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RESEARCH ARTICLE

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New mutual coupling compensation method and its application in DOA estimation

Abstract A new mutual coupling compensation method based on a new mutual impedance matrix, as well as its application to dipole arrays, are proposed. This new mutual impedance matrix is deduced by electromotive force (EMF) method, based on the current distribution obtained by the characteristic basis function method. It appears in a concise and explicit formulation that facilitates the numerical calculation. The compensation performance is demonstrated and evaluated through its application in direction of arrival (DOA) estimation. Numerical results show that the proposed method exhibits excellent compensation performance compared with conventional mutual impedance matrix approaches.

Keywords mutual coupling compensation, characteristic basis function, electromotive force (EMF) method, MUSIC algorithm

1 Introduction

Mutual coupling among array elements can affect the phase distribution of array currents, thus decreasing the performance of the direction of arrival (DOA) estimation significantly [1,2]. Therefore, mutual coupling compensation plays an important role in the society of DOA estimation. Existing methods to tackle this difficulty can be divided into two kinds. The first kind considers mutual coupling by modifying the DOA algorithm [1]. The other method uses a coupling matrix, based on the electromagnetic method, to relate the received signals with coupling to those without coupling [2–8]. The method used in this paper belongs to the latter.

Among various coupling matrices, the most popular one is the mutual impedance matrix proposed in Ref. [3]. In this method, the open-circuit signals are considered as ideal coupling free signals. Significant improvement can be shown when applying this method in DOA estimation [2]. However, there are several disadvantages to using this method. First, the sinusoidal current distribution is assumed for the dipole. Second, self-impedance in the receiving mode is considered the same as that in the transmitting mode, while it is proven to be different in Ref. [9]. Finally, the scattering of the open-circuit dipole is neglected. In Ref. [4], the inverted Fourier transformation is used to calculate the mutual coupling matrix based on the far-field pattern. However, the accuracy is only guaranteed when the element distance is less than half wavelength. The method of moment (MoM) is used in Refs. [5,6] to calculate the coupling. However, this requires the accurate current distribution or accurate incident direction, which are unknown in practice. Moreover, the calculation burden becomes extensively large when the number of elements becomes large. A new definition of the mutual impedance is proposed in Refs. [7,8]. It can alleviate the disadvantage of the conventional open-circuit model, and exhibits good compensation performance. However, the calculations for the mutual coupling between every two elements are required, where one acts as the transmitter and the other as the receiver. Thus, large calculations are needed for large arrays. A new mutual coupling compensation method is proposed in this paper. It adopts the characteristic basis function (CBF) as the current distribution in the array and employs the electromotive force (EMF) to calculate the coupling matrix composed of self and mutual impedances. It has an explicit formula and does not require solving the MoM equation as in Refs. [7,8]. The applications of this new mutual coupling compensation method in DOA estimation demonstrates that compensation accuracy can be significantly improved compared with the conventional open-circuit model.

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2 Characteristic basis functions

CBFs are proposed to solve the large MoM [10]. It can decrease the size of matrix equations without losing accuracy. Therefore, it is suitable for the numerical analysis of large arrays. The principle of CBF is to divide the whole structure into M sub-structures, which can be the elements for arrays. There are M CBFs for every element, where the primary CBF (PCBF) reflects the self-action of each element, while the other $M-1$ CBFs, which are called second CBFs (SCBF), reflect the mutual coupling between elements. Additionally, higher orders of CBFs can be used to describe high order mutual coupling, although they can be usually neglected. In general, only M^2 CBFs are needed to model the whole structure. Thus, the size of the matrix equation can be decreased significantly.

The received signals of a uniform linear array with M dipoles will be solved below. This array is aligned along x-axis, as depicted in Fig. 1. The polarization of each element is z-axis, and loaded with the impedance of Z_L. The radius of element is $\lambda/200$, while the spacing of the neighbor element is d. Figure 2 illustrates the division of each element. A piece-wise sinusoidal basis function and Galerkin match are adopted. The PCBF can then be calculated as

$$\boldsymbol{Z}^{ii}_{(N\times N)}\boldsymbol{I}^{i}_{(N\times 1)} = \boldsymbol{V}^{i}_{(N\times 1)}, \quad i = 1,2,...,M, \tag{1}$$

where

$$\boldsymbol{Z}^{ii} = \begin{bmatrix} Z^{ii}_{11} & \cdots & Z^{ii}_{1,(N+1)/2} & \cdots & Z^{ii}_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ Z^{ii}_{(N+1)/2,1} & \cdots & Z^{ii}_{(N+1)/2,(N+1)/2} + Z_L & \cdots & Z^{ii}_{(N+1)/2,N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ Z^{ii}_{N1} & \cdots & Z^{ii}_{N,(N+1)/2} & \cdots & Z^{ii}_{NN} \end{bmatrix}, \tag{2}$$

and N is the number of the basis functions. $\boldsymbol{I}$ and $\boldsymbol{V}$ are the current and excitation vectors, respectively. By calculating Eq. (1), the current distribution in the absence of other elements, i.e., PCBF, can be calculated as

$$i^{i}(z) = I^{i}_{b}\sum_{m=1}^{N} I^{i}_{m} f^{i}_{m}(z), \quad i = 1,2,...,M, \tag{3}$$

where I^i_m is the mth coefficient of $\boldsymbol{I}$, and is normalized by the current of the load, denoted as I^i_b. $f^i_m(z)$ is the mth sinusoidal function. It can be seen that PCBF is a numerical current distribution. In addition, the sinusoidal distributed current that has been widely used for dipoles is just a particular case as $N=1$.

The above analysis shows that PCBFs are determined by both the structure of antenna and excitations. However, in the DOA estimation, the incoming directions of incident signals are unknown, and thus the exact PCBF of each element is also unknown. This is why an under-determined equation was deduced in Ref. [6]. Here, the approximation assumes that the incident wave along the x-axis, as shown in Fig. 1, will be adopted below. This assumption is based on the fact that the current distribution of the dipole is stable when the incident wave does not significantly depart from the xoy plane [9]. This assumption will be verified by the numeric example that follows, in which the loading current of a 100-dipole array will be calculated. The load impedance Z_L of each element is 50 Ω. The MoM, CBF method with PCBF and SCBF, CBF method with only PCBF and sinusoidal current distribution are used respectively, where the PCBF and SCBF are obtained based on the above assumption. It can be seen from Fig. 3 that when $\theta = 90°$, i.e., the incident wave lies in the xoz plane, the result with PCBF + SCBF is almost the same as that of MoM, while the result with PCBF only is also nearly the same as that of MoM. Since the unknown quantity for the method with PCBF is only 100, calculation time is nearly 1 percent of the time with MoM. However, the results with sinusoidal current distribution depart from MoM considerably. When θ is further turned to 70°, the result with PCBF also agrees well with that of MoM.

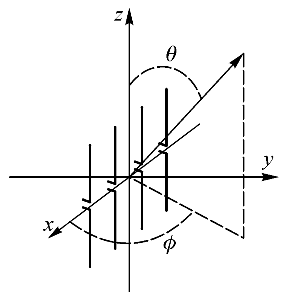

Fig. 1 Illustration of dipole array

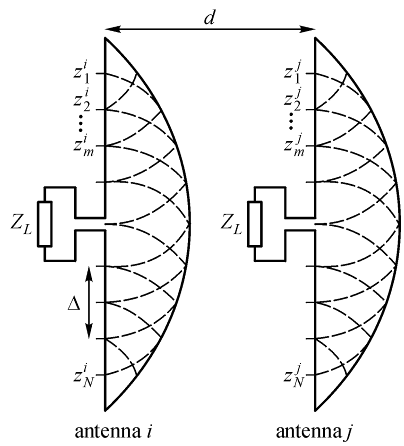

Fig. 2 Detailed division of element

Through the analysis above, it can be seen that using PCBF as the current in the array is not only more advantageous than a sinusoidal current, but also has adequate accuracy. The current adopted in Refs. [7,8] is actually the same as PCBF, but a detailed analysis and verification have not been presented.

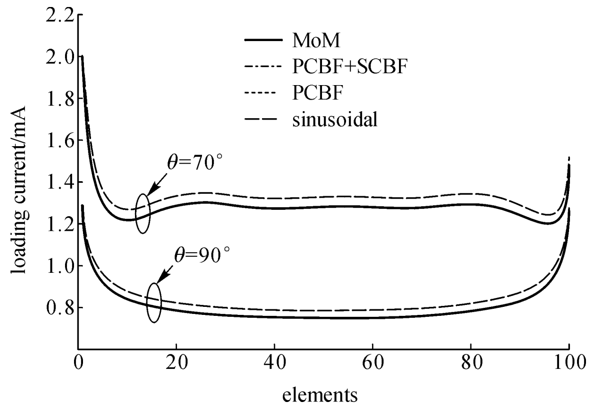

Fig. 3 Amplitude of loading current of 100-dipole array

3 Calculation of coupling matrix

When the current distribution of a dipole element is known, the mutual impedance of elements i and j can be calculated using EMF method [11], i.e.,

$$Zt^{ij}=\frac{1}{I_b^{i*}I_b^j}\int_{-L/2}^{L/2}E_z^j(z)i^{i*}(z)\mathrm{d}z, \tag{4}$$

where L is length of the dipole, $E_z^j(z)$ is the electric field along the axis of the jth element excited by the ith element. I_b^i and I_b^j are the load currents of the ith and jth elements respectively, where superscript * denotes the conjugate. A minus sign is reduced in Eq. (4) when compared with that in Ref. [11] because a different current direction is adopted. Using Eq. (3), and considering that $E_z^j(z)$ is the sum of each basis function f(z), the mutual impedance of elements i and j can be formulated as

$$Zt^{ij}=\sum_{m=1}^{N}I_m^{i*}\sum_{n=1}^{N}I_n^j\int_{z_m^i-\Delta/2}^{z_m^i+\Delta/2}E_{zn}^j(z)f_m^i(z)\mathrm{d}z, \tag{5}$$

where the meaning of z_m^i and Δ are shown in Fig. 2. $E_{zn}^j(z)$ is the electric field along the axis of the jth element excited by the nth basis function of the ith element. Defining the mutual impedance between the mth basis function and nth basis function as

$$M_{mn}^{ij}=\int_{z_m^i-\Delta/2}^{z_m^i+\Delta/2}E_{zn}^j(z)f_m^i(z)\mathrm{d}z. \tag{6}$$

The explicit expression of Eq. (6) had been presented in Ref. [12], i.e.,

$$M_{mn}^{ij}=\frac{15}{\sin^2(k\Delta/2)}\sum_{m=-2}^{2}\sum_{n=-1}^{1,2}A(m)\times\exp\left[-\mathrm{j}kn\left(\left|z_m^i-z_n^j\right|+\frac{m\Delta}{2}\right)\right]Ei(k\beta), \tag{7}$$

where k is wave-number; $Ei(x)=Ci(x)-\mathrm{j}Si(x)$, $Ci(x)$ and $Si(x)$ are the cosine integral and sine integral, respectively; $\beta=[d^2+(\left|z_m^i-z_n^j\right|+m\Delta/2)^2]^{1/2}-n(\left|z_m^i-z_n^j\right|+m\Delta/2)$; and d is the spacing between two neighboring elements. For the self-impedance, d is adopted as the radius of the dipole. $A(-2)=A(2)=1, A(-1)=A(1)=-4\cos(k\Delta/2)$, $A(0)=2[1+2\cos^2(k\Delta/2)]$. Using Eq. (6) and considering the PCBF obtained by the approximate method as described in Sect. 2, Eq. (5) can be rewritten as

$$Zt^{ij}=\boldsymbol{I}^{\mathrm{H}}\boldsymbol{M}^{ij}\boldsymbol{I}, \tag{8}$$

where the elements of $\boldsymbol{M}^{ij}$ have been defined in Eq. (7). Since $\boldsymbol{M}^{ij}$ is a Toeplitz matrix, the calculation complexity is relatively low.

Once the mutual impedances of each element have been obtained by Eq. (8), a mutual coupling matrix $\boldsymbol{Zt}$ can be composited. Thus, the mutual coupling compensation matrix can be calculated similar to Ref. [3]:

$$\boldsymbol{C}=(\boldsymbol{Zt}+\boldsymbol{U}Z_L)/(Zt^{11}+Z_L), \tag{9}$$

where $\boldsymbol{U}$ is the unit matrix. The coupling free received signal vector $\boldsymbol{V}_n$ can then be written as

$$\boldsymbol{V}_n=\boldsymbol{C}\boldsymbol{V}_{\mathrm{meas}}, \tag{10}$$

where $\boldsymbol{V}_{\mathrm{meas}}$ is the actual signal measured at the terminal of each dipole. Thus, $\boldsymbol{V}_n$ can be used for the conventional DOA estimation algorithm.

4 Numerical results and discussion

In this section, DOA estimation using MUSIC algorithm is chosen as an application to check the validity of the proposed method. The dipole array illustrated in Fig. 1 will be used. The incident fields, with the incident angle of (θ, φ), are E-Polarized plane waves and 5000 snapshots are collected.

Figure 4 shows an example selected from Ref. [2]. Two incoherent signals with equal power of 3 dB relative to noise come from $(90°, 90°)$ and $(90°, 105°)$ respectively. The number of elements is 5 and the load impedance Z_L is assumed to be the complex conjugate of the self-impedance of the dipole antenna. It can be seen that using the measured voltages with no compensation for the mutual coupling effects had the worst performance. Using the compensation method defined in Ref. [2] shows a significant improvement over the first case. The third and the forth kinds of voltages are the voltages compensated by the method in Ref. [7] and our proposed method respectively. It can be seen that two sharper peaks are accurately located at 90° and 105° when using these voltages.

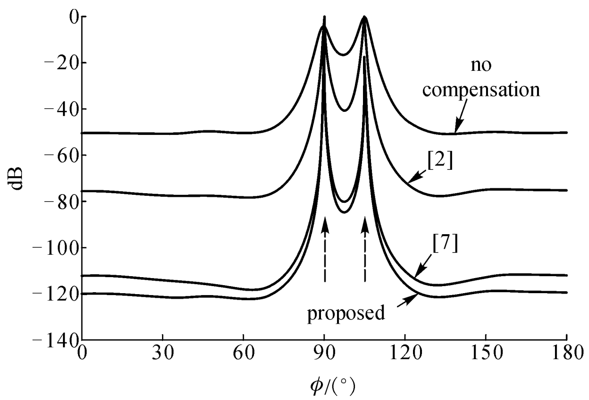

Fig. 4 Spatial spectrum of MUSIC algorithm for the direction of two incoherent signals using different kinds of voltages ($\theta = 90°$, $\phi = 90°$ and 105° respectively)

The second example considers two coherent signals coming from (90°, 30°) and (90°, 57°) respectively. This example has been studied in Ref. [7]. A four-element array is used and $Z_L = 50\ \Omega$ for all elements. The SNR is 10 dB for both signals. The same four kinds of voltages are applied. As shown in Fig. 5, both the proposed method and the method in Ref. [7] outperform the method in Ref. [2], while the case using the voltages without compensation fails to indicate the exact directions.

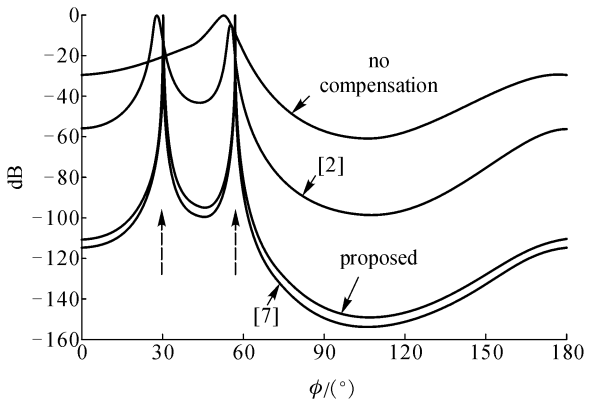

Fig. 5 Spatial spectrum of MUSIC algorithm for the direction of two coherent signals using different kinds of voltages ($\theta = 90°$, $\phi = 30°$ and 57° respectively)

In the last example, to verify the compensation performance when the incident signals are not coming from the normal direction, the incoming directions are changed to (70°, 90°) and (70°, 105°) respectively. It can be seen from Fig. 6 that substantial improvements can be obtained by the proposed method and the method in Ref. [7] even when θ is changed to 70°. Note that the method in Ref. [7] needs to solve the MoM equation, while our method depends only on the PCBF and can be calculated a priori to reduce the computational burden. Thus, it is more convenient than the method proposed by Ref. [7]. When θ is further changed to 40°, it can be seen from Fig. 6 that although the compensation performance is decreased slightly, the incident direction can still be estimated accurately.

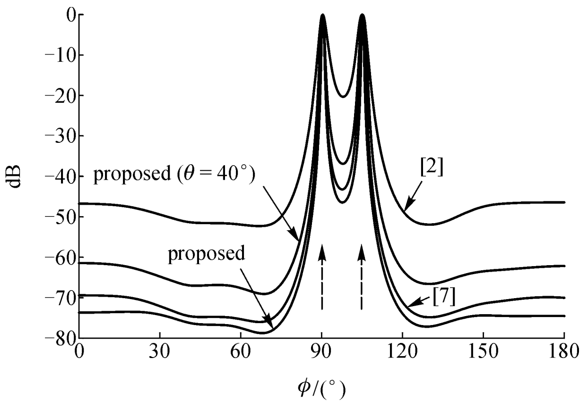

Fig. 6 Spatial spectrum of MUSIC algorithm for the direction of two coherent signals using different kinds of voltages ($\theta = 70°$, $\phi = 90°$ and 105° respectively)

5 Conclusions

To get the accurate mutual coupling information between elements requires the exact current distribution of each element, which can only be solved by the numerical method within the knowledge of excitation. Thus, mutual coupling compensation becomes a difficult problem in the array processing society. The CBF method has been applied in this paper, and the PCBF is adopted as the current distribution of each element. The accuracy of this approximation has also been proven when compared with the results of MoM. Based on the PCBF current distribution, the mutual coupling compensation matrix can be deduced using the EMF method. A simple relationship between the measured voltages and coupling free voltages can thus be built through the obtained mutual impedance compensation matrix. Numerical results show that excellent compensation performance can be achieved by the proposed method compared with the conventional methods, while featuring a concise expression that facilitates the numerical calculation.

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