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# Error-space estimate method for generalized synergic target tracking

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**Abstract** To improve the tracking accuracy and stability of an optic-electronic target tracking system, the concept of generalized synergic target and an algorithm named error-space estimate method is presented. In this algorithm, the motion of target is described by guide data and guide errors, and then the maneuver of the target is separated into guide data and guide errors to reduce the maneuver level. Then state estimate is implemented in target state-space and error-space respectively, and the prediction data of target position are acquired by synthesizing the filtering data from target state-space according to kinematic model and the prediction data from error-space according to guide error model. Differing from typical multi-model method, the kinematic and guide error models work concurrently rather than switch between models. Experiment results show that the performance of the algorithm is better than Kalman filter and strong tracking filter at the same maneuver level.

**Keywords** target tracking, generalized synergic target, position prediction, error-space estimate

## 1 Introduction

To restrain miscellaneous light to improve the detection capability of the optical instrument in target tracking, the visual field of the instrument is generally very small. Moreover, various factors interfere with the extraction of the target, which consequently influence tracking stability and accuracy [1]. Therefore, high accuracy prediction data

of the target position in the tracking process is particularly expected. At present, there are mainly two methods of improving the target tracking accuracy. The first one is to improve the kinematic model of the target and make it accord with the actual maneuver. Then, various kinematic models of target and tracking algorithms [2,3] are presented. However, it is difficult to track the target effectively by single model algorithm when the target is at different maneuver levels. Although tracking accuracy can to some extent be improved by multiple model algorithms [4,5], calculation cost increases considerably and the number of models is also finite that not all situations of the target maneuver can be covered. The multi-model approaches are characterized that working model is switched between multiple models in the tracking system, and at any time only one model is valid rather than a number of models performing in parallel. The second one is to improve the accuracy and validity of observation data, which can be achieved with multi-sensor data fusion [6,7]. However, the data fusion method is only to improve the observation accuracy to obtain estimation of target state with higher accuracy. In predicting and estimating the target state, because prediction error is mainly caused by models [8], the effect of data fusion method is restricted. Because of the unpredictability of target maneuver, it is significant to search for a tracking algorithm that can perform well at various target maneuver levels.

Focused on the characteristic of target tracking of an optical instrument, the concept of generalized synergic target is presented in this article, and a tracking algorithm named error-space estimate method for generalized synergic target is brought forward. In this algorithm, the tracking of the target with different maneuver levels is transformed to weak maneuver target, so that the filtering model can be applied to the target with a wider maneuver range.

## 2 Generalized synergic target

Synergic target is a target that continuously provides tracking instrument with its location information to guide

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the instrument in the tracking process. Because of the small visual field, most optical instruments lack search capability and therefore, external guide data are necessary at the beginning of target tracking. The tracking instrument operates according to the guide data until the target is acquired, and it can then perform the tracking according to the guide data or the observation of the instrument. The synergic target can be classified into the following types:

1) Actual synergic target: The target provides the optical instrument with real-time guide data. A target of this kind is mainly presented in an experimental system. Because of time delay of guide data, to satisfy the demand of the optical instrument, it is required that the target should provide guide data frequently enough or the apparent velocity of the target in the visual field of optical instrument should not be too large. It means that for each time  $k$ , the following relation is tenable:

$$\|\mathbf{Z}(k) - \mathbf{L}(k)\| \leq \frac{d}{2}, \quad (1)$$

where  $\mathbf{Z}(k) \in R^2$  is the actual location of the target,  $\mathbf{L}(k) \in R^2$  is the guide data and  $d \in R^+$  is the visual field of the optical instrument.

2) Target of which the guide data is provided by other observation instruments tracking the target: For the non-synergic target, a tracking instrument with search capability is required to work together with an optical instrument, and the search instrument seeks and supplies real-time observation data of the target location to optical instrument as the guide data. If providers of actual guide data are not stressed and the guide data are regarded as being obtained from the target, it will be in the same situation with type 1). Such a non-synergic target can also be considered as a generalized synergic target. Then observation data provided by the searching instrument is called generalized guide data and are required to satisfy Eq. (1).

3) Equivalent generalized synergic target: When the communication period between the optical instrument and generalized synergic target of types 1) and 2) equates the observation period of the optical instrument, presuming that  $\mathbf{Z}_L(k-1)$  is the measurement provided by the generalized synergic target and  $\mathbf{Z}(k-1)$  is provided by the optical instrument at time  $k-1$ , we can obtain the following equation without consideration of observation noise:

$$\mathbf{Z}_L(k-1) \approx \mathbf{Z}(k-1). \quad (2)$$

Because  $\mathbf{Z}_L(k-1)$  is received by the optical instrument at time  $k$  through the communication interval  $T$ , and it is regarded as guide data  $\mathbf{L}(k)$  of time  $k$ , thus  $\mathbf{Z}_L(k-1) = \mathbf{L}(k)$ . According to Eq. (2), we have

$$\mathbf{L}(k) \approx \mathbf{Z}(k-1). \quad (3)$$

Actually, the generalized guide data equate the observation data of the optical instrument at the last sampling

time. If observation of optical instrument  $\mathbf{Z}(k)$  is always available at each time  $k$ , using  $\mathbf{Z}(k)$  as guide data of time  $k+1$  directly amounts to using  $\mathbf{L}(k+1)$ . Even if there is no actual guide data  $\mathbf{L}(k+1)$ , the tracking can also proceed with the way of synergic target. The target is then regarded as an equivalent generalized synergic target, and the final observation data of the optical instrument become generalized guide data. Likewise, the data should satisfy Eq. (1), which can then be expressed as

$$\|\mathbf{Z}(k) - \mathbf{Z}(k-1)\| \leq \frac{d}{2}. \quad (4)$$

4) Satellite: Satellite a special target. Because the orbit can be predicted by orbit parameters and also serve as the guide data of the observation instrument, based on the characteristic of synergic target, a satellite can be regarded as a generalized synergic target.

To sum up, with regard to an optical instrument with a small visual field, satellites and all other targets satisfying Eqs. (1) or (4) can be described as a generalized synergic target. Thus, the conclusions of this article have universal significance for target tracking of optical instruments.

### 3 Error-space estimate method

#### 3.1 Proposition of the problem

For target tracking of a single optical instrument, the state and observation equations of the target are

$$\mathbf{X}(k) = \mathbf{\Phi}(k-1)\mathbf{X}(k-1) + \mathbf{\Gamma}(k-1)\mathbf{W}(k-1), \quad (5)$$

$$\mathbf{Z}(k) = \mathbf{H}(k)\mathbf{X}(k) + \mathbf{V}(k). \quad (6)$$

where  $\mathbf{X}(k) \in R^n$  is the state vector of the target at time  $k$ ,  $\mathbf{\Phi}(k-1)$  is the state transfer matrix,  $\mathbf{\Gamma}(k-1)$  is the input matrix of noise,  $\mathbf{Z}(k) \in R^2$  is observation vector of the target position at time  $k$ ,  $\mathbf{H}(k)$  is the observation matrix, process noise  $\{\mathbf{W}(k)\}$  and observation noise  $\{\mathbf{V}(k)\}$  are white Gaussian noise sequences with zero average with  $\mathbf{Q}$  and  $\mathbf{R}$  as their covariance matrixes, and  $\mathbf{Q}$  is symmetric non-negative matrix but  $\mathbf{R}$  is symmetric positive matrix.

At present, in target tracking algorithms based on the Kalman filter, a suitable kinematic model of target  $\mathbf{\Phi}(k)$  is selected first, optimal state estimates of the target state  $\hat{\mathbf{X}}(k-1)$  at time  $k-1$  is obtained by filtering according to observation data  $\mathbf{Z}(k-1)$  at time  $k-1$ , then one-step prediction of the target state  $\hat{\mathbf{X}}(k)$  at the next time  $k$  can be obtained from the state Eq. (5):

$$\hat{\mathbf{X}}(k,k-1) = \mathbf{\Phi}(k,k-1)\hat{\mathbf{X}}(k-1). \quad (7)$$

And one-step prediction of the target position  $\hat{\mathbf{Z}}(k,k-1)$  at the next time  $k$  can also be figured out according to

observation Eq. (6):

$$\hat{\mathbf{Z}}(k, k-1) = \mathbf{H}(k)\hat{\mathbf{X}}(k, k-1). \quad (8)$$

Then,  $\hat{\mathbf{Z}}(k, k-1)$  can be used as guide data to drive the instrument to track and point the target.

Because of model error, position prediction accuracy of a high maneuver target may be comparatively low. If prediction error is so high that the target oversteps the visual field of the optical instrument, the target cannot be obtained, as shown in following equation:

$$\|\mathbf{Z}(k) - \hat{\mathbf{Z}}(k, k-1)\| > \frac{d}{2}, \quad (9)$$

where  $\mathbf{Z}(k)$  is the actual position of the target at time  $k$ .

### 3.2 Principle of error-space estimate method

Presume that the motion of the target occurs in the Cartesian coordinate system  $O-xyz$  established at an observation station. If the vector  $\mathbf{Z}$  is observation of the target position and vector  $\mathbf{L}$  is guide data, the guide error is then

$$\mathbf{e} = \mathbf{Z} - \mathbf{L}. \quad (10)$$

Define the coordinate system  $O'-x'y'z'$  obtained by moving  $O-xyz$  along guide data vector  $\mathbf{L}$ , and presume that there is a virtual observation station bound to  $O'-x'y'z'$ , to which guide error  $\mathbf{e}$  can be regarded as observation of the target position in the coordinate system  $O'-x'y'z'$ . As shown in Fig. 1, in the whole tracking process,  $O'-x'y'z'$  moves according to the guide data of the target in  $O-xyz$ , so that the guide error sequence  $\{\mathbf{e}(k)\}$  constitutes the track of the target motion in  $O'-x'y'z'$ .

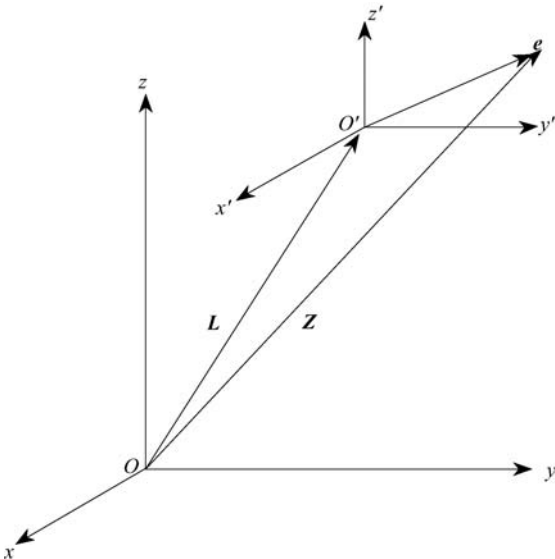


Fig. 1 Principle of error-space estimate method

According to Eq. (10), if guide error  $\mathbf{e}(k)$  is regarded as a state variable that is filtered and estimated in  $O'-x'y'z'$  to obtain optimal estimate  $\hat{\mathbf{e}}(k)$ , and synthesized with guide

data  $\mathbf{L}$ , then the target position estimate  $\hat{\mathbf{Z}}(k)$  which is more accurate than that of pure guide data  $\mathbf{L}$  can also be obtained to drive the instrument to track. For the coordinate system  $O-xyz$ , the method mentioned above is equivalent to filtering and estimating in error-space spanned by  $\mathbf{e}(k)$  and its rate of change. According to Eqs. (1) and (4), the upper bound of the guide error  $\mathbf{e}(k)$  is the visual field of instrument, and  $\mathbf{e}(k)$  will not vary much, which indicates that however the maneuver of the target is in  $O-xyz$ , in  $O'-x'y'z'$  the target always behaves as a weak maneuver target that moves slowly and even not, then a relatively simple target kinematic model and filtering algorithm can be adopted to track and predict, which will not result in high error. The physical meaning of this approach is that by coordinate transforming, the problem of target tracking in  $O-xyz$  is converted to the problem in  $O'-x'y'z'$  which moves along the generalized guide data vector  $\mathbf{L}$  relative to  $O-xyz$ .

If the coordinates mentioned above are not rectangular ones, the physical meaning of coordinate transforming of the method is not tenable any more. However, Eqs. (1), (4) and (10) are still correct, and the guide error  $\mathbf{e}(k)$  will not vary much, either. Therefore,  $\mathbf{e}(k)$  can be filtered and estimated in error-space similarly and utilized to amend the guide data  $\mathbf{L}$ , and target position prediction data  $\hat{\mathbf{Z}}(k)$  with much higher accuracy will be obtained to drive the instrument.

As to types 1) to 3) of generalized synergic target, generalized guide data are obtained by real-time measurement. Before being provided to the optical instrument, optimal estimate of guide data  $\hat{\mathbf{L}}(k)$  will be obtained by filtering and outliers restraining. For a satellite, it is not necessary because the guide data are obtained by calculation beforehand according to orbit parameters rather than measurement.

### 3.3 Calculation procedure of error-space estimate method

Presuming that the state and observation equations of the target are Eqs. (5) and (6), the state and observation equations of the guide error can then be expressed as

$$\mathbf{X}_e(k) = \Phi_e(k-1)\mathbf{X}_e(k-1) + \Gamma_e(k-1)\mathbf{W}_e(k-1), \quad (11)$$

$$\mathbf{e}(k) = \mathbf{H}_e(k)\mathbf{X}_e(k) + \mathbf{V}_e(k), \quad (12)$$

where  $\mathbf{X}_e(k) \in R^n$  is the state vector of the guide error at time  $k$ ,  $\Phi_e(k-1)$  is the state transfer matrix,  $\Gamma_e(k-1)$  is the input matrix of noise,  $\mathbf{e}(k) \in R^2$  is the observation vector of guide error at time  $k$ ,  $\mathbf{H}_e(k)$  is the observation matrix, process noise  $\{\mathbf{W}_e(k)\}$  and observation noise

$\{V_e(k)\}$  are white Gaussian noise sequences with zero average, with  $Q_e$  and  $R_e$  as their covariance matrixes, of which  $Q_e$  is symmetric non-negative matrix but  $R_e$  is symmetric positive matrix.

The calculation procedure of target tracking utilizing error-space estimate method can be described as follows:

1) Determining parameters of the target kinematic model in Eqs. (5) and (6) and the guide error model in Eqs. (11) and (12).

2) At time  $k$ , calculating the generalized guide error  $e(k)$  of time  $k$  according to observation data  $Z(k)$  and optimal estimate guide data  $\hat{L}(k)$  as

$$e(k) = Z(k) - \hat{L}(k). \quad (13)$$

3) Estimating one-step prediction of the generalized guide error  $\hat{e}(k+1, k)$  at time  $k+1$  utilizing the Kalman filter according to the guide error model Eqs. (11), (12) and observation of the generalized guide error  $e(k)$  from Eq. (13).

4) Estimating optimal guide data  $\hat{L}(k+1)$  at time  $k+1$  utilizing the Kalman filter according to the target kinematic model Eqs. (5), (6) and observation of generalized guide data  $L(k+1)$  which is equivalent to observation data  $Z(k)$  of time  $k$  according to Eq. (3).

5) Calculating one-step prediction of target position  $\hat{Z}(k+1, k)$  at time  $k+1$  according to  $\hat{L}(k+1)$  and  $\hat{e}(k+1, k)$ . The final optimal instrument guide data are

$$\hat{Z}(k+1, k) = \hat{L}(k+1) + \hat{e}(k+1, k). \quad (14)$$

## 4 Experiment and discussion

To compare tracking result with current typical filtering method, an experiment is implemented with equivalent generalized synergic target. Horizontal gimbal is adopted in the tracking system, and experiment data are from azimuth observation of the target shown in Fig. 2. It can be seen from 60 to 90 s of the tracking time, because the zenith passes with maximal elevation angle of  $74.947^\circ$ ,

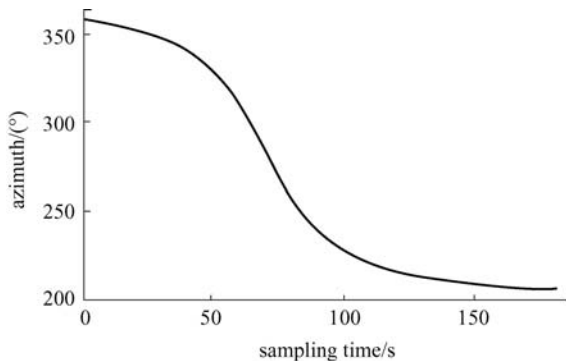


Fig. 2 Observation data of target azimuth

the azimuth data vary drastically and the target maneuvers strongly, but it does not happen at other time frames.

In the experiment, the Kalman filter, the strong tracking filter [9] and the error-space estimate method are utilized to estimate and predict the target position, and one-step prediction errors of them are compared to evaluate tracking accuracy of the methods. The CA model [2] is adopted as target kinematic model and guide error model. The standard deviation of process noise  $\{W(k)\}$  and  $\{W_e(k)\}$  are  $q = 0.000002$ , and the standard deviation of observation noise  $\{V(k)\}$  and  $\{V_e(k)\}$  are  $r = 0.000002$ . Softening factor of strong tracking filter is  $b = 1$ , and forgetting factor is  $\rho = 0.95$ . In the error-space estimate method, for each time  $k$ , the observation  $Z(k-1)$  of time  $k-1$  is performed as guide data  $L(k)$  of current time  $k$ , the generalized guide error  $e(k)$  is calculated according to Eq. (13) to estimate  $\hat{e}(k+1)$  of time  $k+1$ , and the observation  $Z(k)$  is utilized as guide data  $L(k+1)$  to estimate  $\hat{L}(k+1)$ , the optimal prediction of  $\hat{Z}(k+1, k)$  can then be obtained by Eq. (14).

Experiment results are shown in Fig. 3, where the dash-dotted line indicates one-step prediction error of the Kalman filter, the dashed line indicates one of the strong tracking filter and the solid line indicates one of the error-space estimate method. When the target maneuvers greatly from 60 to 90 s, the maximum prediction error of the error-space estimate method is less than that of the Kalman filter by nearly an order of magnitude, and is about 1/3 of the strong tracking filter. Also, the prediction error is also less than that of the Kalman filter and the strong tracking filter at other time frames. It is proved that the error-space estimate method can be applied to wider target maneuver extent than other tracking methods.

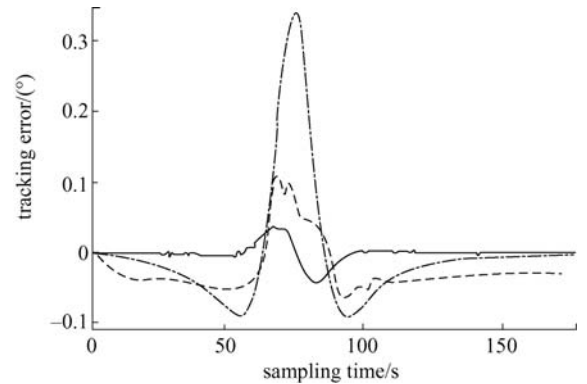


Fig. 3 Comparing of one-step estimate errors of three methods

## 5 Conclusions

Target tracked by a small visual field optical instrument can be depicted as a generalized synergic target that can provide guide data for tracking instruments, so that the filtering and predicting in the state space spanned by the

state parameter of the target motion in the tracking process can be transferred to that in an error-space spanned by the state parameter of the guide error.

The algorithm presented in this article is a tracking method with dual models and dual filters working in coordination. In the tracking system, both filtering of observation data and filtering and prediction of guide error coexist, and the target kinematic model and the guide error model are necessary. Meanwhile, differing from typical multi-model method, the kinematic model and guide error model work concurrently rather than switch between models. When maneuver occurs, it is separated into guide data and guide errors, and estimate is implemented in target state-space and error-space respectively, which amounts to reducing the maneuver level. The essence of the algorithm lies in converting target tracking with different maneuver levels to that of a weak maneuver target, so that the adopted filtering model is with high adaptability and can be applied to targets with wider maneuver range, while calculation cost is only twice the amount. Estimate error is also reduced by only filtering without predicting the generalized guide data of the target, i.e., without considering the maneuver of the next period.

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