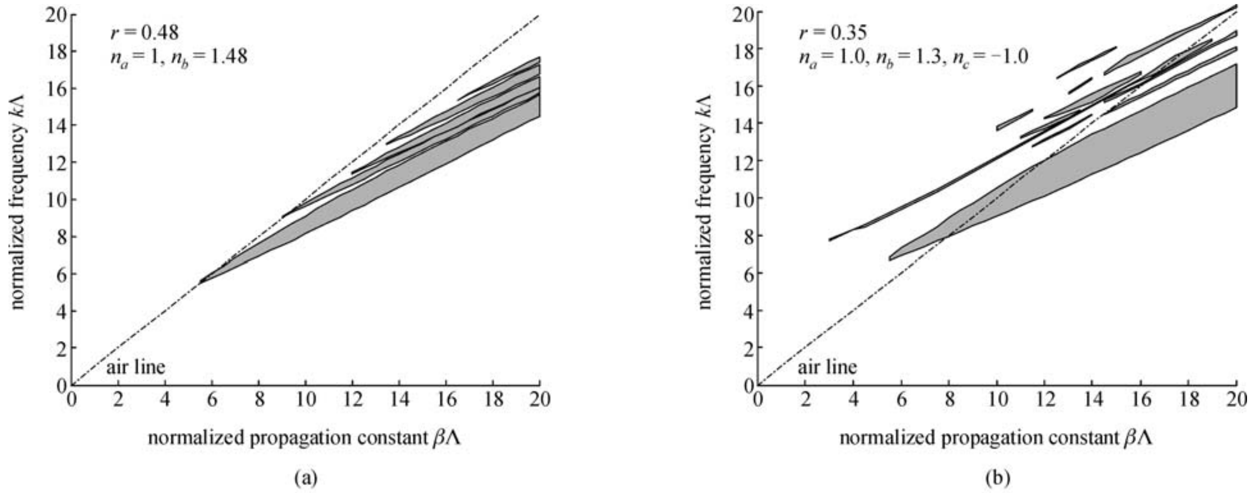
where G_x and G_y are the components of $\boldsymbol{G}$ along the directions X and Y, respectively, and $\boldsymbol{G}$'s value equals to $|\boldsymbol{G}|$.

3 Simulation results and discussion

First, we defined $\boldsymbol{r}$ as R/Λ in the following band-gap diagram, where R is the radius of the air-holes or medium-columns. Figure 2(a) gives the band-gap diagram of the traditional square lattice air-holes photonic crystal fiber, where $R/\Lambda = 0.48$, $n_a = 1.0$, $n_b = 1.48$, and the cladding number is 5. Figure 2(b) gives the band-gap diagram of the non-bravais lattice photonic crystal fiber, where $R_1/\Lambda = R_2/\Lambda = 0.35$, $n_a = 1.0$, $n_b = 1.30$, $n_c = -1.0$, and the cladding number is 5. n_a, n_b, and n_c are the refractive index of air-hole, the cladding substrate, and medium-column respectively. We know that the transmittable optical wavelength range is the overlap of the band-gap and the air-line in the PBG-PCFs. It is clear that the transmittable optical wavelength range of the traditional square lattice PBG-PCFs is very small, while that of the non-bravais lattice PBG-PCFs has a great one. Figure 3 gives the band-gap diagram of the non-bravais lattice photonic crystal fiber with different refractive index of the cladding substrate. We find that the band-gap structure shifts to the right, where $R_1/\Lambda = R_2/\Lambda = 0.35$, $n_a = 1.0$, $n_c = -1.0$, and $n_b = 1.40$, 1.30, respectively. Figure 4 gives the band-gap diagram of the non-bravais lattice photonic crystal fiber with different R/Λ. It is found that the change of the value of R/Λ has a great influence on the band-gap structure.

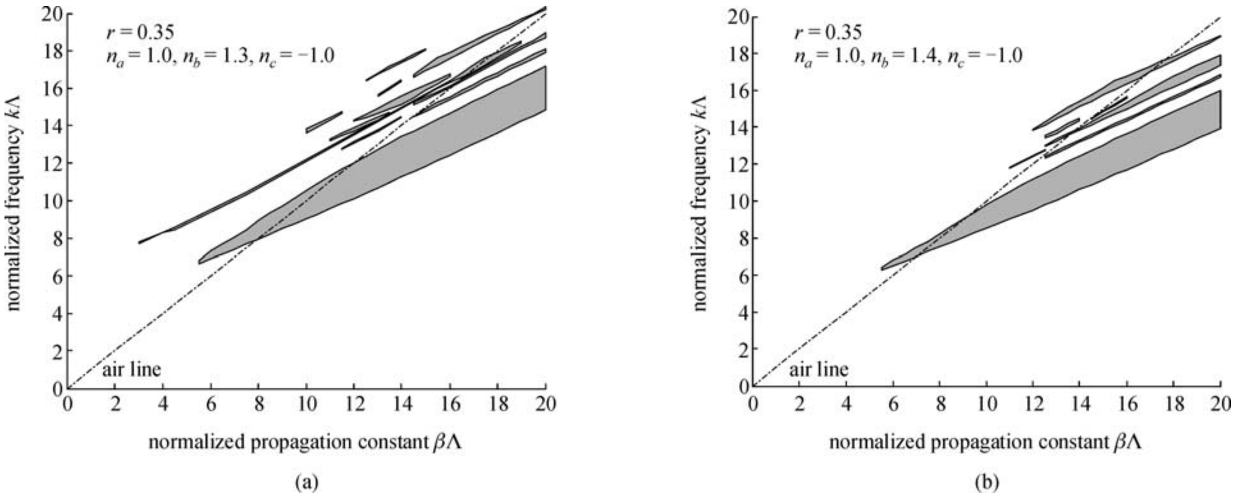
4 Conclusions

In this article, based on three different refractive index materials, we put forward a novel kind of PBG-PCF, i.e., the square non-bravais lattice photonic crystal fiber constituted by the periodic arrangement of the air-hole $n = 1$ and the negative medium-column $n = -1$ ordering on the cladding substrate with $n = 1.30$. We study the photonic band-gap of the PBG-PCF, and compare it with the traditional square lattice air-holes PBG-PCF. The simulation results show that this square non-bravais lattice PBG-PCF has a wide range of optical transmittable wavelength and adjustable characteristics. Besides, it can create conditions for the realization of the full-band single-



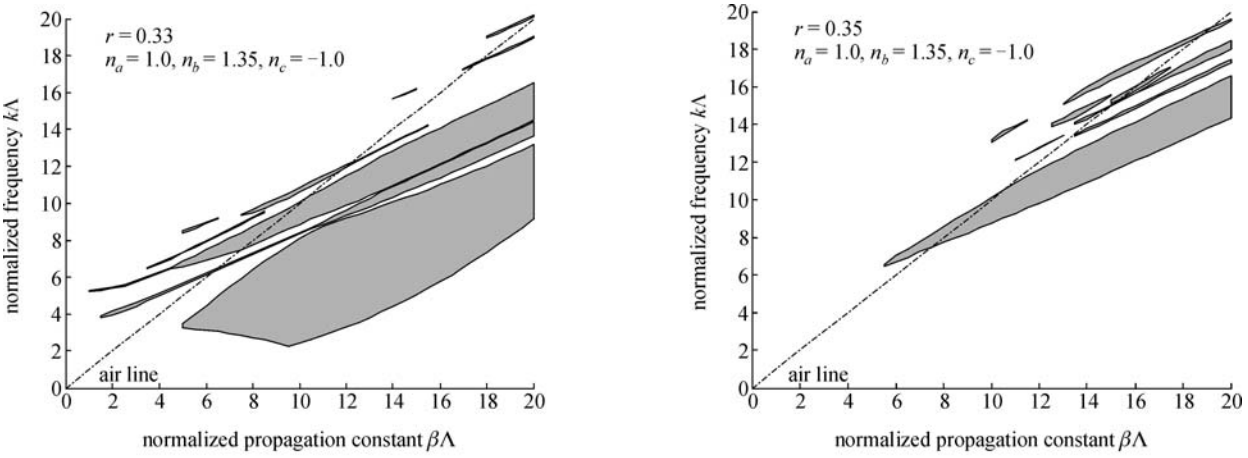
(a) (b)

Fig. 2 Band-gap diagram of photonic crystal fiber. (a) Traditional square lattice; (b) band-gap of non-bravais lattice PCFs



(a) (b)

Fig. 3 Band-gap of non-bravais lattice PCFs with different refractive index of substrate. (a) $n_b = 1.30$; (b) $n_b = 1.40$



(a) (b)

Fig. 4 Band-gap of non-bravais lattice PCFs with different R/Λ. (a) $R/\Lambda = 0.33$; (b) $R/\Lambda = 0.35$

mode working, lay theoretical foundations for an all-optical communications transmission system in the future, and also demonstrates that breaking the symmetry of the photonic crystal will result in a broader photonic band-gap.

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