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RESEARCH ARTICLE

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Self-tuning decoupled fusion Kalman filter based on the Riccati equation

Abstract An online noise variance estimator for multi-sensor systems with unknown noise variances is proposed by using the correlation method. Based on the Riccati equation and optimal fusion rule weighted by scalars for state components, a self-tuning component decoupled information fusion Kalman filter is presented. It is proved that the filter converges to the optimal fusion Kalman filter in a realization by dynamic error system analysis method, so that it has asymptotic optimality. Its effectiveness is demonstrated by simulation for a tracking system with 3 sensors.

Keywords multi-sensor information fusion, decoupled fusion, self-tuning fuser, Kalman filter, convergence in a realization

1 Introduction

The multi-sensor information fusion was a new frontier subject that emerged in the 1970s and has received great attention [1]. Generally, there are two kinds of information fusion methods [2], i.e., the state fusion method and the measurement fusion method. The state fusion method is divided into centralized Kalman filtering and distributed Kalman filtering [3]. Centralized fusion Kalman filtering can globally estimate the optimal fusion state in theory, but requires a large computation with poor fault tolerance. However, these drawbacks can be avoided by distributed information fusion Kalman filtering, which is globally optimal or suboptimal. In practice, because noise statistics are unknown and filtering for the system with unknown model parameters and/or noise statistics is called self-tuning filtering [4], its convergence analysis is very difficult and has not yet been solved. As a result, convergence analysis for the self-tuning fuser is difficult. The dynamic error system analysis method of convergence analysis for the self-tuning filter is presented in Ref. [4]. Its principle lies in attributing the problem where the self-tuning filter converges to the optimal filter to the problem where the dynamic error system converges to zero, i.e., the problem is translated into the stability problem of a non-homogeneous different equation (the bounded input-to-bounded output stability and the infinitesimal input-to-infinitesimal output stability). The identification method for the noise variances presented in Refs. [4,5] requires an online identifier for the autoregressive moving average (ARMA) innovation model of the system. The consistency of the obtained noise variance estimator depends on that of the ARMA innovation model estimation. The consistencies of some parameter estimation algorithms for the ARMA model given in Ref. [6] require stronger conditions, for instance, the sufficient condition for consistency of the recursive extended least square algorithm (RELS) is the positive realness condition. However, this condition cannot be directly verified in application. Since the self-tuning information fusion-filtering problem is seldom reported in literature, we obtain consistent estimation of the unknown noise variances by solving correlation function matrix equations for the multi-sensor system with those variances. Based on the Riccati equation and the optimal weighted fusion rule weighted by scalars for components [7], the self-tuning weighted fusion Kalman filter weighted by scalars for components is then presented. The novel filter realizes component decoupled fusion estimation [7]. Under the assumption that the measurement process is bounded with probability 1, it is proven that the local self-tuning Kalman filter converges to the steady-state optimal Kalman filter with probability 1 in Ref. [4]. Although the assumption is a very strong condition, it cannot be satisfied by many non-stationary measurement processes. Thus, the new concept that the self-tuning estimator converges in a realization is presented in Ref. [8], which is weaker than that with probability 1. Based on the stability of the dynamic error system, it is also proven that the self-tuning fusion Kalman predictor converges to the corresponding optimal fusion Kalman predictor in a realization, hence it is

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asymptotically optimal. The method presented in this article is completely different from that given in Refs. [8,9]. The self-tuning fusion Kalman predictor is designed based on ARMA innovation model identification in Refs. [8,9], and the self-tuning fusion Kalman filter is designed based on the Riccati equation in this article. The consistent estimation of noise variances can be directly obtained by the correlation method. Since an indirect identification of the noise variances by the ARMA innovation model is avoided, it overcomes the difficulty in proving the consistency of noise variance estimation.

2 Problem formulation

Consider the multi-sensor time-invariant linear discrete stochastic system as follows:

$$\begin{cases} x(t+1)=\Phi x(t)+\Gamma w(t), \\ y_i(t)=H_i x(t)+v_i(t), \end{cases} \quad i=1,2,...,L, \tag{1}$$

where t is the discrete time, the state $x(t)\in R^n$, the measurement of the ith sensor $y_i(t)\in R^{m_i}$, Φ, Γ, H_i are constant matrices with compatible dimensions, $w(t)$ is the input noise, and $v_i(t)$ are the measurement noises. Assume that $w(t)\in R^r$, $v_i(t)\in R^{m_i}$ are uncorrelated white noises with zero mean, variance matrices Q_w, Q_{vi}, respectively, and (Φ, H_i) is a completely observable pair, and (Φ, Γ) is a completely controllable pair. The objectives are to find the local self-tuning Kalman filters $\hat{x}_i(t|t)$, $i=1, 2,\ldots, L$, and the self-tuning component decoupled fusion Kalman filter $\hat{x}_0(t|t)$ based on the measurements $(y_i(t), y_i(t-1),\ldots)$.

The self-tuning local and component decoupled fusion Kalman filters are derived by the local steady-state Kalman filter and the optimal component decoupled fusion Kalman filter with the known noise variance, accompanied with the online identifier of noise variances.

Lemma 1 [3] When noise statistics are known, the local steady-state Kalman filters are

$$\hat{x}_i(t|t)=\Psi_{fi}\hat{x}_i(t-1|t-1)+K_{fi}y_i(t), \quad i=1, 2,...,L, \tag{2}$$

$$\Psi_{fi}=\left[I_n-K_{fi}H_i\right]\Phi, \tag{3}$$

$$K_{fi}=\Sigma_{ii}H_i^{\mathrm{T}}\left[H_i\Sigma_{ii}H_i^{\mathrm{T}}+Q_{vi}\right]^{-1}. \tag{4}$$

The prediction error variance matrices Σ_{ii} satisfy the following steady-state Riccati equation:

$$\Sigma_{ii}=\Phi\left[\Sigma_{ii}-\Sigma_{ii}H_i^{\mathrm{T}}\left(H_i\Sigma_{ii}H_i^{\mathrm{T}}+Q_{vi}\right)^{-1}H_i\Sigma_{ii}\right]\Phi^{\mathrm{T}} +\Gamma Q_w\Gamma^{\mathrm{T}}. \tag{5}$$

The local steady-state error cross-covariance matrices P_{ij} satisfy the Lyapunov equation:

$$\begin{aligned} P_{ij}&=\Psi_{fi}P_{ij}\Psi_{fj}^{\mathrm{T}}+\Delta_{ij}, \quad i,j=1, 2,...,L, \\ \Delta_{ij}&=\left[I_n-K_{fi}H_i\right]\Gamma Q_w\Gamma^{\mathrm{T}}\left[I_n-K_{fj}H_j\right]^{\mathrm{T}}+K_{fi}Q_{vi}\delta_{ij}K_{fj}^{\mathrm{T}}, \end{aligned} \tag{6}$$

where $\delta_{ii}=1$, $\delta_{ij}=0$, $i\neq j$.

Lemma 2 [3] For the multi-sensor Eq. (1), the optimal fusion steady-state Kalman filter weighted by scalars for components is

$$\hat{x}_{0j}(t|t)=\sum_{i=1}^{L}a_{ij}\hat{x}_{ij}(t|t), \quad i=1, 2,...,L, \quad j=1, 2,...,n,$$

$$\hat{x}_0(t|t)=[\hat{x}_{01}(t|t), \hat{x}_{02}(t|t), \ldots, \hat{x}_{0n}(t|t)]^{\mathrm{T}}, \tag{7}$$

$$\hat{x}_i(t|t)=[\hat{x}_{i1}(t|t), \hat{x}_{i2}(t|t), \ldots, \hat{x}_{in}(t|t)]^{\mathrm{T}},$$

and the optimal weights are

$$\left[a_{1j}, a_{2j}, \ldots, a_{Lj}\right]=e^{\mathrm{T}}\left(P^{jj}\right)^{-1}\left[e^{\mathrm{T}}\left(P^{jj}\right)^{-1}e\right]^{-1}, \tag{8}$$

where $e^{\mathrm{T}}=[1, 1,\ldots,1]$, P^{jj} is an $L\times L$ matrix with P_{ki}^{jj} as the (k, i)th element, and P_{ki}^{jj} is the (j, j)th diagonal element of P_{ki}. The optimal fusion estimation error variance for each component is $P_{0j}=[e^{\mathrm{T}}(P^{jj})^{-1}e]^{-1}$ and the accuracy relation is $\mathrm{tr}P_{0j}\leqslant\mathrm{tr}P_{ii}^{jj}$. It realizes the state component decoupled fusion estimation, i.e., only the component estimator $\hat{x}_{ij}(t|t)$ with the same physical meaning is weighted to yield the component fusion estimator $\hat{x}_{0j}(t|t)$, which is uncorrelated with other components $\hat{x}_{ik}(t|t)$, $i=1, 2, \ldots, L$, $k\neq j$.

3 Decoupled fusion self-tuning Kalman filter weighted by scalars for components

When noise variances are unknown, Q_w and Q_{vi} are identified by the correlation method. According to Eq. (1), we have

$$y_i(t)=H_i\left(I_n-q^{-1}\Phi\right)^{-1}\Gamma q^{-1}w(t)+v_i(t), \tag{9}$$

where q^{-1} is the backward shift operator, $q^{-1}x(t)=x(t-1)$ and I_n is the $n\times n$ identity matrix. To reduce the model order, a left co-prime factorization is introduced [3]:

$$H_i\left(I_n-q^{-1}\Phi\right)^{-1}\Gamma q^{-1}=A_i^{-1}\left(q^{-1}\right)B_i\left(q^{-1}\right),$$

where the polynomial matrices $A_i(q^{-1})$ and $B_i(q^{-1})$ have the forms:

$$X_i\left(q^{-1}\right)=X_{i0}+X_{i1}q^{-1}+\cdots+X_{in_{x_i}}q^{-n_{x_i}},$$

where $X_{in_{x_i}}=0,\ i>n_{x_i},\ A_{i0}=I_{m_i},\ B_{i0}=0$ and $A^{-1}(q^{-1})$ is the inverse matrix of $A(q^{-1})$. Hence we have

$$A_i(q^{-1})y_i(t)=B_i(q^{-1})w(t)+A_i(q^{-1})v_i(t). \quad (10)$$

Introducing the new measurement process $z_i(t)=A_i(q^{-1})y_i(t)$ yields

$$z_i(t)=B_i(q^{-1})w(t)+A_i(q^{-1})v_i(t), \quad i=1,2,\ldots,L. \quad (11)$$

The right of Eq. (11) is a sum of two MA processes. Thus, $z_i(t)$ is a stationary stochastic process and its correlation function is defined as $R_{z_i}(k)=\mathrm{E}\left[z_i(t)z_i^{\mathrm{T}}(t-k)\right]$. It is obvious that $R_{z_i}(k)=0,\ k>n_{z_i}$, i.e., $R_{z_i}(k)$ has truncation properties. Computing the correlation functions of the stochastic processes on both sides of Eq. (11) yields

$$R_{z_i}(k)=\sum_{j=k}^{n_{bi}}B_{ij}Q_wB_{i,j-k}^{\mathrm{T}}+\sum_{j=k}^{n_{ai}}A_{ij}Q_{vi}A_{i,j-k}^{\mathrm{T}}, \quad (12)$$

$$k=0,\ldots,n_{z_i},\ n_{zi}\leqslant \max(n_a,n_b),\ i=1,2,\ldots,L,$$

where A_{ij}, B_{ij} are known, and we define $B_{ij}=0$ $(j>n_{bi})$, $A_{ij}=0$ $(j>n_{ai})$. For a fixed i, Eq. (12) can be expanded by elements ($k=0,\ldots,n_{zi}$). Since it is defined that all of the unknown elements of Q_w and Q_{vi} construct the $n_i\times 1$ column vector as θ_i, then Eq. (12) can be rewritten as the linear equations about θ_i:

$$\Omega_i\theta_i=\omega_i,$$

where the coefficient matrices Ω_i are known, and each element of the column vector ω_i is composed of a constant plus an element of $R_{z_i}(k),\ k=0,1,\ldots,n_{z_i}$. Assume that Ω_i is column full rank and rank $\Omega_i=n_i$, then its row rank is also n_i and n_i linear uncorrelated equations can be chosen to construct the new linear equations:

$$\Omega_{i0}\theta_i=\omega_{i0}, \quad i=1,2,\ldots,L,$$

where Ω_{i0} is the $n_i\times n_i$ non-singular matrix. Hence we have

$$\theta_i=\Omega_{i0}^{-1}\omega_{i0}, \quad (13)$$

where each element of the column vector ω_{i0} is composed of a constant plus an element of $R_{z_i}(k),\ k=0,1,\ldots,n_{z_i}$. We define the sample estimation of $R_{z_i}(k)$ at t as

$$\hat{R}_{z_i}^t(k)=\frac{1}{t}\sum_{j=1}^{t}z_i(j)z_i^{\mathrm{T}}(j-k), \quad (14)$$

which has the recursive formula:

$$\hat{R}_{z_i}^t(k)=\hat{R}_{z_i}^{t-1}(k)+\frac{1}{t}\left[z_i(t)z_i^{\mathrm{T}}(t-k)-\hat{R}_{z_i}^{t-1}(k)\right],$$
$$t=2,3,\ldots,\quad k=0,1,\ldots,n_{z_i}, \quad (15)$$

with the initial value $\hat{R}_{z_i}^1(k)=z_i(1)z_i^{\mathrm{T}}(1-k)$. According to the ergodicity [6], we have

$$\hat{R}_{z_i}^t(k)\to R_{z_i}(k),\ \text{as}\ t\to\infty,\ \text{w.p.1}, \quad (16)$$

where the symbol "w.p.1" means "with probability 1".

Putting Eq. (14) into Eq. (13) yields the estimator of θ_i at t:

$$\hat{\theta}_i=\Omega_{i0}^{-1}\hat{\omega}_{i0}, \quad (17)$$

i.e., the estimators $\hat{Q}_{wi}$ and $\hat{Q}_{vi}$ of Q_w and Q_{vi} at t can be obtained. We define the estimator of Q_w at time t as

$$\hat{Q}_w=\frac{1}{L}\sum_{i=1}^{L}\hat{Q}_{wi}. \quad (18)$$

Remark 1 When Ω_i is not column full rank, the linear equations have infinite solutions. To obtain the unique solution, the number of the unknown elements in Q_w and Q_{vi} must be reduced. For example, they are usually considered in diagonal form and only elements at the diagonal line are unknown.

Theorem 1 For the multi-sensor system Eq. (1) with unknown noise variance, the noise variance estimator is consistent, i.e.,

$$\hat{Q}_w\to Q_w,\ \hat{Q}_{vi}\to Q_{vi},\ \text{as}\ t\to\infty,\ \text{w.p.1}. \quad (19)$$

Proof Since each element of ω_{i0} is composed of a constant and an element of $R_{z_i}(k)$, Eq. (16) yields

$$\hat{\omega}_{i0}\to\omega_{i0},\ \text{as}\ t\to\infty,\ \text{w.p.1}. \quad (20)$$

According to Eq. (13), each element of θ_i is the continuous function for the elements of ω_{i0}, and from Eq. (17) we obtain

$$\hat{\theta}_i\to\theta_i,\ \text{as}\ t\to\infty,\ \text{w.p.1}, \quad (21)$$

i.e., $\hat{Q}_{wi}\to Q_{wi}$, $\hat{Q}_{vi}\to Q_{vi}$, and according to Eq. (18), we have $\hat{Q}_w\to Q_w$, w.p.1.

The self-tuning decoupled fusion Kalman filter is realized by the following three steps.

Step 1 Online identification of unknown noise variances yields the estimators $\hat{Q}_w$ and $\hat{Q}_{vi}$ at time t.

Step 2 Put $\hat{Q}_w$ and $\hat{Q}_{vi}$ into Eqs. (2)–(6). The ith sensor subsystem has the local self-tuning Kalman filter:

$$\hat{x}_i^s(t|t)=\hat{\Psi}_{fi}\hat{x}_i^s(t-1|t-1)+\hat{K}_{fi}y_i(t),$$
$$i=1,2,\ldots,L, \quad (22)$$

$$\hat{\Psi}_{fi}=\left[I_n-\hat{K}_{fi}H_i\right]\Phi, \quad (23)$$

$$\hat{K}_{fi}=\hat{\Sigma}_{ii}H_i^{\mathrm{T}}\left[H_i\hat{\Sigma}_{ii}H_i^{\mathrm{T}}+\hat{Q}_{vi}\right]^{-1}, \quad (24)$$

where the estimators $\hat{\Sigma}_{ii}$ satisfy the Riccati equation:

$$\hat{\Sigma}_{ii} = \Phi\left[\hat{\Sigma}_{ii} - \hat{\Sigma}_{ii}H_i^{\mathrm{T}}\left(H_i\hat{\Sigma}_{ii}H_i^{\mathrm{T}} + \hat{Q}_{vi}\right)^{-1}H_i\hat{\Sigma}_{ii}\right]\Phi^{\mathrm{T}} + \Gamma\hat{Q}_{vi}\Gamma^{\mathrm{T}}. \tag{25}$$

Step 3 According to Eqs. (7) and (8), the self-tuning weighted fusion Kalman filter weighted by scalars for components is

$$\hat{x}_{0j}^s(t|t) = \sum_{i=1}^{L} \hat{a}_{ij}\hat{x}_{ij}^s(t|t), \quad j = 1, 2, \ldots, n, \tag{26}$$

$$\hat{x}_i^s(t|t) = \left[\hat{x}_{i1}^s(t|t), \hat{x}_{i2}^s(t|t), \ldots, \hat{x}_{in}^s(t|t)\right]^{\mathrm{T}}, \tag{27}$$

$$\left[\hat{a}_{1j}, \hat{a}_{2j}, \ldots, \hat{a}_{Lj}\right] = e^{\mathrm{T}}\left(\hat{P}^{ii}\right)^{-1}\left[e^{\mathrm{T}}\left(\hat{P}^{ii}\right)^{-1}e\right]^{-1}, \tag{28}$$

where $\hat{P}_{ij}$ satisfies the Lyapunov equation:

$$\begin{aligned} \hat{P}_{ij} &= \hat{\Psi}_{fi}\hat{P}_{ij}\hat{\Psi}_{fj} + \hat{\Delta}_{ij}, \\ \hat{\Delta}_{ij} &= \left[I_n - \hat{K}_{fi}H_i\right]\Gamma\hat{Q}_w\Gamma^{\mathrm{T}}\left[I_n - \hat{K}_{fj}H_j\right]^{\mathrm{T}} + \hat{K}_{fi}\hat{Q}_{vi}\delta_{ij}\hat{K}_{fj}^{\mathrm{T}}, \end{aligned} \tag{29}$$

and $\hat{P}^{jj}$ is the $L \times L$ matrix with $\hat{P}_{ki}^{jj}$ as the (k, i)th element, and $\hat{P}_{ki}^{jj}$ is the (j, j)th diagonal element of $\hat{P}_{ki}$.

The above three steps are repeated at each time t.

Remark 2 The estimators $\hat{K}_{fi}$ and $\hat{\Psi}_{fi}$ in Eq. (22) are updated at each time t. However, there is no need to simultaneously compute the estimators $\hat{\Sigma}_{ii}$ and $\hat{P}_{ii}$ at the time t, because they require solving the Riccati equation (Eq. (25)) and the Lyapunov equation (Eq. (29)) by the iterative method to compute $\hat{\Sigma}_{ij}$ and $\hat{P}_{ij}$. We can select a computing period about $\hat{\Sigma}_{ii}$ and $\hat{P}_{ij}$ as the fixed computing period T_d, i.e., dead band. In a period T_d, the estimators $\hat{\Sigma}_{ii}$ and $\hat{P}_{ij}$ are not changed. Only at time $t = T_d, 2T_d, \ldots$, we compute $\hat{\Sigma}_{ii}$ and $\hat{P}_{ij}$, which can reduce the online computational burden.

The known measurement data $y_i(t)$ can be viewed as a realization (a sample function) of the measurement stochastic process $y_i(t)$. A corresponding realization of the local filters $\hat{x}_i(t|t)$ and $\hat{x}_i^s(t|t)$ can also be obtained, and a realization of the fusion filters $\hat{x}_0(t|t)$ and $\hat{x}_0^s(t|t)$ can be obtained by all of the measurement data $y_i(t)$, $i = 1,2,\ldots,L$.

Definition 1 [9] If $\left[\hat{x}_0^s(t|t) - \hat{x}_0(t|t)\right] \to 0$, $t \to \infty$ for a realization, the self-tuning fusion filter $\hat{x}_0^s(t|t)$ converges to the optimal fusion filter $\hat{x}_0(t|t)$ in a realization, which is defined as

$$\left[\hat{x}_0^s(t|t) - \hat{x}_0(t|t)\right] \to 0, \text{ as } t \to \infty, \text{ i.a.r.}, \tag{30}$$

where the symbol "i.a.r." means "in a realization".

Lemma 3 [4,9] (bounded input-to-bounded output stability) Consider a dynamic system:

$$\delta(t) = \Psi(t)\delta(t-1) + u(t),$$

where $t \geqslant 0$, the output $\delta(t) \in R^n$ and the input $u(t) \in R^n$. Assume that $\Psi(t) \to \Psi$, as $t \to \infty$, Ψ is the $n \times n$ stable matrix, and $u(t)$ is bounded, then $\delta(t)$ is bounded.

Lemma 4 [4,9] (infinitesimal input-to-infinitesimal output stability) Consider the dynamic system:

$$\delta(t) = \Psi\delta(t-1) + u(t),$$

where $\delta(t) \in R^n$, $u(t) \in R^n$. Assume that Ψ is the $n \times n$ stable matrix and $u(t) \to 0$, as $t \to \infty$, then $\delta(t) \to 0$, $t \to \infty$.

Theorem 2 For the multi-sensor Eq. (1) with unknown noise variances, if the measurement data of each sensor are bounded, the self-tuning component decoupled information fusion Kalman filter $\hat{x}_{0j}^s(t|t)$ given by Eq. (26) converges to the optimal information fusion Kalman filter $\hat{x}_{0j}(t|t)$ given by Eq. (7) in a realization, i.e., Eq. (30) holds.

Proof According to Eq. (5) and the implicit function theorem [10], in a sufficiently small neighborhood, the elements of Σ_{ii} are the continuous function of the elements of Q_w and Q_{vi}, which is defined as

$$\Sigma_{ii} = f_i(Q_w, Q_{vi}), \tag{31}$$

where f_i is the $n \times n$ continuous matrix function, whose each element is the continuous function of Q_w and Q_{vi}, and we have the relation

$$\hat{\Sigma}_{ii} = f_i\left(\hat{Q}_w, \hat{Q}_{vi}\right). \tag{32}$$

According to Eq. (19) and the statistical inference principle, the event with probability 1 in an experiment will be inferred as a sure event. Therefore, taking a realization of the stochastic process implies that the convergence in a realization holds:

$$\hat{Q}_w \to Q_w, \quad \hat{Q}_{vi} \to Q_{vi}, \quad \text{as } t \to \infty, \text{ i.a.r.} \tag{33}$$

According to Eqs. (31)–(33) and continuity of f_i, we have

$$\hat{\Sigma}_{ii} \to \Sigma_{ii}, \text{ as } t \to \infty, \text{ i.a.r.} \tag{34}$$

According to Eqs. (3), (4), (23) and (24), we have

$$\hat{K}_{fi} \to K_{fi}, \quad \hat{\Psi}_{fi} \to \Psi_{fi}, \quad \text{as } t \to \infty, \text{ i.a.r.} \tag{35}$$

According to Eq. (29) and the implicit function theorem, in a sufficiently small neighborhood, the elements of P_{ij} are the continuous function of Q_w, Q_{vi}, Σ_{ii} and Σ_{jj}. Applying Eqs. (34) and (35), we have

$$\hat{P}_{ij} \to P_{ij}, \text{ as } t \to \infty, \text{ i.a.r.} \tag{36}$$

Considering Eq. (22), we obtain from Eq. (35) that $\hat{K}_{fi}$ is bounded. Thus, the assumption that $y_i(t)$ is bounded yields that $\hat{K}_{fi}y_i(t)$ is bounded. Because Ψ_{fi} is a stable matrix, applying Eq. (35) and Lemma 3 to Eq. (22), we obtain that $\hat{x}_i^s(t|t)$ is bounded. Setting $\hat{\Psi}_{fi}=\Psi_{fi}+\Delta\hat{\Psi}_{fi}$, $\hat{K}_{fi}=K_{fi}+\Delta\hat{K}_{fi}$, according to Eq. (35), we have

$$\Delta\hat{\Psi}_{fi}\to 0, \quad \Delta\hat{K}_{fi}\to 0, \quad t\to\infty, \text{ i.a.r.} \tag{37}$$

Defining $\delta_i(t)=\hat{x}_i^s(t|t)-\hat{x}_i(t|t)$ and subtracting Eq. (2) from Eq. (22) yields the dynamic error system:

$$\begin{aligned}\delta_i(t)&=\Psi_{fi}(t)\delta_i(t-1)+u_i(t),\\ u_i(t)&=\Delta\hat{\Psi}_{fi}\hat{x}_i^s(t-1|t-1)+\Delta\hat{K}_{fi}y_i(t).\end{aligned} \tag{38}$$

Applying the boundeness of $y_i(t)$ and $\hat{x}_i^s(t|t)$ and Eq. (37) yields $u_i(t)\to 0$, $t\to\infty$, i.a.r. Applying Lemma 4 to Eq. (38) and noting that Ψ_{fi} is a stable matrix, we have $\delta_i(t)\to 0$, $t\to\infty$, i.a.r., i.e.,

$$\left[\hat{x}_i^s(t|t)-\hat{x}_i(t|t)\right]\to 0, \text{ as } t\to\infty, \text{ i.a.r.} \tag{39}$$

holds. This proves the convergence of the local self-tuning Kalman filter. In Eq. (26), we define $\hat{a}_{ij}=a_{ij}+\Delta\hat{a}_{ij}$. Equations (28) and (36) yield $\Delta\hat{a}_{ij}\to 0$. Subtracting Eq. (7) from Eq. (26) yields

$$\hat{x}_{0j}^s(t|t)-\hat{x}_{0j}(t|t)=\sum_{i=1}^{L}a_{ij}[\hat{x}_{ij}^s(t|t)-\hat{x}_{ij}(t|t)]+\sum_{i=1}^{L}\Delta\hat{a}_{ij}\hat{x}_{ij}^s(t|t).$$

Applying Eq. (39), $\Delta\hat{a}_{ij}\to 0$ and the boundeness of $\hat{x}_i^s(t|t)$ yields

$$\left[\hat{x}_{0j}^s(t|t)-\hat{x}_{0j}(t|t)\right]\to 0,\ t\to\infty, \text{ i.a.r.}, \quad j=1,2,\ldots,n,$$

i.e., Eq. (30) holds. This completes the proof.

Note that the convergence in a realization has theoretical and implicational significance. The convergence in a realization is weaker than that with probability 1 in theory. If the convergence with probability 1 holds, according to the statistical inference principle, the convergence in a realization can be obtained. While it may not be the case on the other side, only when the convergence in a realization holds for each realization, except the realization set with probability zero, can we maintain the convergence with probability 1. Only weaker conditions are needed to prove it, i.e., based on the assumption that the measured data (a realization of the measurement process) are bounded. It always holds in application. But the assumption that the measurement process is bounded with probability 1 is required to prove the convergence with probability one, which cannot be satisfied for the non-stationary measurement processes in theory. In practice, only a realization of the measurement process is generally known, such as hydrology, meteorology, and astronomy. Therefore, convergence in a known realization is emphasized in research.

4 Simulation

Consider the target tracking system Eq. (1) with 3 sensors, where

$$\Phi=\begin{bmatrix}1 & T_0 & 0.5T_0^2\\ 0 & 1 & T_0\\ 0 & 0 & 1\end{bmatrix}, \quad \Gamma=\begin{bmatrix}0\\0\\1\end{bmatrix},$$

$$H=[1 \quad 0 \quad 0], \quad x(t)=[x_1(t) \quad x_2(t) \quad x_3(t)]^{\mathrm{T}},$$

T_0 is the sample period, $x_1(t)$, $x_2(t)$ and $x_3(t)$ are the position and the target velocity and acceleration at the sample time tT_0 respectively. $w(t)$, $v_i(t)$ are independent white noises with zero mean and variances $Q_w=\sigma_w^2$, $Q_{vi}=\sigma_{vi}^2$, and σ_w^2, σ_{vi}^2 are unknown. The aim is to find the self-tuning and optimal decoupled fusion Kalman filters $\hat{x}_{0j}^s(t|t)$ and $\hat{x}_{0j}(t|t)$. In the simulation, we set $T_0=1.5$, $\sigma_w^2=0.64$, $\sigma_{v1}^2=0.1$, $\sigma_{v2}^2=0.2$, and $\sigma_{v3}^2=0.3$. The simulation results are shown in Figs. 1 and 2. Figure 1 shows the convergence of the noise variance estimations, where the beeline denotes the true value and the curve denotes the estimators, which shows that the parameter estimation is consistent. The error curves between the self-tuning and the optimal fuser shows that the self-tuning fusion filter converges to the optimal fusion filter in a realization, as shown in Fig. 2.

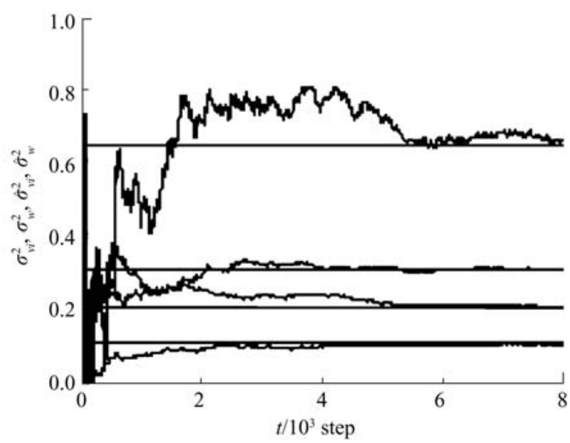

Fig. 1 Convergence of noise variance estimators $\hat{\sigma}_{vi}^2$, $\hat{\sigma}_w^2$

Because of the slow convergence velocity for the noise variances of the method in this article, it can be first operated offline, and then switched to real time application for higher accuracy of the self-tuning fusion filtering in engineering applications.

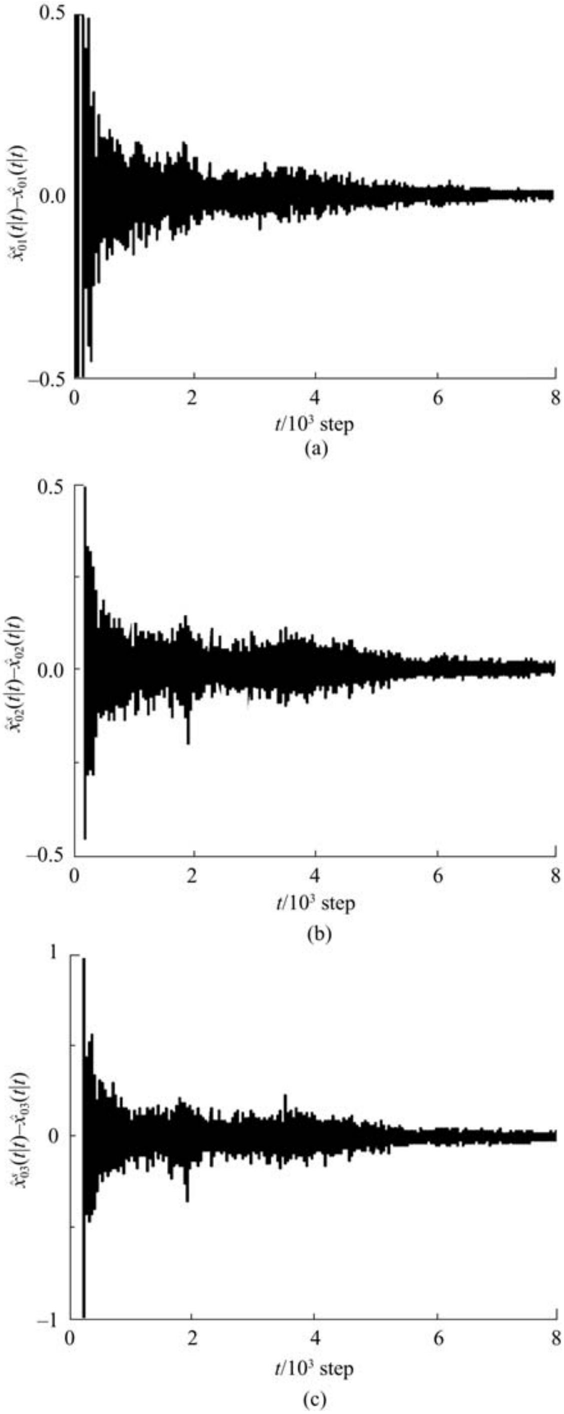

Fig. 2 Error between self-tuning and optimal fusion Kalman filters. (a) Fusion error curves for position; (b) fusion error curves for velocity; (c) fusion error curves for acceleration

5 Conclusions

For multi-sensor systems with unknown noise variances, the online estimators of the noise variances are presented in this article using the correlation method. It avoids identification of the ARMA innovation model and can ensure the consistency of the noise variance estimators. Under the optimal fusion rule weighted by scalars for components, the self-tuning component decoupled fusion Kalman filter based on the Riccati equation is presented. Moreover, dynamic error system analysis method is applied to prove that the self-tuning fusion Kalman filter converges to the optimal fusion Kalman filter in a realization. The simulation shows that the self-tuning fusion Kalman filter is asymptotically optimal.

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