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Parameter estimation for MIMO system based on MUSIC and ML methods

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Abstract The frequency offset and channel gain estimation problem for multiple-input multiple-output (MIMO) systems in the case of flat-fading channels is addressed. Based on the multiple signal classification (MUSIC) and the maximum likelihood (ML) methods, a new joint estimation algorithm of frequency offsets and channel gains is proposed. The new algorithm has three steps. A subset of frequency offsets is first estimated with the MUSIC algorithm. All frequency offsets in the subset are then identified with the ML method. Finally, channel gains are calculated with the ML estimator. The algorithm is a one-dimensional search scheme and therefore greatly decreases the complexity of joint ML estimation, which is essentially a multi-dimensional search scheme.

Keywords multiple-input multiple-output (MIMO), multiple signal classification (MUSIC), frequency offsets, channel estimation, maximum likelihood (ML) estimation

1 Introduction

Recent research on information theory has shown that a multiple-input multiple-output (MIMO) communication system can efficiently increase system capacity in a rich scattering wireless channel environment without additional power and frequency spectrum requirements needed by traditional techniques [1]. Space-time coding has been proposed to achieve both diversity gain and coding gain in MIMO systems [2,3]. However, the effectiveness of space-time coding schemes is based on the assumption that accurate channel state information (CSI) [4,5] is known perfectly at the receiver, which is impossible in practice. Furthermore, the performance of such MIMO systems may

be seriously degraded in the presence of frequency offsets that are unavoidably present because of poor synchronization between the transmitter and the receiver as well as motion-induced Doppler shifts. Therefore, channel estimation and frequency offset estimation are crucial to MIMO systems.

The authors assume that only a single frequency offset exists between transmitting and receiving antennas in many previous published articles. However, in most cases this assumption is unrealistic, because carrier frequency mismatches or Doppler shifts are different. Thus, the frequency offsets between different transmitting-receiving pairs are not identical. In this article, we consider a more general model in which $T \times R$ frequency offsets are assumed to exist in an MIMO system with T transmitting antennas and R receiving antennas. The parameter estimation methods for such a model have been addressed in Refs. [6,7]. In Ref. [6], Besson and Stoica discussed maximum likelihood (ML) estimation and showed that the optimal technique is a T -dimensional maximization problem, which is rather computationally complex. They proposed a low-complexity method. However, this method is bandwidth-wasting since it requires that training sequences (TSs) are not simultaneously transmitted across all transmitting antennas. Furthermore, the power of the sequences needs to be increased to achieve performance similar to the case when the training sequences are transmitted across all transmitting antennas, which greatly increases the dynamic range requirement of the power amplifier. In Ref. [7], Yao and Ng presented a correlation-based estimator with low computation complexity. However, the estimation range is smaller than the ML method and the mean-squared error (MSE) of the estimator exhibits an error floor because of multi-antenna interference. The ‘error floor’ means that when signal-to-noise ratio (SNR) is greater than some values, the estimate performance cannot be improved any more with higher SNR.

In this article, based on the multiple signal classification (MUSIC) and ML methods, we propose a new frequency offset and channel gain estimation algorithm for the

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general model. This algorithm has a lower computation complexity than the method in Ref. [6] and relaxes the constraint of training sequences in Ref. [6]. Furthermore, it extends the frequency offset estimation range to $[-\pi, \pi]$ without the error floor in Ref. [7].

2 System model

The base band MIMO system considered in this article has T transmitting antennas and R receiving antennas in a flat-fading channel, and perfect timing synchronization is assumed. The output of the k th receive antenna at time n can be written as

$$y_k(n) = \sum_{l=1}^T h_{k,l} e^{j n \omega_{k,l}} x_l(n) + z_k(n), \quad n = 0, \dots, N-1, \quad (1)$$

where $x_l(n)$ is the n th transmitting symbol via the l th transmitting antenna. $h_{k,l}$ and $\omega_{k,l}$ are the channel gain and the frequency offset between the l th transmitting antenna and the k th receiving antenna respectively. The channel gains and the frequency offsets are assumed to be unknown constants that do not change over the interval $[0, 1, \dots, N-1]$. $z_k(n)$ is complex Gaussian noise with zero mean and variance σ^2 . Equation (1) can be written in compact matrix form as

$$\mathbf{Y}_k = \mathbf{X}_{\omega_k} \mathbf{h}_k + \mathbf{Z}_k, \quad (2)$$

where

$$\mathbf{Y}_k = [y_k(0) \ y_k(1) \ \dots \ y_k(N-1)]^T, \quad \mathbf{h}_k = [h_{k,1} \ h_{k,2} \ \dots \ h_{k,T}]^T,$$

$$\mathbf{Z}_k = [z_k(0) \ z_k(1) \ \dots \ z_k(N-1)]^T, \quad \boldsymbol{\omega}_k = [\omega_{k,1} \ \omega_{k,2} \ \dots \ \omega_{k,T}]^T$$

and

$$\mathbf{X}_{\omega_k} = \begin{bmatrix} x_1(0) & x_2(0) & \dots & x_T(0) \\ x_1(1)e^{j\omega_{k,1}} & x_2(1)e^{j\omega_{k,2}} & \dots & x_T(1)e^{j\omega_{k,T}} \\ \vdots & \vdots & \ddots & \vdots \\ x_1(N-1)e^{j(N-1)\omega_{k,1}} & x_2(N-1)e^{j(N-1)\omega_{k,2}} & \dots & x_T(N-1)e^{j(N-1)\omega_{k,T}} \end{bmatrix}$$

The ML estimation of $\boldsymbol{\omega}_k$ and \mathbf{h}_k with the known training sequence $\{x_l(n)\}$ is equivalent to minimizing the following metric:

$$\Lambda = \sum_{k=1}^R \|\mathbf{Y}_k - \mathbf{X}_{\omega_k} \mathbf{h}_k\|^2. \quad (3)$$

Since the term in the sum of Eq. (3) depends only on $\boldsymbol{\omega}_k$ and \mathbf{h}_k , each of these can be minimized independently. For each receiving antenna, the following item is to be minimized:

$$\Lambda_k = \|\mathbf{Y}_k - \mathbf{X}_{\omega_k} \mathbf{h}_k\|^2. \quad (4)$$

And for a given $\boldsymbol{\omega}_k$, the minimizer of Eq. (4) with respect to \mathbf{h}_k is

$$\hat{\mathbf{h}}_k = (\mathbf{X}_{\omega_k}^H \mathbf{X}_{\omega_k})^{-1} \mathbf{X}_{\omega_k}^H \mathbf{Y}_k. \quad (5)$$

By inserting Eq. (5) into Eq. (4), the frequency offset estimation is obtained as

$$\hat{\boldsymbol{\omega}}_k = \arg \max_{\boldsymbol{\omega}} \mathbf{Y}_k^H \mathbf{X}_{\omega_k} (\mathbf{X}_{\omega_k}^H \mathbf{X}_{\omega_k})^{-1} \mathbf{X}_{\omega_k}^H \mathbf{Y}_k. \quad (6)$$

Note that Eq. (6) is a complicated T -dimensional minimization problem. Therefore, we need to find a simpler method to solve this problem.

3 Joint frequency offsets and channel estimation based on MUSIC and ML methods

In this section, we briefly discuss the TS design and then introduce the proposed joint frequency offset and channel gain estimation algorithm. First, we design a special TS based on space time equivalence theory of the MUSIC algorithm [8]. The structure of TS is shown in Fig. 1.

In Fig. 1, j_BLOCK_p is the p th transmitted symbol block via the j th transmitting antenna, L is the length of each symbol block and N is the length of TS. The number of symbol blocks is $M = N/L$. Tx $_i$ denotes the i th transmitting antenna. According to the space time equivalence theory, the following conclusions can be drawn:

1) L is equivalent to the number of elements for the uniform linear array (ULA). It is well known that the number of signals must be less than that of elements according to the subspace-based method, which means that

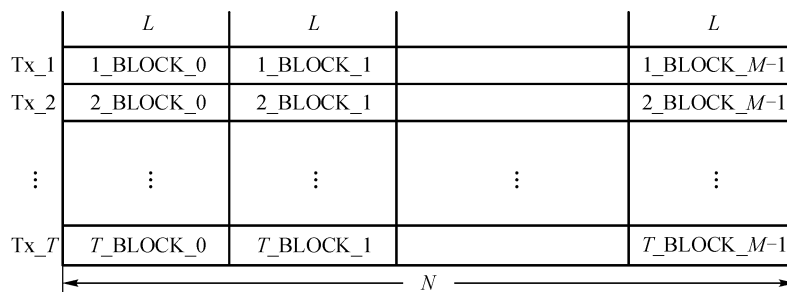


Fig. 1 Structure of training sequence

L should be greater than the number of transmitting antenna, i.e., $L > T$.

2) Signal is assumed to be narrowband and non-coherent in spatial spectrum estimation, which means that the received signals belonging to the same signal source are coherent at all elements and those belonging to different signal sources are non-coherent. Thus, it is necessary that at any given time the transmitted training symbols via a certain transmitting antenna in a symbol block should be identical and that those are different when transmitted via different transmitting antennas for a MIMO system. This condition can be satisfied, for instance, if we choose such training sequences as

$$x_l(mL) = x_l(mL + 1) = \dots = x_l(mL + L - 1),$$

$$x_l(mL) \neq x_q(mL), \quad l \neq q, \quad m = 0, 1, \dots, M-1, \quad l = 1, 2, \dots, T.$$

According to the designed TS, the m th symbol block received by the k th receiving antenna can be written as

$$\mathbf{Y}_{km} = \mathbf{A}_{\omega_k} \mathbf{S}_m + \mathbf{Z}_{km} = \mathbf{A}_{\omega_k} (\mathbf{X}_m \odot \mathbf{h}_k \times \mathbf{W}_m) + \mathbf{Z}_{km}, \quad (7)$$

where

$$\mathbf{Y}_{km} = [y_k(mL) \quad y_k(mL + 1) \quad \dots \quad y_k(mL + L - 1)]^T,$$

$$\mathbf{W}_m = e^{j m L (\omega_{k,1} + \omega_{k,2} + \dots + \omega_{k,T})},$$

$$\mathbf{Z}_{km} = [z_k(mL) \quad z_k(mL + 1) \quad \dots \quad z_k(mL + L - 1)]^T,$$

$$\mathbf{h}_k = [h_{k,1} \quad h_{k,2} \quad \dots \quad h_{k,T}]^T,$$

$$\mathbf{X}_m = [x_1(mL) \quad x_2(mL) \quad \dots \quad x_T(mL)]^T,$$

$$\boldsymbol{\alpha}(\omega) = [1 \quad e^{j\omega} \quad \dots \quad e^{j(L-1)\omega}]^T,$$

and

$$\begin{aligned} \mathbf{A}_{\omega_k} &= [\boldsymbol{\alpha}(\omega_{k,1}) \quad \boldsymbol{\alpha}(\omega_{k,2}) \quad \dots \quad \boldsymbol{\alpha}(\omega_{k,T})] \\ &= \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{j\omega_{k,1}} & e^{j\omega_{k,2}} & \dots & e^{j\omega_{k,T}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j(L-1)\omega_{k,1}} & e^{j(L-1)\omega_{k,2}} & \dots & e^{j(L-1)\omega_{k,T}} \end{bmatrix} \end{aligned}$$

is a Vandermonde matrix.

Define $\bar{\mathbf{Y}}_k = [\mathbf{Y}_{k0} \quad \mathbf{Y}_{k1} \quad \dots \quad \mathbf{Y}_{k(M-1)}]$, $\bar{\mathbf{Z}}_k = [\mathbf{Z}_{k0} \quad \mathbf{Z}_{k1} \quad \dots \quad \mathbf{Z}_{k(M-1)}]$ and $\mathbf{S} = [\mathbf{S}_0 \quad \mathbf{S}_1 \quad \dots \quad \mathbf{S}_{M-1}]$. We have

$$\bar{\mathbf{Y}}_k = \mathbf{A}_{\omega_k} \mathbf{S} + \bar{\mathbf{Z}}_k. \quad (8)$$

The covariance matrix of \mathbf{Y}_{km} is given by

$$\mathbf{R}_k = E[\mathbf{Y}_{km} \mathbf{Y}_{km}^H] = \mathbf{A}_{\omega_k} \boldsymbol{\Phi} \mathbf{A}_{\omega_k}^H + \sigma^2 \mathbf{I}, \quad (9)$$

where \mathbf{R}_k is an $L \times L$ Hermitian matrix, $\boldsymbol{\Phi} = E[\mathbf{S}_m \mathbf{S}_m^H]$ is

the covariance matrix of \mathbf{S}_m and \mathbf{I} is an $L \times L$ identity matrix. The Eigenvalue decomposition (EVD) of \mathbf{R}_k is

$$\mathbf{R}_k = [\mathbf{U}_S \quad \mathbf{U}_Z] \begin{bmatrix} \boldsymbol{\Sigma}_S & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_Z \end{bmatrix} \begin{bmatrix} \mathbf{U}_S^H \\ \mathbf{U}_Z^H \end{bmatrix}, \quad (10)$$

where $\boldsymbol{\Sigma}_S = \text{Diag}[\lambda_1, \lambda_2, \dots, \lambda_T]$, $\boldsymbol{\Sigma}_Z = \text{Diag}[\lambda_{T+1}, \lambda_{T+2}, \dots, \lambda_L]$ and $\lambda_{T+1} = \lambda_{T+2} = \dots = \lambda_L = \sigma^2$. \mathbf{U}_S is an $L \times T$ matrix composed of T eigenvectors corresponding to $\lambda_1, \lambda_2, \dots, \lambda_T$ and \mathbf{U}_Z is an $L \times (L-T)$ matrix composed of $L-T$ eigenvectors corresponding to $\lambda_{T+1}, \lambda_{T+2}, \dots, \lambda_L$. Since \mathbf{U}_S and \mathbf{U}_Z are both unitary matrices and orthogonal to each other, $\mathbf{U}_S \mathbf{U}_Z^H = \mathbf{0}$. Multiplying Eq. (10) by \mathbf{U}_Z , we have

$$\begin{aligned} \mathbf{R}_k \mathbf{U}_Z &= [\mathbf{U}_S \quad \mathbf{U}_Z] \begin{bmatrix} \boldsymbol{\Sigma}_S & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_Z \end{bmatrix} \begin{bmatrix} \mathbf{U}_S^H \\ \mathbf{U}_Z^H \end{bmatrix} \mathbf{U}_Z \\ &= [\mathbf{U}_S \quad \mathbf{U}_Z] \begin{bmatrix} \boldsymbol{\Sigma}_S & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_Z \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} = \sigma^2 \mathbf{U}_Z. \end{aligned} \quad (11)$$

Since $\mathbf{R}_k = \mathbf{A}_{\omega_k} \boldsymbol{\Phi} \mathbf{A}_{\omega_k}^H + \sigma^2 \mathbf{I}$, we can get $\mathbf{R}_k \mathbf{U}_Z = \mathbf{A}_{\omega_k} \boldsymbol{\Phi} \mathbf{A}_{\omega_k}^H \mathbf{U}_Z + \sigma^2 \mathbf{U}_Z$. According to Eq. (11), we have

$$\mathbf{A}_{\omega_k} \boldsymbol{\Phi} \mathbf{A}_{\omega_k}^H \mathbf{U}_Z = \mathbf{0}. \quad (12)$$

Multiplying Eq. (12) by \mathbf{U}_Z^H :

$$\mathbf{U}_Z^H \mathbf{A}_{\omega_k} \boldsymbol{\Phi} \mathbf{A}_{\omega_k}^H \mathbf{U}_Z = \mathbf{0}, \quad (13)$$

which means [8]

$$\mathbf{A}_{\omega_k}^H \mathbf{U}_Z = \mathbf{0}. \quad (14)$$

Inserting $\mathbf{A}_{\omega_k} = [\boldsymbol{\alpha}(\omega_{k,1}) \quad \boldsymbol{\alpha}(\omega_{k,2}) \quad \dots \quad \boldsymbol{\alpha}(\omega_{k,T})]$ into Eq. (14), we have

$$\boldsymbol{\alpha}^H(\omega) \mathbf{U}_Z = \mathbf{0}, \quad \omega = \omega_{k,1}, \omega_{k,2}, \dots, \omega_{k,T}. \quad (15)$$

Considering that the length of the received data is finite, the covariance matrix can be estimated by

$$\hat{\mathbf{R}}_k = \frac{1}{M} \bar{\mathbf{Y}}_k \bar{\mathbf{Y}}_k^H = \frac{1}{M} \sum_{m=1}^M \mathbf{Y}_{km} \mathbf{Y}_{km}^H. \quad (16)$$

The covariance matrix $\hat{\mathbf{R}}_k$ needs to be preprocessed by the Toeplitz method to improve its estimation performance [9], and the estimation of \mathbf{U}_Z , $\hat{\mathbf{U}}_Z$ can be derived by introducing EVD to $\hat{\mathbf{R}}_k$. Thus, according to Eq. (15), a subset of frequency offsets $\{\omega^{(i)}\}_{i=1}^T$ at the k th receiving antenna correspond to the largest T local maximum values of

$$\frac{1}{\|\boldsymbol{\alpha}^H(\omega) \hat{\mathbf{U}}_Z \hat{\mathbf{U}}_Z^H \boldsymbol{\alpha}(\omega)\|^2}, \quad (17)$$

where $\boldsymbol{\alpha}(\omega) = [1 \quad e^{j\omega} \quad \dots \quad e^{j(L-1)\omega}]^T$.

Table 1 Comparison of four estimators

method	range	complexity	performance	requirement for TS
ML	$[-\pi, \pi]$	T -dimensional search	asymptotic optimal	no requirement
Besson's	$[-\pi, \pi]$	T 1-dimensional search	asymptotic optimal	no overlapping in time domain
Yao's	$[-\pi/P, \pi/P]$	close-form solution	have error floor	allow overlapping (Walsh sequence)
MUSIC + ML	$[-\pi, \pi]$	1-dimensional search	suboptimal	allow overlapping

Note: $P = 2ik$, $i \geq 1$ and k is the length of the correlator in Ref. [7].

Although we can obtain the T estimated frequency offsets by MUSIC at the k th receiving antenna, the match between the estimated frequency offset and each transmitting antenna has not yet been identified. To solve this match problem we resort to ML estimation method in this article. Thus, the proposed estimator is based on the combination of the MUSIC and the ML estimation methods to estimate the frequency offsets. First, the T effective frequency offsets $\{\omega^{(i)}\}_{i=1}^T$ can be estimated by MUSIC at the k th receiving antenna. Second, these frequency offsets are mapped to the T transmitting antenna according to Eq. (6), which is different from T -dimensional search by the ML in a direct way. Here, only finite frequency offsets are searched by the ML to solve the match problem.

In summary, frequency offsets and channel gains are estimated using the proposed algorithm as shown in the following steps.

Step 1 Formulate the received signal samples into matrix form $\bar{\mathbf{Y}}_k$ and calculate $\hat{\mathbf{R}}_k$.

Step 2 Introduce EVD to $\hat{\mathbf{R}}_k$ to find $\hat{\mathbf{U}}_Z$.

Step 3 Find the largest T peaks according to Eq. (17) to estimate $\{\omega^{(i)}\}_{i=1}^T$.

Step 4 Using Eq. (6), the frequency offset of each transmit antenna can be estimated.

Step 5 Substitute the estimated frequency offsets into Eq. (5) to estimate channel gains.

In Table 1, the comparison of the ML method [6], Besson's method [6], Yao's method [7] and the proposed method (MUSIC + ML) is given.

4 Simulation results

In this section, we provide the simulation results of the proposed algorithm. We evaluate a 2×2 MIMO system in a flat fading environment. Without loss of generality, we only present the results associated with the 1st transmitting antenna and the 1st receiving antenna pair. In our simulation, the length of training sequence N is 60, the length of each symbol block L is 5, and the frequency offsets are selected as $\omega_1 = 2\pi[-0.05 \ 0.05]^T$.

Figure 2 shows the mean-square error (MSE) of the frequency offset estimation in the Rayleigh flat fading channel and additive white Gaussian noise (AWGN)

channel for a 2×2 MIMO system. Figure 3 shows the MSE of the channel estimation in the Rayleigh flat fading channel for a 2×2 MIMO system. It can be observed from Figs. 2 and 3 that the performance of frequency offset and channel estimation of the proposed estimator outperforms that of Yao's estimator [7]. It is noted that the MSE of

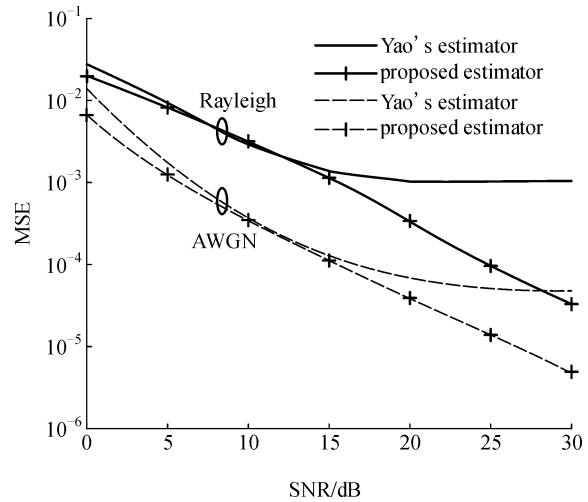


Fig. 2 MSE of frequency offset estimation versus SNR for different estimators

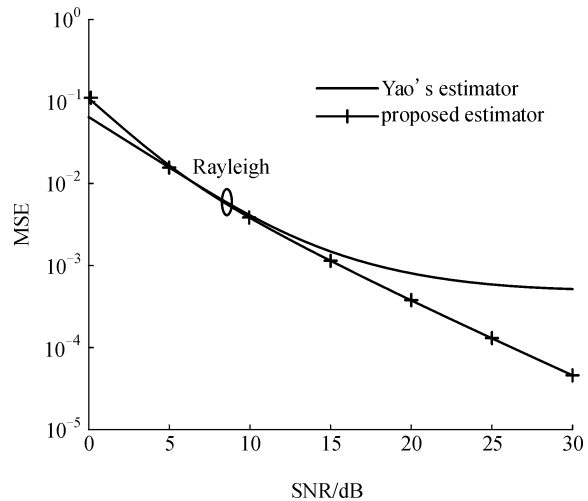


Fig. 3 MSE of channel estimation versus SNR for different estimators

Yao's estimator exhibits an error floor caused by multi-antenna interference in high SNR cases.

Figure 4 shows the MSE of the estimated frequency offset with different lengths of L in the Rayleigh flat fading channel when the length of training sequence N is 60 and the number of transmitting antenna is 2. It can be seen that the MSE of the estimated frequency offset improves with the increase of L . This happens because the resolution of different frequency offsets is raised with the increase of L . Note that given the length of training sequence, the number of transmitted symbol block M will decrease with the increase of L , which will affect estimation accuracy of the covariance matrix. Therefore, the length of symbol block L should be considered well as a tradeoff.

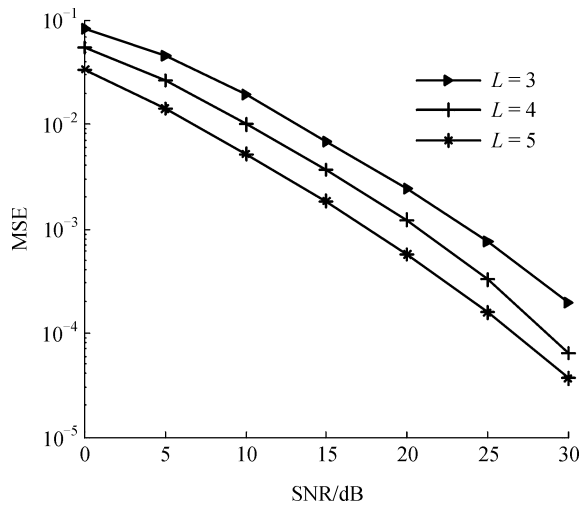


Fig. 4 MSE of frequency offset estimation with different lengths of L

Figure 5 shows the MSE of the estimated channel gain with different lengths of L in the Rayleigh flat fading

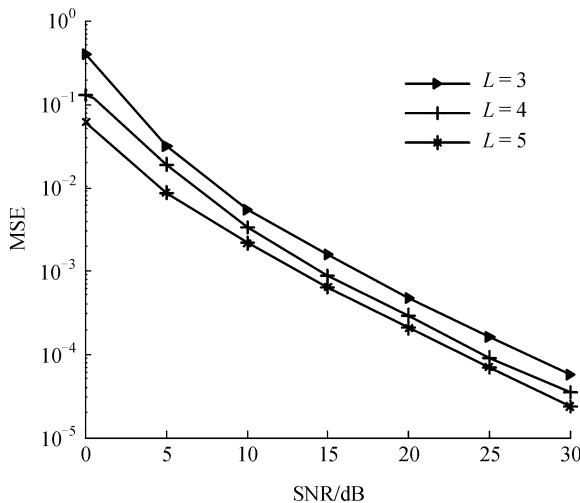


Fig. 5 MSE of channel estimation with different lengths of L

channel when the length of training sequence N is 60 and the number of transmitting antenna is 2. It can be observed that the MSE of the estimated channel gain improves with the increase of L due to the same reason explained above.

5 Conclusions

This article addresses the problem of frequency offset and channel gain estimation for MIMO systems in flat-fading channels. A new frequency offset and channel gain estimation algorithm based on the combination of the MUSIC and the ML estimation methods is proposed and training sequences are designed correspondingly. The proposed algorithm greatly decreases the complexity of the joint ML estimation method and the performance of this algorithm is evaluated by Monte Carlo simulations.

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