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RESEARCH ARTICLE

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Fault detection for a class of Markov jump systems with unknown disturbances

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Abstract An optimized fault detection observer is designed for a class of Markov jump systems with unknown disturbances. By reconstructing the system, the residual error dynamic characteristics of unknown input and fault signals, including unknown disturbances and modeling error are obtained. The energy norm indexes of disturbance and fault signals of the residual error are selected separately to reflect the restraint of disturbance and the sensitivity of faults, and the design of the fault detection observer is described as an optimization problem. By using the constructed Lyapunov function and linear matrix inequalities, a sufficient condition that the solution to the fault detection observer exists is given and proved, and an optimized design approach is presented. The designed observer makes the systems have stochastic stability and better capability of restraining disturbances, and the given norm index is satisfied. Simulation results demonstrate that the proposed observer can detect the faults sensitively, and the influence of unknown disturbance on residual error can be restrained to a given range.

Keywords Markov jump systems, fault detection, optimized observer, stochastic stability, linear matrix inequalities

1 Introduction

A lot of dynamical systems are highly relevant to processes whose parameters are subject to random abrupt changes because of, for example, sudden environment changes, subsystem switching, system noises, failures occurring in components or interconnections and executor's faults, etc. Markov jump systems (MJSs) are a special class of hybrid systems that involve both time-evolving and event-driven mechanisms, and can be employed to model the above system phenomenon. In MJSs, the dynamics of the jump modes and continuous states are respectively modeled by finite state Markov chains and differential equations. Each operation mode refers to some dynamics and the corresponding mode transitions from one to another are governed by the Markov process as well. The existing results about MJSs cover a large variety of problems such as stochastic stability [1,2], stochastic H_∞ control [3], filtering problem [4] and references therein. Compared with these, however, very few literatures [5] consider the fault detection problems for MJSs.

In this article, the authors discuss the fault detection problem for a class of MJSs with unknown norm-bounded disturbances. With the aid of the related fault detection theory of linear system [6–9], the augmented dynamic system is remodeled and an optimized observer is constructed. By using the constructed Lyapunov function and linear matrix inequalities (LMIs), a sufficient condition that the solution to the fault detection observer exists is given and proved, and an optimized design approach is presented.

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2 System description

Consider a class of linear MJSs over the space $(\Omega,\ F,\ P)$ by

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}(\boldsymbol{r}_t)\boldsymbol{x}(t) + \boldsymbol{B}(\boldsymbol{r}_t)\boldsymbol{u}(t) + \boldsymbol{B}_f(\boldsymbol{r}_t)\boldsymbol{f}(t) + \boldsymbol{B}_d(\boldsymbol{r}_t)\boldsymbol{d}(t), \\ \boldsymbol{y}(t) = \boldsymbol{C}(\boldsymbol{r}_t)\boldsymbol{x}(t) + \boldsymbol{D}_d(\boldsymbol{r}_t)\boldsymbol{d}(t) + \boldsymbol{D}_f(\boldsymbol{r}_t)\boldsymbol{f}(t), \\ \boldsymbol{x}(t) = \boldsymbol{x}_0, \boldsymbol{r}_t = \boldsymbol{r}_0, t = 0, \end{cases} \tag{1}$$

where $\boldsymbol{x}(t) \in R^n$ is the state, $\boldsymbol{u}(t) \in R^m$ is the control input, $\boldsymbol{y}(t) \in R^l$ is the measured output, $\boldsymbol{f}(t) \in R^p$ is the additional fault signals, $\boldsymbol{d}(t) \in L_2^q[0, +\infty)$ is the unknown inputs, including unknown disturbances and noises. $\boldsymbol{x}_0$, $\boldsymbol{r}_0$ are respectively the initial state and mode. $\boldsymbol{A}(\boldsymbol{r}_t)$, $\boldsymbol{B}(\boldsymbol{r}_t)$, $\boldsymbol{B}_d(\boldsymbol{r}_t)$, $\boldsymbol{B}_f(\boldsymbol{r}_t)$, $\boldsymbol{C}(\boldsymbol{r}_t)$, $\boldsymbol{D}_d(\boldsymbol{r}_t)$, $\boldsymbol{D}_f(\boldsymbol{r}_t)$ are known mode-dependent constant matrices with appropriate dimensions. Let the random form process $\{\boldsymbol{r}_t, t \geqslant 0\}$ be the Markov stochastic process taking values on a finite set $M = \{1,2,...,N\}$ with transition rate matrix $\boldsymbol{\Pi} = \{\pi_{ij}\}$, $i,j \in M$, and define the following transition probability from mode i at time t to mode j at time $t + \Delta t$ as

$$\boldsymbol{P}_{ij} = P_r\{\boldsymbol{r}_{t+\Delta t} = j | \boldsymbol{r}_t = i\} = \begin{cases} \pi_{ij}\Delta t + o(\Delta t), & i \neq j, \\ 1 + \pi_{ii}\Delta t + o(\Delta t), & i = j, \end{cases} \tag{2}$$

with transition probability rates $\pi_{ij} \geqslant 0$ for $i, j \in M$, $i \neq j$ and $\sum_{j=1, j\neq i}^{N} \pi_{ij} = -\pi_{ii}$, where $\Delta t > 0$ and $\lim_{\Delta t \to 0} o(\Delta t)/\Delta t = 0$.

For notational simplicity, when $\boldsymbol{r}_t = i$, $i \in M$, $\boldsymbol{A}(\boldsymbol{r}_t)$, $\boldsymbol{B}(\boldsymbol{r}_t)$, $\boldsymbol{B}_d(\boldsymbol{r}_t)$, $\boldsymbol{B}_f(\boldsymbol{r}_t)$, $\boldsymbol{C}(\boldsymbol{r}_t)$, $\boldsymbol{D}_d(\boldsymbol{r}_t)$ and $\boldsymbol{D}_f(\boldsymbol{r}_t)$ are respectively denoted as $\boldsymbol{A}_i$, $\boldsymbol{B}_i$, $\boldsymbol{B}_{di}$, $\boldsymbol{B}_{fi}$, $\boldsymbol{C}_i$, $\boldsymbol{D}_{di}$ and $\boldsymbol{D}_{fi}$. For the sake of clarity, MJSs (1) are assumed to be stochastically stable, $(\boldsymbol{C}_i, \boldsymbol{A}_i)$ is assumed to be observable and $\boldsymbol{B}_{fi}$ is a full rank matrix.

Definition 1 The MJSs (1) (setting $\boldsymbol{u}(t), \boldsymbol{f}(t), \boldsymbol{d}(t) \equiv 0$) are said to be stochastically stable, if for any initial $\boldsymbol{x}_0$ and mode $\boldsymbol{r}_0$, the following relation holds:

$$\lim_{T\to\infty} \boldsymbol{E}\left\{ \int_0^T \left\| \boldsymbol{x}(t,\ \boldsymbol{x}_0,\ \boldsymbol{r}_0) \right\|^2 \mathrm{d}t \middle| \boldsymbol{x}_0,\ \boldsymbol{r}_0 \right\} < \infty, \tag{3}$$

where $E\{\cdot\}$ denotes the mathematics statistical expectation of the stochastic process or vector.

Definition 2 In the Euclidean space $\{R^n \times M \times R_+\}$, the authors introduce the stochastic Lyapunov function of system (2) as $\boldsymbol{V}(\boldsymbol{x}(t),\ \boldsymbol{r}_t = i,\ t > 0) = \boldsymbol{V}(\boldsymbol{x},\ i)$, the weak infinitesimal operator of which satisfies

$$\Gamma \boldsymbol{V}(\boldsymbol{x},\ i) = \lim_{\Delta t \to 0} \frac{1}{\Delta t}\{\boldsymbol{E}\{\boldsymbol{V}(\boldsymbol{x}(t+\Delta t),\ \boldsymbol{r}_{t+\Delta t},\ t+\Delta t)\} - \boldsymbol{V}(\boldsymbol{x}(t),\ i,\ t)\}. \tag{4}$$

In general, for model-based fault detection, the remaining important task is the evaluation of the generated residual. Furthermore, the evaluation of the generated residual includes residual evaluation function, choosing an appropriate threshold and fault detection logic.

As to MJSs (1), the authors set up the following full-rank fault detection observer:

$$\begin{cases} \dot{\hat{\boldsymbol{x}}}(t) = \boldsymbol{A}_i \hat{\boldsymbol{x}}(t) + \boldsymbol{H}_i[\boldsymbol{y}(t) - \hat{\boldsymbol{y}}(t)], \\ \hat{\boldsymbol{y}}(t) = \boldsymbol{C}_i \hat{\boldsymbol{x}}(t), \end{cases} \tag{5}$$

where $\hat{\boldsymbol{x}}(t)$ and $\hat{\boldsymbol{y}}(t)$ are the estimated state and output, and $\boldsymbol{H}_i$ is the observer gain to be designed. For further analysis, the authors first introduce the state estimate error $\boldsymbol{\varepsilon}(t) = \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)$ and the output error $\boldsymbol{\gamma}(t) = \boldsymbol{y}(t) - \hat{\boldsymbol{y}}(t)$. Here, $\boldsymbol{\gamma}(t)$ is selected as the residual signal because it is undetectable, and then the following residual generator can be obtained:

$$\boldsymbol{\gamma} = \boldsymbol{y}(t) - \boldsymbol{C}_i \hat{\boldsymbol{x}}(t) = \boldsymbol{C}_i \boldsymbol{\varepsilon} + \boldsymbol{D}_{fi}\boldsymbol{f} + \boldsymbol{D}_{di}\boldsymbol{d}, \tag{6}$$

which describes the influences of additional faults and unknown disturbances to the system residual. For convenience, the robustness of disturbances to residual is denoted as $\boldsymbol{\gamma}_d$ and the sensitivity of faults to residual as $\boldsymbol{\gamma}_f$. Similarly, the influence of disturbances and faults on estimate errors is denoted as $\boldsymbol{\varepsilon}_d$ and $\boldsymbol{\varepsilon}_f$ respectively.

Thus, the overall dynamic observer systems can then be presented according to systems (5) and (1):

$$\begin{cases} \dot{\boldsymbol{\varepsilon}} = \overline{\boldsymbol{A}}_i \boldsymbol{\varepsilon} + \overline{\boldsymbol{B}}_{fi}\boldsymbol{f} + \overline{\boldsymbol{B}}_{di}\boldsymbol{d}, \\ \boldsymbol{\gamma} = \boldsymbol{C}_i \boldsymbol{\varepsilon} + \boldsymbol{D}_{fi}\boldsymbol{f} + \boldsymbol{D}_{di}\boldsymbol{d}, \end{cases} \tag{7}$$

where $\overline{\boldsymbol{A}}_i = \boldsymbol{A}_i - \boldsymbol{H}_i\boldsymbol{C}_i$, $\overline{\boldsymbol{B}}_{fi} = \boldsymbol{B}_{fi} - \boldsymbol{H}_i\boldsymbol{D}_{fi}$, $\overline{\boldsymbol{B}}_{di} = \boldsymbol{B}_{di} - \boldsymbol{H}_i\boldsymbol{D}_{di}$.

Lemma 1 [6] The MJSs (1) are said to be almost asymptotically stable if there exists a set of mode-dependent symmetric positive-definite matrices $\boldsymbol{P}_i$, $i \in \boldsymbol{M}$, such that

$$\boldsymbol{\Xi}_i = \overline{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{P}_i + \boldsymbol{P}_i\overline{\boldsymbol{A}}_i + \sum_{j=1}^{N}\pi_{ij}\boldsymbol{P}_j < 0. \tag{8}$$

To minimize the effects of disturbances on the residual, the fault detection problem is formulated in the case $\boldsymbol{f} = 0$ to design the observer parameter $\boldsymbol{H}_i$ such that the dynamic error system (7) is stochastically stable and for all non-zero $\boldsymbol{d} \in L_2[0,\ \infty)$,

$$E\left\{\int_0^t \boldsymbol{\gamma}_d^{\mathrm{T}}\boldsymbol{\gamma}_d \mathrm{d}t\right\} \leqslant r_1^2 E\left\{\int_0^t \boldsymbol{d}^{\mathrm{T}}\boldsymbol{d}\mathrm{d}t\right\}\Bigg|_{\boldsymbol{f}=0}. \tag{9}$$

Condition (9) gives the method of minimizing the effects of disturbances on the residual and the next target is to make the difference between the residual and the disturbances as small as possible.

Similarly, the authors introduce another performance index that can be described as designing the observer parameter $\boldsymbol{H}_i$ such that the dynamic error system (7) is stochastically stable and for all non-zero $\boldsymbol{f}(t) \in L_2[0,\ \infty)$,

$$E\left\{\int_0^t \boldsymbol{\gamma}_f^{\mathrm{T}}\boldsymbol{\gamma}_f \mathrm{d}t\right\} \geqslant r_2^2 E\left\{\int_0^t \boldsymbol{f}^{\mathrm{T}}\boldsymbol{f}\mathrm{d}t\right\}\Bigg|_{\boldsymbol{d}=0}. \tag{10}$$

Condition (10) gives the schemes of enhancing the effects of faults on the residual and then the difference between the residual and the faults is made as large as possible.

According to the above analysis, in the process of robust fault detection, it is expected that the effect of the robust fault detection observer will make the unknown inputs as small as possible while the fault signal should be large. To detect the faults, a widely adopted approach is used to choose an appropriate threshold J_{th} and determine the evaluation function $f(\gamma)$.

Assume $||\boldsymbol{d}||_2 \leqslant \Delta d$, and then the threshold J_{th} can be set based on the unknown disturbances:

$$J_{\text{th}} = \sup_{\boldsymbol{d}\in L_2, \boldsymbol{f}=0} E\left\{\int_0^t \gamma^{\text{T}}\gamma \text{d}t\right\} = r_1^2 \Delta d. \tag{11}$$

Therefore, the following logic can be made for the fault detection:

$$\begin{cases} f(\gamma) = E\left\{\int_0^t \gamma^{\text{T}}\gamma \text{d}t\right\} > J_{\text{th}} \rightarrow \text{with fault}, \\ f(\gamma) = E\left\{\int_0^t \gamma^{\text{T}}\gamma \text{d}t\right\} \leqslant J_{\text{th}} \rightarrow \text{fault-free}, \end{cases} \tag{12}$$

where $(0,\ t]$ is the finite-time window. The evaluation time window t is limited because the evaluation of residual signal over the whole time range is impractical.

Condition (9) represents the worst-case criterion for the effect of disturbances on the residuals, while condition (10) stands for the worst-case criterion for the sensitivity of residuals to faults. Both provide a directly quantitative measure for robustness and sensitivity of fault detection. Generally speaking, to get the appropriate fault detection observer that reflects the restraining of disturbances on residuals and sensitivity of the increase of faults to residuals, the gain matrix $\boldsymbol{H}_i$ should satisfy condition (9) and condition (10) simultaneously. Therefore, to achieve the optimal trade-off between the robustness against disturbances and the sensitivity to faults, the fault detection observer design problem for stochastic MJSs (1) can be formulated to find $\boldsymbol{H}_i$ such that the dynamic error system (7) is stochastically stable under zero initial conditions and satisfies

$$J_{\text{e}} = r_1/r_2. \tag{13}$$

3 Observer analysis and design of jump system

Theorem 1 Considering MJSs (1), the dynamic error system (7) is almost asymptotically stable and satisfies condition (9), if there exists a set of mode-dependent symmetric positive-definite matrices $\boldsymbol{P}_i > 0$ and mode-dependent matrix $\overline{\boldsymbol{H}}_i$ such that the following relation holds:

$$\begin{bmatrix} \boldsymbol{\Xi}_i + \boldsymbol{C}_i^{\text{T}}\boldsymbol{C}_i & \boldsymbol{P}_i\overline{\boldsymbol{B}}_{di} + \boldsymbol{C}_i^{\text{T}}\boldsymbol{D}_{di} \\ \overline{\boldsymbol{B}}_{di}^{\text{T}}\boldsymbol{P}_i + \boldsymbol{D}_{di}^{\text{T}}\boldsymbol{C}_i & -r_1^2\boldsymbol{I} + \boldsymbol{D}_{di}^{\text{T}}\boldsymbol{D}_{di} \end{bmatrix} < 0, \tag{14}$$

or

$$\begin{bmatrix} \boldsymbol{A}_i^{\text{T}}\boldsymbol{P}_i + \boldsymbol{P}_i\boldsymbol{A}_i + \sum_{j=1}^{N}\pi_{ij}\boldsymbol{P}_j + \boldsymbol{C}_i^{\text{T}}\boldsymbol{C}_i - \boldsymbol{C}_i^{\text{T}}\overline{\boldsymbol{H}}_i^{\text{T}} - \overline{\boldsymbol{H}}_i\boldsymbol{C}_i & \boldsymbol{P}_i\boldsymbol{B}_{di} + \boldsymbol{C}_i^{\text{T}}\boldsymbol{D}_{di} - \overline{\boldsymbol{H}}_i\boldsymbol{D}_{di} \\ \boldsymbol{B}_{di}^{\text{T}}\boldsymbol{P}_i + \boldsymbol{D}_{di}^{\text{T}}\boldsymbol{C}_i - \boldsymbol{D}_{di}^{\text{T}}\overline{\boldsymbol{H}}_i^{\text{T}} & -r_1^2\boldsymbol{I} + \boldsymbol{D}_{di}^{\text{T}}\boldsymbol{D}_{di} \end{bmatrix} < 0. \tag{15}$$

Proof For given mode-dependent symmetric positive-definite matrices $\boldsymbol{P}_i \in R^{n\times n}$, $i\in M$, considering system (7), the Lyapunov function is defined as $\boldsymbol{V}(\boldsymbol{\varepsilon}_d,\ i) = \boldsymbol{\varepsilon}_d^{\text{T}}\boldsymbol{P}_i\boldsymbol{\varepsilon}_d \geqslant 0$.

Let $\boldsymbol{f} = 0$. Equation (7) is equivalent to

$$\begin{cases} \dot{\boldsymbol{\varepsilon}}_d = \overline{\boldsymbol{A}}_i\boldsymbol{\varepsilon}_d + \overline{\boldsymbol{B}}_{di}\boldsymbol{d}, \\ \gamma_d = \boldsymbol{C}_i\boldsymbol{\varepsilon}_d + \boldsymbol{D}_{di}\boldsymbol{d}. \end{cases} \tag{16}$$

From Definition 2, it follows that

$$\Gamma\boldsymbol{V}(\boldsymbol{\varepsilon}_d,\ i) = \boldsymbol{\varepsilon}_d^{\text{T}}\boldsymbol{\Xi}_i\boldsymbol{\varepsilon}_d + \boldsymbol{d}^{\text{T}}\overline{\boldsymbol{B}}_{di}^{\text{T}}\boldsymbol{P}_i\boldsymbol{\varepsilon}_d + \boldsymbol{\varepsilon}_d^{\text{T}}\boldsymbol{P}_i\overline{\boldsymbol{B}}_{di}\boldsymbol{d}. \tag{17}$$

By integrating Eq. (17) between 0 to t, the following can be obtained:

$$E\left\{\int_0^t \Gamma\boldsymbol{V}(\boldsymbol{\varepsilon}_d, i)\text{d}t\right\} = E\{\boldsymbol{V}(\boldsymbol{\varepsilon}_d, i)\} - \boldsymbol{V}(\boldsymbol{\varepsilon}_0, i_0). \tag{18}$$

In zero initial condition, considering the Dynkin formula, $E\left\{\int_0^t \Gamma\boldsymbol{V}(\boldsymbol{\varepsilon}_d,\ i)\text{d}t\right\} = E\{\boldsymbol{V}(\boldsymbol{\varepsilon}_d,\ i)\}$ can be obtained. By combining Eq. (9) and defining $J_1 = E\left\{\int_0^t \gamma_d^{\text{T}}\gamma_d\text{d}t\right\} - r_1^2E\left\{\int_0^t \boldsymbol{d}^{\text{T}}\boldsymbol{d}\text{d}t\right\}$, the following relation can be obtained by putting $\gamma_d = \boldsymbol{C}_i\boldsymbol{\varepsilon}_d + \boldsymbol{D}_{di}\boldsymbol{d}$ into J_1:

$$\begin{aligned} &J_1 + E\{\boldsymbol{V}(\boldsymbol{\varepsilon}_d,\ i)\} \\ &= E\left\{\int_0^t [\gamma_d^{\text{T}}\gamma_d - r_1^2\boldsymbol{d}^{\text{T}}\boldsymbol{d} + \Gamma\boldsymbol{V}(\boldsymbol{\varepsilon}_d, i)]\text{d}t\right\} \\ &= E\left\{\int_0^t \{\boldsymbol{\varepsilon}_d^{\text{T}}[\boldsymbol{\Xi}_i + \boldsymbol{C}_i^{\text{T}}C_i]\boldsymbol{\varepsilon}_d + \boldsymbol{\varepsilon}_d^{\text{T}}\left[\boldsymbol{P}_i\overline{\boldsymbol{B}}_{di} + \boldsymbol{C}_i^{\text{T}}\boldsymbol{D}_{di}\right]\boldsymbol{d}\right. \\ &\quad \left. + \boldsymbol{d}^{\text{T}}\left[\overline{\boldsymbol{B}}_{di}^{\text{T}}\boldsymbol{P}_i + \boldsymbol{D}_{di}^{\text{T}}\boldsymbol{C}_i\right]\boldsymbol{\varepsilon}_d + \boldsymbol{d}^{\text{T}}[-r_1^2\boldsymbol{I} + \boldsymbol{D}_{di}^{\text{T}}\boldsymbol{D}_{di}]\boldsymbol{d}\}\text{d}t\right\}. \end{aligned} \tag{19}$$

By applying Schur complements, the following can be obtained:

$$\begin{aligned} J_1 + E\{\boldsymbol{V}(\boldsymbol{\varepsilon}_d,\ i)\} = E\Bigg\{\int_0^t \Bigg\{ &(\boldsymbol{\varepsilon}_d^{\text{T}},\ \boldsymbol{d}^{\text{T}}) \\ &\times \begin{bmatrix} \boldsymbol{\Xi}_i + \boldsymbol{C}_i^{\text{T}}\boldsymbol{C}_i & \boldsymbol{P}_i\overline{\boldsymbol{B}}_{di} + \boldsymbol{C}_i^{\text{T}}\boldsymbol{D}_{di} \\ \overline{\boldsymbol{B}}_{di}^{\text{T}}\boldsymbol{P}_i + \boldsymbol{D}_{di}^{\text{T}}\boldsymbol{C}_i & -r_1^2\boldsymbol{I} + \boldsymbol{D}_{di}^{\text{T}}\boldsymbol{D}_{di} \end{bmatrix} \\ &\times \begin{pmatrix} \boldsymbol{\varepsilon}_d \\ \boldsymbol{d} \end{pmatrix} \Bigg\}\text{d}t\Bigg\}. \end{aligned} \tag{20}$$

Thus, $J_1 \leqslant 0$ can be guaranteed by matrix inequality (14) or (15). Without considering unknown disturbances, i.e., $\boldsymbol{d} = 0$, $\Gamma\boldsymbol{V}(\boldsymbol{\varepsilon}_d,\ i) < 0$ will reduce to $\boldsymbol{\Xi}_i < 0$. In terms of inequality (10), the augmented system (7) is asymptotically stochastically stable according to Lemma 1. This completes the proof.

Theorem 2 Considering MJSs (1), the dynamic error system (7) is almost asymptotically stable and satisfies condition (10), if there exists a set of mode-dependent symmetric positive-definite matrices $\boldsymbol{P}_i > 0$ and mode-dependent matrix $\overline{\boldsymbol{H}}_i$ such that the following relation holds:

$$\begin{bmatrix} \boldsymbol{\Xi}_i - \boldsymbol{C}_i^{\mathrm{T}}\boldsymbol{C}_i & \boldsymbol{P}_i\overline{\boldsymbol{B}}_{fi} - \boldsymbol{C}_i^{\mathrm{T}}\boldsymbol{D}_{fi} \\ \overline{\boldsymbol{B}}_{fi}^{\mathrm{T}}\boldsymbol{P}_i - \boldsymbol{D}_{fi}^{\mathrm{T}}\boldsymbol{C}_i & r_2^2\boldsymbol{I} - \boldsymbol{D}_{fi}^{\mathrm{T}}\boldsymbol{D}_{fi} \end{bmatrix} < 0, \tag{21}$$

or

$$\begin{bmatrix} \boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P}_i + \boldsymbol{P}_i\boldsymbol{A}_i + \sum_{j=1}^{N}\pi_{ij}\boldsymbol{P}_j - \boldsymbol{C}_i^{\mathrm{T}}\boldsymbol{C}_i - \boldsymbol{C}_i^{\mathrm{T}}\overline{\boldsymbol{H}}_i^{\mathrm{T}} - \overline{\boldsymbol{H}}_i\boldsymbol{C}_i & \boldsymbol{C}_i^{\mathrm{T}}\boldsymbol{D}_{fi} + \overline{\boldsymbol{H}}_i\boldsymbol{D}_{fi} - \boldsymbol{P}_i\boldsymbol{B}_{fi} \\ \boldsymbol{D}_{fi}^{\mathrm{T}}\boldsymbol{C}_i + \boldsymbol{D}_{fi}^{\mathrm{T}}\overline{\boldsymbol{H}}_i^{\mathrm{T}} - \boldsymbol{B}_{fi}^{\mathrm{T}}\boldsymbol{P}_i & r_2^2\boldsymbol{I} - \boldsymbol{D}_{fi}^{\mathrm{T}}\boldsymbol{D}_{fi} \end{bmatrix} < 0. \tag{22}$$

Proof For given mode-dependent symmetric positive-definite matrices $\boldsymbol{P}_i \in R^{n\times n}$, $i \in M$, considering system (7), the Lyapunov function is defined as $\boldsymbol{V}(\boldsymbol{\varepsilon}_f,\ i) = \boldsymbol{\varepsilon}_f^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{\varepsilon}_f \geqslant 0$.

Let $\boldsymbol{d} = 0$. Equation (7) is equivalent to

$$\begin{cases} \dot{\boldsymbol{\varepsilon}}_f = \overline{\boldsymbol{A}}_i\boldsymbol{\varepsilon}_f + \overline{\boldsymbol{B}}_{fi}\boldsymbol{f}, \\ \boldsymbol{\gamma}_f = \boldsymbol{C}_i\boldsymbol{\varepsilon}_f + \boldsymbol{D}_{fi}\boldsymbol{f}. \end{cases} \tag{23}$$

By defining $J_2 = E\left\{\int_0^t \boldsymbol{\gamma}_f^{\mathrm{T}}\boldsymbol{\gamma}_f \mathrm{d}t\right\} - r_2^2 E\left\{\int_0^t \boldsymbol{f}^{\mathrm{T}}\boldsymbol{f}\mathrm{d}t\right\}$ and following the similar proof in Theorem 1, matrix inequalities (21) and (22) can be acquired. This completes the proof.

For MJSs (1) without jump parameters, Theorems 1 and 2 reduce to the main results of linear time-invariant system [6].

Theorem 3 Considering MJSs (1), the dynamic error system (7) is almost asymptotically stochastically stable and satisfies condition (13), if there exists a set of mode-dependent symmetric positive-definite matrices $\boldsymbol{P}_i > 0$ and mode-dependent matrix $\boldsymbol{H}_i = \boldsymbol{P}_i^{-1}\overline{\boldsymbol{H}}_i$ such that matrix inequalities (14) and (21) hold or matrix inequalities (15) and (22) hold for $r_2 > r_1 > 0$.

According to the above Theorems, the design algorithm of the optimized observer can be summarized as follows:

1) Obtain $r_{1\min}$, $r_{2\max}$ by solving Theorem 1 and Theorem 2, respectively.

2) Let $r_2 = r_{2\max}$. If $r_{11} = r_{1\min}$ and r_2 are feasible for LMIs (15) and (22), the optimized $\boldsymbol{H}_i = \boldsymbol{P}_i^{-1}\overline{\boldsymbol{H}}_i$ can be obtained. Otherwise, denote $r_{1i} = r_{1(i-1)} + \eta_1$, $i = 1,2,\ldots$, $r_{2i} = r_{2(i-1)} - \eta_2$, $i = 1,2,\ldots$, where $\eta_1 > 0$, $\eta_2 > 0$ are sufficient small scalars and $i = 1,2,\ldots$ is the ith iteration. With the new r_{1i} and r_{2i}, minimize $J_{1\mathrm{e}}$ that is subject to LMIs (15) and (22) to test the feasibility. Repeat the operation until $J_{1\mathrm{e}} = \min\limits_{i=1,2,\ldots}(r_{1i}/r_{2i})$, and calculate the solution r_1, r_2, $\boldsymbol{P}_i$, and $\boldsymbol{H}_i = \boldsymbol{P}_i^{-1}\overline{\boldsymbol{H}}_i$.

3) Let $r_1 = r_{1\min}$. If $r_{21} = r_{2\max}$ and r_1 are feasible for LMIs (15) and (22), the optimized $\boldsymbol{H}_i = \boldsymbol{P}_i^{-1}\overline{\boldsymbol{H}}_i$ can be obtained. Otherwise, denote $r_{2i} = r_{2(i-1)} - \xi_1$, $i = 1,2,\ldots$, $r_{1i} = r_{1(i-1)} + \xi_2$, $i = 1,2,\ldots$, where $\xi_1 > 0$, $\xi_2 > 0$ are sufficient small scalars and $i = 1,2,\ldots$ is the ith iteration. With the new r_{1i} and r_{2i}, minimize $J_{2\mathrm{e}}$ that is subject to LMIs (15) and (22) to test the feasibility. Repeat the operation until $J_{2\mathrm{e}} = \min\limits_{i=1,2,\ldots}(r_{1i}/r_{2i})$, and calculate the solution, r_1, r_2, $\boldsymbol{P}_i$ and $\boldsymbol{H}_i = \boldsymbol{P}_i^{-1}\overline{\boldsymbol{H}}_i$.

4) Choose the corresponding matrices r_1, r_2, $\boldsymbol{P}_i$ and $\boldsymbol{H}_i = \boldsymbol{P}_i^{-1}\overline{\boldsymbol{H}}_i$ that correspond to $J_{\min} = \min\{J_{1\mathrm{e}},\ J_{2\mathrm{e}}\}$ in the above steps 1), 2) and 3), by which the optimized observer can be obtained. By setting appropriate parameters, the numeral simulation for the optimized observer demonstration will be described in Sect. 4.

4 Numeral example

Consider a class of continuous uncertain MJSs (2) with parameters given by

Mode 1:

$$\boldsymbol{A}_1 = \begin{bmatrix} 2 & 5 \\ -2 & -3 \end{bmatrix}, \boldsymbol{B}_1 = \begin{bmatrix} 0.4 \\ 0.2 \end{bmatrix}, \boldsymbol{C}_1 = [1 \quad 1],$$

$$\boldsymbol{B}_{f1} = \begin{bmatrix} 4.3 \\ 2.8 \end{bmatrix}, \boldsymbol{D}_{f1} = [2.2], \boldsymbol{B}_{d1} = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}, \boldsymbol{D}_{d1} = [0.5];$$

Mode 2:

$$\boldsymbol{A}_2 = \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix}, \boldsymbol{B}_2 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}, \boldsymbol{C}_2 = [1 \quad 1],$$

$$\boldsymbol{B}_{f2} = \begin{bmatrix} -3.6 \\ 2.1 \end{bmatrix}, \boldsymbol{D}_{f2} = [2.2], \boldsymbol{B}_{d2} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}, \boldsymbol{D}_{d2} = [0.4].$$

And the transition rate matrix is defined by

$$\boldsymbol{\Pi} = \begin{bmatrix} -2 & 2 \\ 1 & -2 \end{bmatrix}.$$

By solving LMIs (15) and (22), $r_{1\min} = 0.25$ and $r_{2\max} = 2.19$ can be obtained. From the optimal algorithm formulated in Sect. 3, the optimal scalars will be $r_1 = 0.92$, $r_2 = 1.48$, thus the mode-dependent optimized observer gain matrices are as follows:

$$\boldsymbol{H}_1 = \begin{bmatrix} 1.4771 \\ 1.1297 \end{bmatrix}, \ \boldsymbol{H}_2 = \begin{bmatrix} -1.0385 \\ 0.9960 \end{bmatrix}.$$

Assume external reference input is the unit step signal with 1 V, and the fault signal is a square wave signal with unit amplitude (1 V) that occurs from the 8th second to the 12th second. The unknown disturbance is a random white noise sequence with variance 0.05, as shown in Fig. 1. The system mode, residual signal and residual evaluation function are shown in Figs. 2, 3 and 4 respectively.

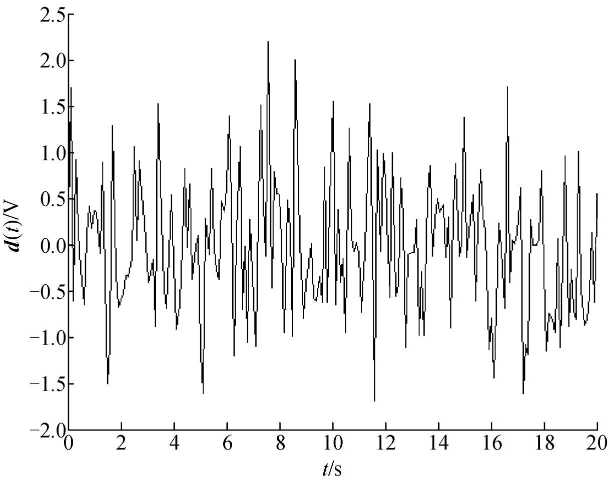

Fig. 1 Unknown disturbance

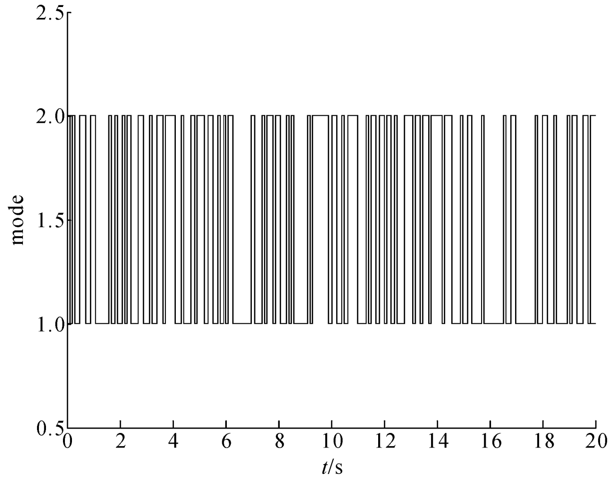

Fig. 2 System mode

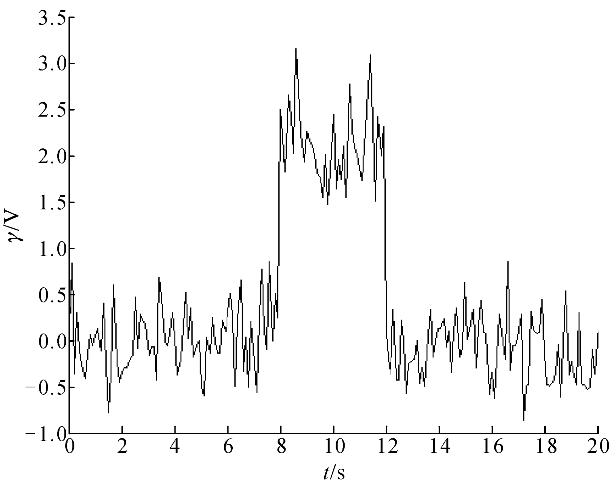

Fig. 3 Residual signal

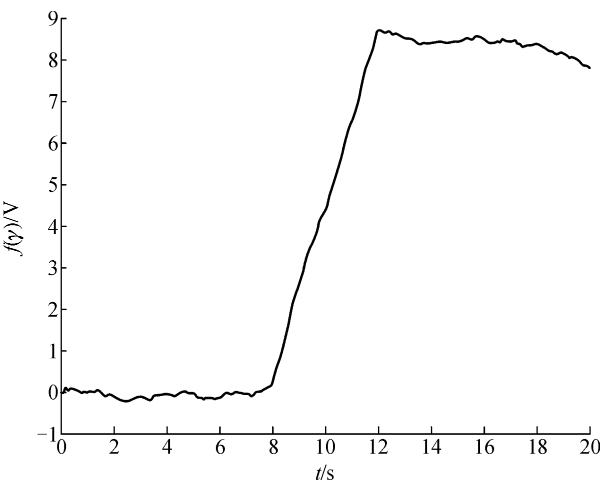

Fig. 4 Residual evaluation function

It can be seen from Fig. 4 that $J_{\mathrm{th}} = r_1^2 \Delta d = 2.54$ when assuming $\Delta d = 3.0$ V. It can be concluded that, when $t = 9.1$ s, $f(\gamma) = E\left\{\int_0^t \gamma^{\mathrm{T}} \gamma \mathrm{d}t\right\} = 2.75 > J_{\mathrm{th}}$. Thus, the faults that appear will be detected 1.1 s later after the faults happen.

5 Conclusions

Based on the norm theory, the authors design an optimized observer for a class of MJSs with unknown disturbances. By selecting the appropriate performance index, a sufficient condition that the solution to the fault detection observer exists is given and proved, and an optimized design approach is presented. Simulation results demonstrate the effectiveness of the developed approaches.

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