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RESEARCH ARTICLE

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Joint optimization in MIMO multiple relay channels

Abstract The joint optimization of cooperative relays in a multiple-input multiple-output (MIMO) multiple relay aided communication system is discussed in this article. A simple linear optimization solution is designed and a multi-user scheduling algorithm is proposed. The new algorithm chooses users with larger cooperation diversity gains from the waiting list to serve; it can reduce inter-user interference and maintain space diversity caused by parallel transmission of multiple relays, and achieve higher system throughput.

Keywords multi-relay cooperation, joint optimization, multi-user scheduling, space diversity

1 Introduction

Wireless relaying has emerged as a powerful technique for high throughput and broad coverage of wireless networks in wide applications [1–3]. By exploiting spatial diversity in a distributed fashion, the relay network has the potential to improve system performance. Different approaches and protocols have been studied in early work. The relay may employ a so-called amplify-and-forward (AF) strategy, which simply amplifies a received signal and forwards it to the destination. The decode-and-forward (DF) strategy is also needed so that the transmitted signals are first decoded at the relay. Other relays such as the selection relay, the estimating relay, the compressing relay and the code cooperation relay were introduced in Refs. [4–7].

Different building blocks of relay networks have emerged for different conditions. A three-node relay network was first introduced in Refs. [1,2]. More recent activities on relays focus on the single-antenna system [8,9] or the multi-antenna system [10,11], respectively. In addition, the model of relay networks has become more complicated. Networks that include multiple relay nodes to simultaneously relay information to the destination and those with multiple levels of relay nodes are hot topics.

It has been shown that system performance can be improved when the channel state information (CSI) is available on the transmitter side. Joint optimization of the relay precoders and decoders has been the subject of many studies [12–14]. In Ref. [15], Yi et al. jointly optimize the relay precoders and decoders of a cooperative network when the CSI is available on the transmitter side. However, the multiple relays are all equipped with a single antenna and the case where a multi-antenna is employed has not been considered. In Ref. [11], the optimal design of non-regenerative multiple-input multiple-output (MIMO) wireless relays is developed to maximize the capacity between the source and the destination. However, the optimization developed in the traditional three-node relay network is not suitable for a multi-relay channel.

In this article, we study a MIMO multi-relay system which employs multiple relays to forward information from the base station to the mobile simultaneously. Multiple antennas are installed at the base station and relays. Because the mobiles are less likely to be equipped with multiple antennas than the base station and relays, only a single antenna communication environment is provided on the mobile.

This article aims to design the precoding, decoding and power loading of relays. The optimization is designed respectively under total CSI (all base-relay channel and relay-user channel state information) or local CSI assumption on the relay side. To fully exploit system capacity, multiple users should be served by multiple relays simultaneously. The optimization of relays under the total CSI and the local CSI assumption is also developed in a multi-user communication environment. Multi-user scheduling technique is also adopted here to further improve system performance.

The article is organized as follows: in Sect. 2 the system model is described and joint optimization of relays with total CSI is developed with many relays and a single source-destination communication pair. The multi-user communication environment is considered in Sect. 3. A joint solution is given with total CSI and a sub-optimal algorithm combined with multi-user scheduling is devel-

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oped under local CSI. Section 4 shows the numerical results which justify the performance gain our protocol can achieve, and the conclusions are given in Sect. 5.

2 System model

In this section, a multi-user MIMO multiple relay channel will be introduced. Based on this model, a relay protocol will be proposed to utilize the space diversity for multi-relay and multi-user transmission in the next section.

The model of a multi-user MIMO relay channel is illustrated in Fig. 1. There is one base station, multiple relays and several users. K relays are employed to assist communication between the base station and users simultaneously. The base station is equipped with M antennas, while the relays are equipped with N antennas. The mobile is equipped with one antenna only. Q is the maximum number of users that can be supported by the system mentioned above and $\overline{Q}$ is the total number of users waiting to be served.

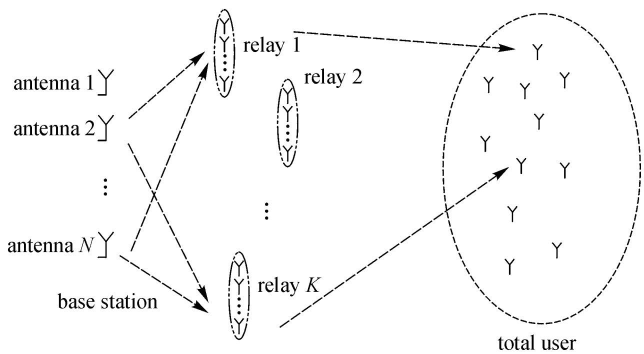

Fig. 1 Multi-user MIMO relay channel model with K relays, a single source and several destinations

The data stream $\boldsymbol{S}=[s_1,s_2,\ldots,s_Q]^{\mathrm{T}}$ is multiplied by a transmit beam-forming matrix $\boldsymbol{F}$ of size $M\times Q$ ($M>Q$). Then the transmit signal $\boldsymbol{X}$ from the base station is $\boldsymbol{X}=\boldsymbol{FS}=[x_1,x_2,\ldots,x_M]^{\mathrm{T}}$.

Let $\boldsymbol{H}_k$ be the channel responses between the source and the kth relay, $\boldsymbol{G}_{qk}$ be the channel responses between the kth relay and the qth user, and $\boldsymbol{W}_k$ be the weighting matrix at the kth relay. The system model is set to be a spatially uncorrelated Rayleigh channel. The signal received by the kth relay is given by

$$\boldsymbol{r}_k=\boldsymbol{H}_k\boldsymbol{X}+\boldsymbol{n}_{1k},\quad k=1,2,\ldots,K, \tag{1}$$

where $\boldsymbol{n}_{1k}$ is Gaussian noise at the kth relay. And the signal received by the qth user is given by

$$Y_q=\sum_{k=1}^{K}\boldsymbol{G}_{qk}\boldsymbol{W}_k\boldsymbol{r}_k+n_{2q}$$

$$=\sum_{k=1}^{K}\boldsymbol{G}_{qk}\boldsymbol{W}_k\boldsymbol{H}_k\boldsymbol{FS}+\sum_{k=1}^{K}\boldsymbol{G}_{qk}\boldsymbol{W}_k\boldsymbol{n}_{1k}+n_{2q},\quad q=1,2,\ldots,Q, \tag{2}$$

where n_{2q} is Gaussian noise at the qth user.

We still assume that the noise at each relay and each user is mutually independent and distributed as $E\{|\boldsymbol{n}_{1k_1}\boldsymbol{n}_{1k_2}^{\mathrm{H}}|\}=\delta_{k_1k_2}\boldsymbol{I}_N$, where $\boldsymbol{I}_N$ is an unitary matrix of size $N\times N$. As long as the channel $\boldsymbol{G}_{qk}$ is an uncorrelated Rayleigh channel, the equivalent noise $\tilde{n}_q=\sum_{k=1}^{K}\boldsymbol{G}_{qk}\boldsymbol{W}_k\boldsymbol{n}_{1k}+n_{2q}$ is a Gaussian variable.

Equation (2) can be reduced as

$$Y_q=\left(\sum_{k=1}^{K}\boldsymbol{G}_{qk}\boldsymbol{W}_k\boldsymbol{H}_k\boldsymbol{F}\right)\boldsymbol{S}+\tilde{n}_q, \tag{3}$$

and the equivalent channel matrix $\left(\sum_{k=1}^{K}\boldsymbol{G}_{qk}\boldsymbol{W}_k\boldsymbol{H}_k\boldsymbol{F}\right)$ must satisfy $\sum_{k=1}^{K}\boldsymbol{G}_{qk}\boldsymbol{W}_k\boldsymbol{H}_k\boldsymbol{F}=[0,\ldots,0,\lambda_q,0,\ldots,0]_{1\times Q}$ to avoid inter-user interference. λ_q is the equivalent channel gain of the qth user. Let $[0,\ldots0,1,0,\ldots,0]_{1\times Q}=\boldsymbol{e}_q$, we can then obtain:

$$\sum_{k=1}^{K}\boldsymbol{G}_{qk}\boldsymbol{W}_k\boldsymbol{H}_k\boldsymbol{F}=\lambda_q\boldsymbol{e}_q. \tag{4}$$

3 Several cooperative designs

In the traditional point to multi-point communication environment, because the received antennas are at different users and receiving has to be operated separately, the pre-coding strategy must be employed on the transmitter side. For a relay-enhanced system, pre-coding should be adopted jointly at the relays.

3.1 Joint solution with total CSI

If the channel state information of $\boldsymbol{H}_k$ and $\boldsymbol{G}_{qk}$ for $k=1,2,\ldots,K, q=1,2,\ldots,Q$ are known at every relay, a series optimal matrix $\boldsymbol{W}_k^{\mathrm{opt}}$ can be found to satisfy $\sum_{k=1}^{K}\boldsymbol{G}_{qk}\boldsymbol{W}_k^{\mathrm{opt}}\boldsymbol{H}_k\boldsymbol{F}=\lambda_q\boldsymbol{e}_q$. In the following, the optimal matrix $\boldsymbol{W}_k^{\mathrm{opt}}$ will be given in detail. A sub-optimal one will also be presented if the kth relay only knows $\boldsymbol{H}_k$, $\boldsymbol{G}_{qk}$ and has no knowledge of $\boldsymbol{H}_{k'}$, $\boldsymbol{G}_{qk'}$, for $q=1,2,\ldots,Q$, $k'=1,2,\ldots,K$, and $k'\neq k$.

$\boldsymbol{G}_{qk}=[G_{qk}(1),G_{qk}(2),\ldots,G_{qk}(N)]$ is a vector of $1\times N$. Let $\tilde{\boldsymbol{G}}=[\boldsymbol{G}_1,\boldsymbol{G}_2,\ldots,\boldsymbol{G}_Q]^{\mathrm{T}}$, $\tilde{\boldsymbol{P}}=[\boldsymbol{P}_1,\boldsymbol{P}_2,\ldots,\boldsymbol{P}_K]^{\mathrm{T}}$ and

$$\tilde{\boldsymbol{G}}_q = [\boldsymbol{G}_{q1},\boldsymbol{G}_{q2},...,\boldsymbol{G}_{qK}] = [G_{q1}(1),...,G_{q1}(N),G_{q2}(1),...,G_{q2}(N),..., G_{qK}(1),...,G_{qK}(N)], \tag{5}$$

where $\boldsymbol{P}_k = \boldsymbol{W}_k\boldsymbol{H}_k\boldsymbol{F}$ is a matrix of $N \times Q$. Equation (4) can be stated as

$$\tilde{\boldsymbol{G}}_q\tilde{\boldsymbol{P}} = \lambda_q\boldsymbol{e}_q,\ q = 1,2,...,Q, \tag{6}$$

i.e., $\tilde{\boldsymbol{G}}\tilde{\boldsymbol{P}} = \boldsymbol{\Lambda}\boldsymbol{I}_Q$. $\boldsymbol{\Lambda}$ is a diagonal with $\boldsymbol{\Lambda} = \mathrm{diag}(\lambda_1,\lambda_2,..., \lambda_Q)$.

To avoid inter-user interference, $\tilde{\boldsymbol{P}}$ must satisfy $\tilde{\boldsymbol{P}} = \tilde{\boldsymbol{G}}^{+}\boldsymbol{\Lambda}$. $\tilde{\boldsymbol{G}}^{+}$ is the pseudo inverse of matrix $\tilde{\boldsymbol{G}}$.

$\tilde{\boldsymbol{G}}$ is a matrix of $Q \times KN$, and $\tilde{\boldsymbol{G}}^{+}$ is a matrix of $KN \times Q$, and $\tilde{\boldsymbol{G}}^{+}$ can be divided into a series of matrix $\left(\tilde{\boldsymbol{G}}^{+}\right)_{(k)}$ of $N \times Q$ with $\tilde{\boldsymbol{G}}^{+} = \left[\left(\tilde{\boldsymbol{G}}^{+}\right)_{(1)},\left(\tilde{\boldsymbol{G}}^{+}\right)_{(2)},...,\left(\tilde{\boldsymbol{G}}^{+}\right)_{(K)}\right]^{\mathrm{T}}$. Because $\boldsymbol{\Lambda}$ is a diagonal, $\boldsymbol{P}_k$ can be expressed as

$$\boldsymbol{P}_k = \left(\tilde{\boldsymbol{G}}^{+}\right)_{(k)}\boldsymbol{\Lambda}. \tag{7}$$

Inserting $\boldsymbol{P}_k = \boldsymbol{W}_k\boldsymbol{H}_k\boldsymbol{F}$ into Eq. (7), we can obtain the matrix $\boldsymbol{W}_k^{\mathrm{opt}}$:

$$\boldsymbol{W}_k^{\mathrm{opt}} = \left(\tilde{\boldsymbol{G}}^{+}\right)_{(k)}\boldsymbol{\Lambda}(\boldsymbol{H}_k\boldsymbol{F})^{+}. \tag{8}$$

The matrix $\boldsymbol{W}_k^{\mathrm{opt}}$ could be solved if every relay knows the total CSI, but this condition is always hard to satisfy. In most cases, the kth relay only knows $\boldsymbol{H}_k$, $\boldsymbol{G}_{qk}$ and has no knowledge of $\boldsymbol{H}_{k'}$, $\boldsymbol{G}_{qk'}$, for $q = 1,2,...,Q$, $k' = 1,2,...,K$, and $k' \neq k$. The solution to a sub-optimal strategy under that condition is applicable.

3.2 Sub-optimal solution with local CSI

From Eq. (8), we can see that the operation on the relay side can be divided into three independent processes: signal receiving, power allocating and pre-processing, which correspond to the matrixes $(\boldsymbol{H}_k\boldsymbol{F})^{+}$, $\boldsymbol{\Lambda}$ and $\left(\tilde{\boldsymbol{G}}^{+}\right)_{(k)}$ respectively. We consider the case that the kth relay only knows $\boldsymbol{H}_k$, $\boldsymbol{G}_{qk}$ and has no knowledge of $\boldsymbol{H}_{k'}$, $\boldsymbol{G}_{qk'}$, for $q = 1,2,...,Q$, $k' = 1,2,...,K$, and $k' \neq k$. Because each relay receives the data stream parallel, the receiving matrix remains as $(\boldsymbol{H}_k\boldsymbol{F})^{+}$. According to different objectives, the power allocating matrix differs considerably. For example, the matrix $\boldsymbol{\Lambda}$ can be designed to minimize the bit error rate the system can get. In another case, it can be designed to maximize the average rate the system can achieve. Specially, if each relay transmits the streams with equal power, the $\boldsymbol{\Lambda}$ can be reduced to a identity matrix $\boldsymbol{I}$.

Let

$$\overline{\boldsymbol{G}}_k = \begin{bmatrix} G_{1k}(1) & G_{1k}(2) & \cdots & G_{1k}(N) \\ G_{2k}(1) & G_{2k}(2) & \cdots & G_{2k}(N) \\ \vdots & \vdots & \ddots & \vdots \\ G_{\overline{Q}k}(1) & G_{\overline{Q}k}(2) & \cdots & G_{\overline{Q}k}(N) \end{bmatrix}$$

be the channel response from the kth relay to $\overline{Q}$ users. Using singular value decomposition (SVD), we can get

$$\overline{\boldsymbol{G}}_k = \boldsymbol{U}_k\boldsymbol{\Lambda}_k\boldsymbol{V}_k. \tag{9}$$

Let

$$\tilde{\boldsymbol{G}}_k = \begin{bmatrix} G_{1k}(1) & G_{1k}(2) & \cdots & G_{1k}(N) \\ G_{2k}(1) & G_{2k}(2) & \cdots & G_{2k}(N) \\ \vdots & \vdots & \ddots & \vdots \\ G_{Qk}(1) & G_{Qk}(2) & \cdots & G_{Qk}(N) \end{bmatrix}$$

be the channel response from the kth relay to Q users that are simultaneously supported by the system. The channel response matrix $\tilde{\boldsymbol{G}}_k$ can be expressed as

$$\tilde{\boldsymbol{G}}_k = \tilde{\boldsymbol{U}}_k\boldsymbol{\Lambda}_k\tilde{\boldsymbol{V}}_k, \tag{10}$$

where $\tilde{\boldsymbol{U}}_k$ satisfies $\tilde{\boldsymbol{U}}_k\tilde{\boldsymbol{U}}_k^{\mathrm{H}} = \boldsymbol{I}$.

In the appendix we demonstrate that $\boldsymbol{V}_k = \tilde{\boldsymbol{V}}_k$, as long as $\overline{Q}$, Q and N satisfy $\overline{Q} > Q > N$.

If we use $\boldsymbol{V}_k$ as the pre-processing matrix, the $\boldsymbol{W}_k$ can be expressed as

$$\boldsymbol{W}_k = \boldsymbol{V}_k\boldsymbol{\Lambda}(\boldsymbol{H}_k\boldsymbol{F})^{-1}. \tag{11}$$

It can be seen that $\boldsymbol{V}_k$ is the matched beam-forming matrix from the kth relay to Q users. In a single user environment, $\boldsymbol{V}_k$ is the equivalent optimal pre-processing matrix. In a multi-user environment, because of multi-user interference, $\boldsymbol{V}_k$ is not the optimal pre-processing matrix.

3.3 Multi-user scheduling for sub-optimal solution with local CSI

We have demonstrated that in a single user environment, the processing matrix $\boldsymbol{W}_k$ can be designed with the local CSI. It will be more complicated in a multi-user environment. Although $\boldsymbol{V}_k$ refers to the matched beam-forming matrix from the kth relay to Q users, the system gain will not be achieved because of multi-user interference.

From Eq. (2), the transmission from the base station to the qth user can be described as

$$Y_q = \left(\sum_{k=1}^{K} \boldsymbol{G}_{qk}\boldsymbol{W}_k\boldsymbol{H}_k\boldsymbol{F}\right)_q s(q) + \sum_{k=1}^{K} \boldsymbol{G}_{qk}\boldsymbol{W}_k\boldsymbol{n}_{1k} + n_{2q} + \left[\sum_{k=1}^{K} \boldsymbol{G}_{qk}\boldsymbol{W}_k\boldsymbol{H}_k\boldsymbol{F}\boldsymbol{S} - \left(\sum_{k=1}^{K} \boldsymbol{G}_{qk}\boldsymbol{W}_k\boldsymbol{H}_k\boldsymbol{F}\right)_q s(q)\right], \quad (12)$$

where $(\cdot)_q$ refers to the qth element of the vector.

Let

$$\boldsymbol{J} = \sum_{k=1}^{K} \boldsymbol{G}_{qk}\boldsymbol{W}_k\boldsymbol{H}_k\boldsymbol{F},$$

$$\boldsymbol{J}(q) = \left(\sum_{k=1}^{K} \boldsymbol{G}_{qk}\boldsymbol{W}_k\boldsymbol{H}_k\boldsymbol{F}\right)_q,$$

$$\overline{\boldsymbol{S}}_q = [s(1), s(2), \ldots, s(q-1), s(q+1), \ldots, s(Q)],$$

and

$$\overline{\boldsymbol{J}}_q = \left[\left(\sum_{k=1}^{K} \boldsymbol{G}_{qk}\boldsymbol{W}_k\boldsymbol{H}_k\boldsymbol{F}\right)_1, \left(\sum_{k=1}^{K} \boldsymbol{G}_{qk}\boldsymbol{W}_k\boldsymbol{H}_k\boldsymbol{F}\right)_2, \ldots, \left(\sum_{k=1}^{K} \boldsymbol{G}_{qk}\boldsymbol{W}_k\boldsymbol{H}_k\boldsymbol{F}\right)_{q-1}, \left(\sum_{k=1}^{K} \boldsymbol{G}_{qk}\boldsymbol{W}_k\boldsymbol{H}_k\boldsymbol{F}\right)_{q+1}, \ldots, \left(\sum_{k=1}^{K} \boldsymbol{G}_{qk}\boldsymbol{W}_k\boldsymbol{H}_k\boldsymbol{F}\right)_Q\right].$$

Eq. (12) can be reduced to

$$Y_q = \boldsymbol{J}(q)s(q) + \overline{\boldsymbol{J}}_q\overline{\boldsymbol{S}}_q + \tilde{n}_q. \quad (13)$$

The instantaneous capacity of the system is

$$C = \sum_{q=1}^{Q} \log_2\left|1 + (\boldsymbol{J}(q)\boldsymbol{J}^*(q) \times E_s)/\left(\overline{\boldsymbol{J}}_q\overline{\boldsymbol{J}}_q^{\mathrm{H}} \times E_s + \sigma_{\tilde{n}}^2\right)\right|.$$

Define the signal to interference and noise ratio (SINR) of the qth user $\mathrm{SINR}_q = (\boldsymbol{J}(q)\boldsymbol{J}^*(q) \times E_s)/\left(\overline{\boldsymbol{J}}_q\overline{\boldsymbol{J}}_q^{\mathrm{H}} \times E_s + \sigma_{\tilde{n}}^2\right)$, which will be feedback from the qth user to the relays. At the relays, the users with larger SINR will be selected. As long as $\overline{Q}$ is large enough, Q users can be selected to avoid multi-user interference efficiently.

4 Numerical results and discussion

In this section, we will present the numerical results on the performance of the joint solution (JO) with the total CSI and sub-optimal solution with multi-user scheduling (SO) with the local CSI by means of Monte-Carlo simulations. We model each MIMO channel as an uncorrelated Rayleigh fading channel. The number of total users waiting for service is 40 and the maximum number of users simultaneously supported by the system is 8. The base station is equipped with 8 antennas and every relay is equipped with 4 antennas. The mobiles are all equipped with a single antenna. We define a reference $\mathrm{SNR}_{\mathrm{relay}}$ as the average received signal to noise ratio (SNR) at the relay and a reference $\mathrm{SNR}_{\mathrm{user}}$ as the average received SNR at the mobiles.

In Fig. 2, the ergodic capacity of the JO, sub-optimal solution (SO), and sub-optimal solution with multi-user scheduling (MSSO) versus $\mathrm{SNR}_{\mathrm{user}}$ is shown. The 8-relay and 4-relay enhanced systems are shown in real line and dashed line respectively. The average received SNR at the relay is 30 dB. At lower $\mathrm{SNR}_{\mathrm{user}}$, the sub-optimal solution with multi-user scheduling with the local CSI is superior, while at higher $\mathrm{SNR}_{\mathrm{user}}$ the joint solution with the total CSI can achieve higher system capacity. In an 8-relay enhanced system, the sub-optimal solution with multi-user scheduling is better when the average received SNR at the mobiles is lower than 12 dB. This also happens to a 4-relay enhanced system when the average received SNR at the mobiles is lower than 13 dB.

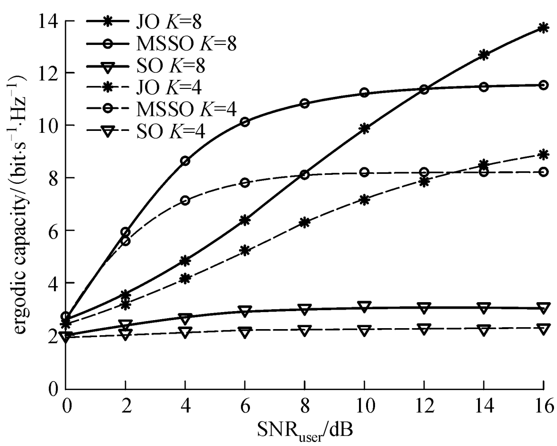

Fig. 2 Ergodic capacity vs $\mathrm{SNR}_{\mathrm{user}}$

Figure 3 shows the cumulative distribution function (CDF) of instantaneous capacity for different systems with different numbers of relays employed. Here SNR in the figure refers to $\mathrm{SNR}_{\mathrm{user}}$. The average received SNR at the relay is 30 dB. System performance can be improved if the number of relays employed is increased. The capacity gap between different systems with different relays increases with $\mathrm{SNR}_{\mathrm{user}}$.

As shown in Fig. 4, the average received SNR at the mobile is 20 dB. The average received SNR at the relay of 5 dB and 20 dB are considered, which are denoted by dashed line and real line respectively. The capacity gap between different systems with different relays in joint

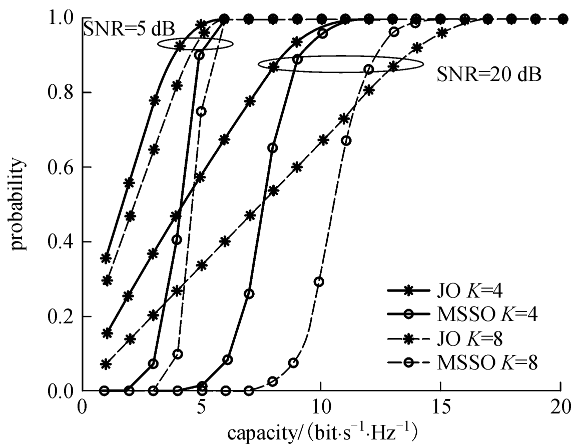

Fig. 3 CDF of capacity for different systems with different numbers of relays employed

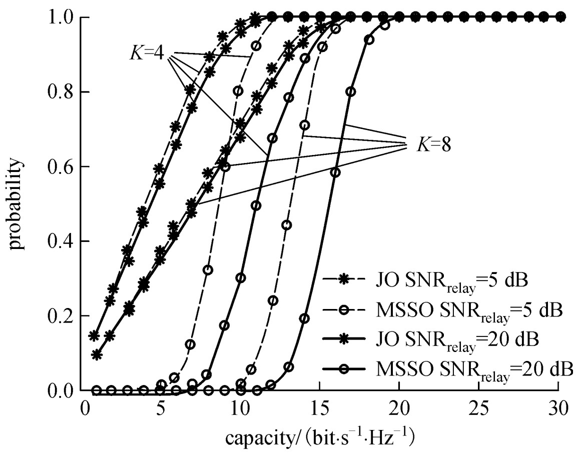

Fig. 4 CDF of capacity for different systems with different numbers of relays employed

solution with the total CSI is 0.5 bit/(s·Hz). In the sub-optimal solution with multi-user scheduling with the local CSI, the capacity gap is about 2–3 dB.

5 Conclusions

The next generation wireless communication systems need to support data rates much greater than 3 G systems. Thus, the newly emerging relay enhanced wireless network has received increasing attention. In this article, we give a joint optimal solution for multiple relays in a relay-enhanced multi-user wireless system with the total CSI. A sub-optimal solution with multi-user scheduling strategy is also proposed with the local CSI at the relays. With a multi-user scheduling strategy, the spatial diversity caused by multiple relays can be achieved efficiently. The instantaneous capacity for different systems with different strategies is considered. The simulation results show that at low $\mathrm{SNR}_{\mathrm{user}}$, better system throughput can be achieved in a sub-optimal solution with multi-user scheduling. At high $\mathrm{SNR}_{\mathrm{user}}$, a joint solution is superior.

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Appendix A

If $\overline{Q} > N$, using singular value decomposition (SVD), we can obtain:

$$\overline{\boldsymbol{G}}_k = \begin{bmatrix} G_{1k}(1) & G_{1k}(2) & \cdots & G_{1k}(N) \\ G_{2k}(1) & G_{2k}(2) & \cdots & G_{2k}(N) \\ \vdots & \vdots & \ddots & \vdots \\ G_{\overline{Q}k}(1) & G_{\overline{Q}k}(2) & \cdots & G_{\overline{Q}k}(N) \end{bmatrix} = \boldsymbol{U}_k \boldsymbol{\Lambda} \boldsymbol{V}_k$$

$$= \begin{bmatrix} U_{1k}(1) & U_{2k}(2) & \cdots & U_{1k}(\overline{Q}) \\ U_{2k}(1) & U_{2k}(2) & \cdots & U_{2k}(\overline{Q}) \\ \vdots & \vdots & \ddots & \vdots \\ U_{\overline{Q}k}(1) & U_{\overline{Q}k}(2) & \cdots & U_{\overline{Q}k}(\overline{Q}) \end{bmatrix} \begin{bmatrix} \boldsymbol{\Lambda}'_N \\ \boldsymbol{\theta}_{(\overline{Q}-N)\times N} \end{bmatrix} \times \begin{bmatrix} V_{1k}(1) & V_{1k}(2) & \cdots & V_{1k}(N) \\ V_{2k}(1) & V_{2k}(2) & \cdots & V_{2k}(N) \\ \vdots & \vdots & \ddots & \vdots \\ V_{Nk}(1) & V_{Nk}(2) & \cdots & V_{Nk}(N) \end{bmatrix}. \quad \text{(A1)}$$

Let $\boldsymbol{D}' = \left[\mathrm{diag}(\boldsymbol{\Lambda}'_N), \boldsymbol{\theta}_{1\times(\overline{Q}-N)}\right]$, $\boldsymbol{U}_{ik} = [U_{ik}(1), U_{ik}(2), \dots, U_{ik}(\overline{Q})]$, where $\boldsymbol{\theta}_{1\times(\overline{Q}-N)}$ is a zero vector with $(\overline{Q}-N)$ dimension. Then

$$\boldsymbol{U}_k \boldsymbol{\Lambda}_k = \begin{bmatrix} \boldsymbol{U}_{1k} \odot \boldsymbol{D}' \\ \boldsymbol{U}_{2k} \odot \boldsymbol{D}' \\ \vdots \\ \boldsymbol{U}_{\overline{Q}k} \odot \boldsymbol{D}' \end{bmatrix},$$

and

$$\overline{\boldsymbol{G}}_k = \begin{bmatrix} \boldsymbol{U}_{1k} \odot \boldsymbol{D}' \\ \boldsymbol{U}_{2k} \odot \boldsymbol{D}' \\ \vdots \\ \boldsymbol{U}_{\overline{Q}k} \odot \boldsymbol{D}' \end{bmatrix} \boldsymbol{V}_k.$$

If the elements of the matrix $\overline{\boldsymbol{G}}_k$ are set to be zero in the jth row, that is

$$\boldsymbol{G}_k^{'} = \begin{bmatrix} G_{1k}(1) & \cdots & G_{1k}(N) \\ \vdots & \ddots & \vdots \\ G_{(j-1)k}(1) & \cdots & G_{(j-1)k}(N) \\ 0 & \cdots & 0 \\ G_{(j+1)k}(1) & \cdots & G_{(j+1)k}(N) \\ \vdots & \ddots & \vdots \\ G_{\overline{Q}k}(1) & \cdots & G_{\overline{Q}k}(N) \end{bmatrix}$$

$$= \begin{bmatrix} \boldsymbol{U}_{1k} \odot \boldsymbol{D}' \\ \vdots \\ \boldsymbol{U}_{(j-1)k} \odot \boldsymbol{D}' \\ \boldsymbol{0} \\ \boldsymbol{U}_{(j+1)k} \odot \boldsymbol{D}' \\ \vdots \\ \boldsymbol{U}_{\overline{Q}k} \odot \boldsymbol{D}' \end{bmatrix} \boldsymbol{V}_k. \quad \text{(A2)}$$

Let

$$\boldsymbol{G}_k'' = \begin{bmatrix} G_{1k}(1) & \cdots & G_{1k}(N) \\ \vdots & \ddots & \vdots \\ G_{(j-1)k}(1) & \cdots & G_{(j-1)k}(N) \\ G_{(j+1)k}(1) & \cdots & G_{(j+1)k}(N) \\ \vdots & \ddots & \vdots \\ G_{\overline{Q}k}(1) & \cdots & G_{\overline{Q}k}(N) \end{bmatrix},$$

$$\boldsymbol{U}_k'' = \begin{bmatrix} \boldsymbol{U}_{1k} \odot \boldsymbol{D}' \\ \vdots \\ \boldsymbol{U}_{(j-1)k} \odot \boldsymbol{D}' \\ \boldsymbol{U}_{(j+1)k} \odot \boldsymbol{D}' \\ \vdots \\ \boldsymbol{U}_{\overline{Q}k} \odot \boldsymbol{D}' \end{bmatrix},$$

we can obtain

$$\boldsymbol{G}_k'' = \boldsymbol{U}_k'' \boldsymbol{\Lambda}_k \boldsymbol{V}_k. \quad \text{(A3)}$$

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