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RESEARCH ARTICLE

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Third-order trajectory planning for high accuracy point-to-point motion

Abstract This article studies a third-order trajectory planning method for point-to-point motion. All available instances for third-order trajectory planning are first analyzed. To distinguish those, three criteria are presented relying on trajectory characteristics. Following that, a fast preprocessing approach considering the trajectory as a whole is given based on the criteria constructed and system constraints. Also, the time-optimality of the trajectory is obtained. The relevant formulas are derived with the combination of geometrical symmetry of trajectory and area method. As a result, an accurate algorithm and its implementation procedure are proposed. The experimental results show the effectiveness and precision of the proposed method. The presented algorithm has been applied in semiconductor manufacturing equipment successfully.

Keywords trajectory planning, point-to-point motion, limitation criteria

1 Introduction

Trajectory planning should meet the following demands: minimal residual vibration, time optimization and high accuracy. In real applications, second-order position trajectory can achieve fast motions, but tend to evoke vibration because of compliance [1,2]. With the increasing demands of accuracy and efficiency, third-order trajectory planning has been widely applicable because of motion fastness and minimal residual vibration [3]. At present, many feasible methods have been developed to plan reference trajectory and shaping [4–6]. However, an increase of execution time of the trajectory will be caused by applying these methods. Furthermore, there is no clear useful mechanism for trajectory planning and time-optimization. It degrades the effectiveness of trajectory planning and limits its extension ability toward higher-order trajectory. Moreover, the prediction of trajectory data on time is not allowable in existing planning methods [7–11]. Hence, these methods commonly are not suitable for the motions with the synchronization requirements. This article presents a straight-forward planning method of the reference trajectory. Several criteria are constructed based on system constraints. The shape of the trajectory will be predetermined by applying the presented criteria. In addition, the common characteristics of different trajectory shapes are concluded. Base on that, the equations needed in the trajectory planning are derived. As a result, the planning algorithm of third-order trajectory can be given.

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2 Trajectory instances and limitation criteria

Point-to-point motion requires high positioning accuracy without concerning the process. Consider the trajectory with zero-constraints, that is, zero velocity, acceleration and jerk. Figure 1 is a typical third-order motion trajectory. Part Ⅰ denotes the duration of maximal jerk, Part Ⅱ the duration of maximal acceleration and Part Ⅲ the duration of maximal velocity. By varying the given constraints, the time intervals of Part Ⅱ and Part Ⅲ could disappear. This means that various possible shapes will occur in the trajectory. Summing up the constraint conditions corresponding to each possible instance, the limitation criteria and relative equations can be obtained. These are used to plan the reference trajectory.

2.1 Eigenvalue definition

For convenience, the following limitation values are defined first.

1) When acceleration reaches its given maximal value,

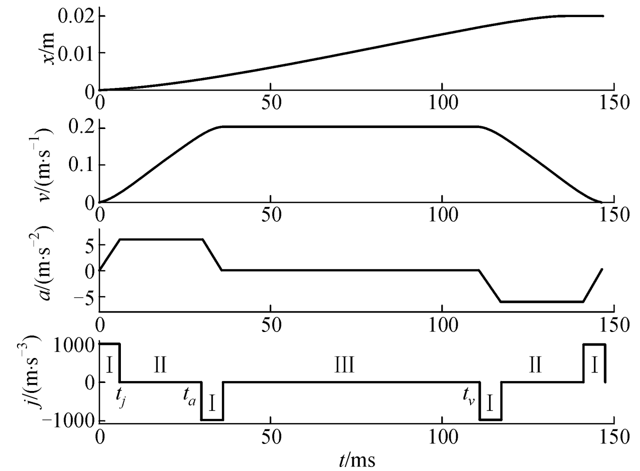

Fig. 1 Typical third-order trajectory

discarding given maximal velocity and distance, the motion runs down to a standstill as soon as possible with symmetrical trajectory shape, and the resulting value of the velocity and the distance is defined as v_a and s_a, respectively.

2) When the velocity reaches its given maximal value, discarding the given distance, the motion runs down to a standstill as soon as possible with symmetrical trajectory shape, and the resulting distance is defined as s_v.

3) The maximal jerk value is denoted by $j_{\max}$, the maximal acceleration $a_{\max}$, the maximal velocity $v_{\max}$ and the given motion distance is denoted by s.

2.2 Trajectory instances

Various trajectory shapes will be determined according to different given constraints. For third-order trajectory, there exist six different possible trajectory shapes. The common characteristics are concluded as follows:

1) Case one: $v_{\max}$ is less than v_a and s is larger than s_a, the acceleration trajectory shape is triangular and the positive and negative shapes are non-continuous.

2) Case two: $v_{\max}$ is larger than v_a and s is less than s_a, the acceleration trajectory shape is triangular and the positive and negative shapes are continuous.

3) Case three: $v_{\max}$ is less than v_a and s is less than s_a, the acceleration trajectory shape is triangular. The continuity of the positive and negative shapes is determined by the time interval during which the maximal velocity or the given distance is to be reached in advance. If s is larger than s_v, the positive and negative triangular shapes are non-continuous. Otherwise, the positive and negative triangular shapes are continuous.

4) Case four: $v_{\max}$ is larger than v_a and s is larger than s_a, the acceleration trajectory shape is trapezoidal. The continuity of the positive and negative shapes is determined by the time during which the maximal velocity or the given distance is to be reached in advance. If s is larger than s_v, the positive and negative trapezoidal shapes are non-continuous. Otherwise, the positive and negative trapezoidal shapes are continuous.

2.3 Limitation criteria

To predetermine the specific instance of the trajectory relying on given constraints and obtain a clear mechanism of time-optimization, several criteria are necessary to construct.

1) Velocity-acceleration criteria: If $s > s_a$ and $v_{\max} < v_a$, the time interval of maximal jerk is determined by the given maximal velocity, and the time interval of maximal velocity by the given motion distance.

2) Distance-acceleration criteria: If $s < s_a$ and $v_{\max} > v_a$, the time interval of maximal jerk is determined by the given motion distance.

3) Distance-velocity criteria one: If $s < s_a$ and $v_{\max} < v_a$, the time interval of maximal jerk is determined by either maximal velocity or the given motion distance. If $s > s_v$, the time interval of maximal jerk is determined by maximal velocity. Otherwise, the time interval of maximal jerk is determined by the given motion distance.

4) Distance-velocity criteria two: If $s > s_a$ and $v_{\max} > v_a$, the time interval of maximal jerk is determined by maximal acceleration. The time interval of maximal acceleration is determined by either maximal velocity or given motion distance. If $s > s_v$, the time interval of maximal acceleration is determined by maximal velocity. Otherwise, the time interval of maximal acceleration is determined by the given motion distance.

3 Formulas derivation and trajectory planning

The determination of specific trajectory instance relies on two issues. One is that acceleration trajectory shape is alternatively triangular or trapezoidal. The other is that the positive and negative shapes in the acceleration trajectory are continuous. Based on these, the equations of the trajectory planning can be derived directly.

3.1 Formulas derivation

The combination of the geometry symmetry of trajectory and the area method is used to derive required equations. Three time intervals, denoted by t_j, t_a, t_v respectively, are used to replace the time interval of Part Ⅰ, Part Ⅱ and Part Ⅲ, as shown in Fig. 1.

The time interval, t_{tab}, is denoted by the interval during which the acceleration is accelerated from zero value up to its preset maximal value.

From the earlier statement, the derivation of equations in third-order trajectory planning can be categorized in four situations.

1) Acceleration trajectory shape being triangular with positive and negative shapes connected:

The relation between the given motion distance s and the time interval of maximal jerk t_j can be formulated as

$$s = \frac{2a_{\max} t_j^3}{t_{\text{tab}}}. \tag{1}$$

Then, the time interval t_j can be solved by

$$t_j = \sqrt[3]{\frac{s t_{\text{tab}}}{2a_{\max}}}. \tag{2}$$

2) Acceleration trajectory shape being triangular with positive and negative shapes not connected:

The relation between maximal velocity $v_{\max}$ and time interval of maximal jerk t_j can be formulated as

$$v_{\max} = \frac{a_{\max} t_j^2}{t_{\text{tab}}}. \tag{3}$$

Then, the time interval t_j can be obtained by

$$t_j = \sqrt{\frac{v_{\max} t_{\text{tab}}}{a_{\max}}}. \tag{4}$$

In this case, the time interval of maximal velocity is determined by the given motion distance s. The relation between given distance s and time interval t_v can be formulated as

$$s = v_{\max} t_v. \tag{5}$$

Then, the time interval t_v can be calculated by

$$t_v = \frac{s}{v_{\max}}. \tag{6}$$

3) Acceleration trajectory shape being trapezoidal with positive and negative shapes connected:

The relation between the maximal acceleration $a_{\max}$ and the time interval of maximal jerk t_j can be formulated as

$$a_{\max} = j_{\max} t_j. \tag{7}$$

Then, the time interval t_j can be determined by

$$t_j = \frac{a_{\max}}{j_{\max}}. \tag{8}$$

In this case, the time interval of maximal acceleration is determined by the given motion distance s. The relation between the given motion distance s and the time interval t_a can be formulated as

$$s = a_{\max}(t_a^2 + t_a t_{\text{tab}}). \tag{9}$$

Then, the time interval t_a is calculated by

$$t_a = \sqrt{\frac{s}{a_{\max}} + \frac{t_{\text{tab}}^2}{4}} - \frac{t_{\text{tab}}}{2}. \tag{10}$$

4) Acceleration trajectory shape being trapezoidal with positive and negative shapes not connected:

The time interval t_j can be calculated by Eq. (8).

In this case, the time interval of maximal acceleration is determined by the maximal velocity $v_{\max}$. The relation between the maximal velocity and the time interval t_a can be formulated as

$$v_{\max} = a_{\max} t_a. \tag{11}$$

Then, the time interval t_a is obtained by

$$t_a = \frac{v_{\max}}{a_{\max}}. \tag{12}$$

The time interval t_v can be obtained by Eq. (6).

3.2 Algorithm

Third-order trajectory planning is to obtain optimal time intervals t_j, t_a, t_v. The algorithm of the planning is presented as follows.

1) Calculate limit values v_a, s_a and s_v.

$$v_a = \frac{a_{\max}^2}{j_{\max}}, \tag{13}$$

$$s_a = \frac{2a_{\max}^3}{j_{\max}^2}, \tag{14}$$

$$s_v = v_{\max}\left[M\left(2\sqrt{\frac{v_{\max}}{j_{\max}}}\right) + N\left(\frac{v_{\max}}{a_{\max}} + \frac{a_{\max}}{j_{\max}}\right)\right], \tag{15}$$

where M and N are two integral coefficients, satisfying the below condition: if $v_{\max} j_{\max} < a_{\max}^2$, then $M=1$, $N=0$; otherwise, $M=0$, $N=1$.

2) If $v_{\max} < v_a$ and $s > s_a$, time interval t_j is calculated by Eq. (4), and time interval t_v is calculated by Eq. (6). In this case, $t_a = t_j$.

3) If $v_{\max} > v_a$ and $s < s_a$, time interval t_j is calculated by Eq. (2). In this case, $t_a = t_j$, $t_v = 2t_j$.

4) If $v_{\max} < v_a$ and $s < s_a$ and $s > s_v$, time interval t_j is calculated by Eq. (4), and time interval t_v is calculated by Eq. (6). In this case, $t_a = t_j$.

5) If $v_{\max} < v_a$ and $s < s_a$ and $s < s_v$, time interval t_j is calculated by Eq. (2). In this case, $t_a = t_j$, $t_v = 2t_j$.

6) If $v_{\max} > v_a$ and $s > s_a$ and $s > s_v$, time interval t_j is calculated by Eq. (8), time interval t_a is calculated by Eq. (12), and time interval t_v is calculated by Eq. (6).

7) If $v_{\max} > v_a$ and $s > s_a$ and $s < s_v$, time interval t_j is calculated by Eq. (8), and time interval t_a is calculated by Eq. (10). In this case, $t_v = t_a + t_j$.

Table 1 Given velocity and distance and resultant data

velocity/($m \cdot s^{-1}$)	distance/m	correcting factor	planned position/m
0.03	0.0005	0.94930520352154	5.00000000000000e–4
0.10	0.0004	0.92592592592593	4.00000000000000e–4
0.02	0.0004	0.85131197817236	4.00000000000000e–4
0.03	0.00032	0.91133299538068	3.20000000000000e–4
0.18	0.02	0.99920063948841	2.00000000000000e–2
0.50	0.02	0.98670708218875	2.00000000000000e–2

4 Examples and validity

To verify the effectiveness and reliability of the proposed method, all possible instances of third-order trajectory are executed in the experiments. The assumed constraints are as follows: the maximal jerk j_{max} is 1000 m/s^3, the maximal acceleration a_{max} is 6 m/s^2 and the sample time is 0.4 ms. The distance and the maximal velocity are shown in Table 1. The results of possible instances are also illustrated in Table 1. From these results, it is obvious that the presented algorithm obtains the high position accuracy of trajectory end point, which also demonstrates that the algorithm can handle all possible instances, hence it is reliable. In this article, graphic curves of the two instances are shown in Figs. 2 and 3.

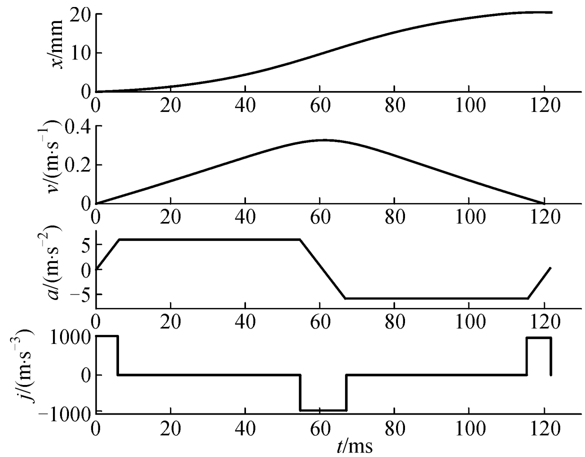

Fig. 3 v_{max} = 0.5 m/s, s = 20 mm

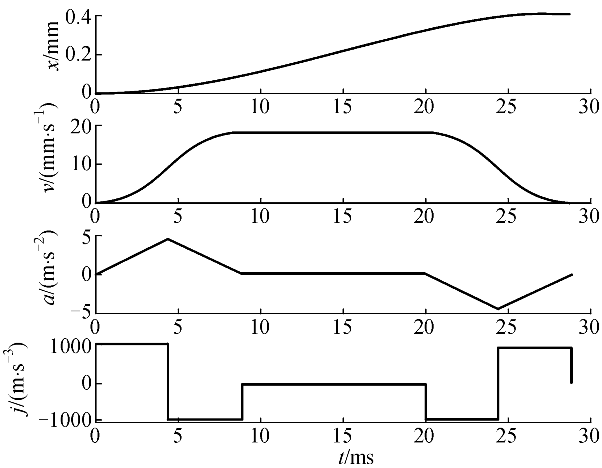

Fig. 2 v_{max} = 20 mm/s, s = 0.4 mm

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