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RESEARCH ARTICLE

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Robust adaptive beamforming algorithm based on Bayesian approach

Abstract The performance of adaptive array beamforming algorithms substantially degrades in practice because of a slight mismatch between actual and presumed array responses to the desired signal. A novel robust adaptive beamforming algorithm based on Bayesian approach is therefore proposed. The algorithm responds to the current environment by estimating the direction of arrival (DOA) of the actual signal from observations. Computational complexity of the proposed algorithm can thus be reduced compared with other algorithms since the recursive method is used to obtain inverse matrix. In addition, it has strong robustness to the uncertainty of actual signal DOA and makes the mean output array signal-to-interference-plus-noise ratio (SINR) consistently approach the optimum. Simulation results show that the proposed algorithm is better in performance than conventional adaptive beamforming algorithms.

Keywords robust adaptive beamforming, signal-to-interference-plus-noise ratio (SINR), Bayesian approach, signal steering vector mismatch

1 Introduction

Adaptive beamforming is used for enhancing a desired signal while suppressing noise and interference at the output of an array of sensor. Adaptive beamforming has wide applications in fields such as radar, sonar and wireless communications [1–5]. In practice, the performance of adaptive beamforming may degrade severely because of violations of underlying assumptions on environment, sources, or sensor arrays. This may cause a mismatch between the assumed array response and actual array response. Therefore, robust approaches to adaptive beamforming have received considerable attention. Several efficient approaches have robustness against the desired signal arrival direction mismatch. One is the linearly constrained minimum variance (LCMV) beamformer [6], which has robustness against uncertainty in the signal arrival direction. To account for signal steering vector mismatches, additional linear constraints (point and derivative constraints) can be imposed to improve robustness of adaptive beamforming [7,8]. However, the beamformers lose degrees of freedom for interference suppression. Diagonal loading [9–12] has been a popular approach of improving robustness against mismatch errors, random perturbations, and small sample support, while its main drawback is the difficulty in deriving a closed-form expression. Hence, these approaches mentioned above cannot be expected to provide sufficient robustness improvements in practice. To deal with uncertainty in signal arrival direction, we propose robust adaptive beamforming algorithm based on a Bayesian approach, which balances the use of observed sample data and a priori knowledge about source direction of arrival (DOA). We can update weight vectors with the recursive method. The proposed algorithm provides excellent robustness to uncertain steering vectors, enhances array system performance under non-ideal conditions and makes the mean output array signal-to-interference-plus-noise ratio (SINR) consistently close to the optimum. In this paper, computational complexity can be efficiently reduced compared with other algorithms since the recursive method is used to obtain inverse matrix. The excellent performance of the proposed algorithm is demonstrated by simulations.

2 Problem formulation

Consider a uniform linear array (ULA) with M omnidirectional sensors spaced by a distance d and D incoherent narrow-band plane waves impinging from directions $\{\theta_0,\theta_1,\ldots,\theta_{D-1}\}$. The observation vector is given by

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$$\boldsymbol{X}(k)=\boldsymbol{s}(k)+\boldsymbol{i}(k)+\boldsymbol{n}(k)=s_0(k)\boldsymbol{a}+\boldsymbol{i}(k)+\boldsymbol{n}(k), \qquad (1)$$

where $\boldsymbol{X}(k) = [x_1(k),x_2(k),\ldots,x_M(k)]^{\mathrm{T}}$ is complex vector of array observations, $s_0(k)$ is signal waveform, $\boldsymbol{a}$ is signal steering vector, and $\boldsymbol{i}(k)$ and $\boldsymbol{n}(k)$ are interference and noise components respectively. The output of a narrowband beamformer is given by

$$y(k)=\boldsymbol{w}^{\mathrm{H}}\boldsymbol{X}(k), \qquad (2)$$

where $\boldsymbol{w} = [w_1,w_2,\ldots,w_M]^{\mathrm{T}}$ is complex vector of beamformer weights, $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ denote transpose and Hermitian transpose respectively. The SINR has the following form:

$$\mathrm{SINR}=\frac{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{s}}\boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{i+n}}\boldsymbol{w}}. \qquad (3)$$

where

$$\boldsymbol{R}_{\mathrm{s}}=\mathrm{E}\{\boldsymbol{s}(k)\boldsymbol{s}^{\mathrm{H}}(k)\}, \qquad (4)$$

$$\boldsymbol{R}_{\mathrm{i+n}}=\mathrm{E}\left\{(\boldsymbol{i}(k)+\boldsymbol{n}(k))(\boldsymbol{i}(k)+\boldsymbol{n}(k))^{\mathrm{H}}\right\} \qquad (5)$$

are the $M \times M$ signal and interference-plus-noise covariance matrices respectively. $\mathrm{E}\{\cdot\}$ denotes statistical expectation.

Find the solution for the weight vector by maintaining a distortionless response toward the desired signal and minimizing the output power. Hence, the maximum of Eq. (3) is equal to the following optimization problem:

$$\min_{\boldsymbol{w}}\ \boldsymbol{w}^{\mathrm{H}}\hat{\boldsymbol{R}}\boldsymbol{w} \text{ subject to } \boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}(\theta)=1, \qquad (6)$$

where $\hat{\boldsymbol{R}}=\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{X}(i)\boldsymbol{X}^{\mathrm{H}}(i)$ is the sample covariance matrix, N is the number of snapshots, and $\boldsymbol{a}(\theta)$ is the desired signal steering vector.

Equation (6) yields the version of a minimum variance distortionless response (MVDR) beamformer [13]:

$$\boldsymbol{w}_{\mathrm{MV}}=\frac{\hat{\boldsymbol{R}}^{-1}\boldsymbol{a}(\theta)}{\boldsymbol{a}^{\mathrm{H}}(\theta)\hat{\boldsymbol{R}}^{-1}\boldsymbol{a}(\theta)}. \qquad (7)$$

When the DOA of a signal is exactly known, MVDR beamformer provides a distortionless response in the direction of the desired signal, while suppressing noise and interferences. However, if there is uncertainty in the DOA of the desired signal, the performance of MVDR beamformer is known to degrade severely.

To avoid DOA uncertainty, additional linear constraints can be imposed to reduce sensitivity to pointing errors. The constraints are on the beamformer output at k values of θ near the presumed DOA. The weight vector can be found from the following constrained minimization problem:

$$\min_{\boldsymbol{w}}\ \boldsymbol{w}^{\mathrm{H}}\hat{\boldsymbol{R}}\boldsymbol{w} \text{ subject to } \boldsymbol{C}^{\mathrm{H}}\boldsymbol{w}=\boldsymbol{f}, \qquad (8)$$

where $\boldsymbol{C}=[\boldsymbol{a}(\theta_1),\boldsymbol{a}(\theta_2),\ldots,\boldsymbol{a}(\theta_k)]$ is the $M \times k$ matrix of steering vectors for the constrained DOA and $\boldsymbol{f}=[f_1, f_2,\ldots,f_k]^{\mathrm{T}}$ is the $k \times 1$ vector of constraints. By Lagrange multiplier method, we can obtain the following weight vector:

$$\boldsymbol{w}_{\mathrm{LC}}=\hat{\boldsymbol{R}}^{-1}\boldsymbol{C}\left(\boldsymbol{C}^{\mathrm{H}}\hat{\boldsymbol{R}}^{-1}\boldsymbol{C}\right)^{-1}\boldsymbol{f}. \qquad (9)$$

When the observed data are sufficient to yield good estimates of the DOA, the LCMV beamformer works well. However, when the estimates are poor, there can be degradation in performance. Moreover, the additional constraints protect the desired signal but lose degrees of freedom used for noise and interference suppression.

3 Robust adaptive beamforming algorithm

To avoid uncertain steering vectors, we develop a novel robust adaptive beamforming based on a Bayesian approach, which is robust to uncertainty in source DOA. The proposed algorithm balances the use of observed data and a priori knowledge about the source DOA. In the proposed algorithm, the DOA is assumed to be a discrete random variable with a known priori probability density function (pdf) $q(\theta)$, which reflects the level of uncertainty about the source DOA. The Bayesian approach has been used for detecting signals under directional uncertainty in Ref. [14], which averages over the a priori pdf $q(\theta)$. For computational simplicity, we assume that $q(\theta)$ is defined only on a discrete set of P points: $\Theta = \{\theta_1,\theta_2,\ldots,\theta_P\}$ in the a priori parameter space.

For each θ_i, the a posteriori pdf is given by

$$p(\theta_i|\boldsymbol{X}_N)=\frac{q(\theta_i)p(\boldsymbol{X}_N|\theta_i)}{\sum_{j=1}^{P}q(\theta_j)p(\boldsymbol{X}_N|\theta_j)}, \qquad (10)$$

where $p(\boldsymbol{X}_N|\theta_i)$ is the pdf of the observation given θ_i. If it is assumed that the source and noise waveforms are Gaussian random processes of uncorrelated, zero-mean, stationary functions with the variance σ_{s}^2 and covariance $\sigma_{\mathrm{n}}^2\boldsymbol{I}$ respectively, then $p(\boldsymbol{X}_N|\theta_i)$ is a Gaussian density with zero mean with the covariance

$$\boldsymbol{R}_{xx}(\theta_i)=\sigma_{\mathrm{s}}^2\boldsymbol{a}(\theta_i)\boldsymbol{a}^{\mathrm{H}}(\theta_i)+\boldsymbol{R}_{\mathrm{i+n}}. \qquad (11)$$

If there are no interferers $\boldsymbol{R}_{\mathrm{i+n}}=\sigma_{\mathrm{n}}^2\boldsymbol{I}$, by applying the Bayesian approach, $p(\theta_i|\boldsymbol{X}_N)$ has the following form [15]:

$$p(\theta_i|\boldsymbol{X}_N)=\frac{q(\theta_i)\exp\left\{\beta N\boldsymbol{a}^{\mathrm{H}}(\theta_i)\hat{\boldsymbol{R}}_N\boldsymbol{a}(\theta_i)\right\}}{\sum_{j=1}^{P}q(\theta_j)\exp\left\{\beta N\boldsymbol{a}^{\mathrm{H}}(\theta_j)\hat{\boldsymbol{R}}_N\boldsymbol{a}(\theta_j)\right\}},\tag{12}$$

where $\hat{\boldsymbol{R}}_N$ is the sample covariance matrix of $\boldsymbol{X}_N$ and β is a monotonically increasing function of signal-to-noise ratio (SNR).

When interferers are present, $p(\theta_i|\boldsymbol{X}_N)$ is difficult to implement because it is a function of $\boldsymbol{R}_{\mathrm{i+n}}$, which is unknown and hard to estimate. We use the intuition gained from no interferer case to obtain the $p(\theta_i|\boldsymbol{X}_N)$ approximately with a simpler expression [16]

$$\hat{p}(\theta_i|\boldsymbol{X}_N)=\frac{q(\theta_i)\exp\left\{\beta N\left(\boldsymbol{a}^{\mathrm{H}}(\theta_i)\hat{\boldsymbol{R}}_N^{-1}\boldsymbol{a}(\theta_i)\right)^{-1}\right\}}{\sum_{j=1}^{P}q(\theta_j)\exp\left\{\beta N\left(\boldsymbol{a}^{\mathrm{H}}(\theta_j)\hat{\boldsymbol{R}}_N^{-1}\boldsymbol{a}(\theta_j)\right)^{-1}\right\}}.\tag{13}$$

With a high SNR, the a posteriori pdf will be sharply peaked near the true DOA and at a low SNR it will be relatively flat over all DOAs and revert to the priori pdf.

The cost function of the proposed algorithm can be formulated as follows:

$$\min\sum_{i=0}^{K}\delta^{K-i}|e(i)|^2\ \text{subject to}\ \boldsymbol{w}^{\mathrm{H}}\bar{\boldsymbol{a}}=1,\tag{14}$$

where $0<\delta<1$ is a forgetting factor, $e(i)=d(i)-\boldsymbol{w}^{\mathrm{H}}\boldsymbol{X}(i)$ denotes the error between the desired signal and the beamformer output, and $\bar{\boldsymbol{a}}$ is an average steering vector averaged over $p(\theta_i|\boldsymbol{X}_N)$:

$$\bar{\boldsymbol{a}}=\sum_{i=1}^{P}\boldsymbol{a}(\theta_i)p(\theta_i|\boldsymbol{X}_N)=\boldsymbol{A}\boldsymbol{P},\tag{15}$$

where $\boldsymbol{A}$ is the $M\times P$ matrix of steering vectors:

$$\boldsymbol{A}=[\boldsymbol{a}(\theta_1),\boldsymbol{a}(\theta_2),\ldots,\boldsymbol{a}(\theta_P)],\tag{16}$$

and $\boldsymbol{P}$ is the $P\times 1$ vector:

$$\boldsymbol{P}=[p(\theta_1|\boldsymbol{X}_N),p(\theta_2|\boldsymbol{X}_N),\ldots,p(\theta_P|\boldsymbol{X}_N)]^{\mathrm{T}}.\tag{17}$$

The optimal weight vector can be found using Lagrange multiplier method by means of minimization of the function

$$H(\boldsymbol{w})=\sum_{i=0}^{K}\delta^{K-i}\left|d(i)-\boldsymbol{w}^{\mathrm{H}}\boldsymbol{X}(i)\right|^2+\lambda\left(1-\boldsymbol{w}^{\mathrm{H}}\bar{\boldsymbol{a}}\right),\tag{18}$$

where λ is a Lagrange multiplier. Taking the gradient of Eq. (18) and equating it to zero, we obtain

$$\sum_{i=0}^{K}\delta^{K-i}\boldsymbol{X}(i)\boldsymbol{X}^{\mathrm{H}}(i)\boldsymbol{w}=\sum_{i=0}^{K}\delta^{K-i}\boldsymbol{X}(i)d^{\mathrm{H}}(i)+\bar{\boldsymbol{a}}\lambda,\tag{19}$$

where

$$\sum_{i=0}^{K}\delta^{K-i}\boldsymbol{X}(i)\boldsymbol{X}^{\mathrm{H}}(i)=\boldsymbol{R}_{xk},\tag{20}$$

$$\sum_{i=0}^{K}\delta^{K-i}\boldsymbol{X}(i)d^{\mathrm{H}}(i)=\boldsymbol{r}_{dk}.\tag{21}$$

Simplify Eq. (19) as

$$\boldsymbol{R}_{xk}\boldsymbol{w}=\boldsymbol{r}_{dk}+\bar{\boldsymbol{a}}\lambda.\tag{22}$$

Hence, we obtain the optimal weight vector

$$\boldsymbol{w}_{\mathrm{R}}=\boldsymbol{R}_{xk}^{-1}\boldsymbol{r}_{dk}+\boldsymbol{R}_{xk}^{-1}\bar{\boldsymbol{a}}\lambda.\tag{23}$$

Inserting Eq. (23) into Eq. (14), we have the Lagrange multiplier

$$\lambda=\frac{1-\bar{\boldsymbol{a}}^{\mathrm{H}}\boldsymbol{R}_{xk}^{-1}\boldsymbol{r}_{dk}}{\bar{\boldsymbol{a}}^{\mathrm{H}}\boldsymbol{R}_{xk}^{-1}\bar{\boldsymbol{a}}}.\tag{24}$$

Using Eqs. (15) and (24), the weight vector can be rewritten as

$$\boldsymbol{w}_{\mathrm{R}}=\boldsymbol{R}_{xk}^{-1}\boldsymbol{r}_{dk}+\boldsymbol{R}_{xk}^{-1}\boldsymbol{A}\boldsymbol{P}\frac{1-\boldsymbol{P}^{\mathrm{T}}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{R}_{xk}^{-1}\boldsymbol{r}_{dk}}{\boldsymbol{P}^{\mathrm{T}}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{R}_{xk}^{-1}\boldsymbol{A}\boldsymbol{P}}.\tag{25}$$

The optimal weight vector $\boldsymbol{w}_{\mathrm{R}}$ is obtained by computing the inverse of sample covariance matrix $\boldsymbol{R}_{xk}$ directly, which leads to a high computational cost. In an environment that yields $\boldsymbol{R}_{xk}$ with a large eigenvalue spread, the sidelobe gains of array beampatterns rise. To solve the problem, we need a recursive algorithm to replace the computing inverse matrix directly. Its inverse may then be combined to update the inverse of $\boldsymbol{R}_{xk}$ from array signal samples using the Matrix Inversion Lemma as follows:

$$\begin{aligned}\boldsymbol{K}(n)&=\boldsymbol{R}_{xk}^{-1}\\&=\frac{1}{\delta}\left[\boldsymbol{K}(n-1)-\frac{\boldsymbol{K}(n-1)\boldsymbol{X}(n)\boldsymbol{X}^{\mathrm{H}}(n)\boldsymbol{K}(n-1)}{\delta+\boldsymbol{X}^{\mathrm{H}}(n)\boldsymbol{K}(n-1)\boldsymbol{X}(n)}\right].\end{aligned}\tag{26}$$

The updating weight vector is written as

$$\boldsymbol{w}(n)=\boldsymbol{w}(n-1)+\boldsymbol{K}(n)\boldsymbol{X}(n)e^{*}(n)+\boldsymbol{K}(n)\bar{\boldsymbol{a}}\lambda_{\mathrm{r}},\tag{27}$$

where $(\cdot)^{*}$ denotes conjugate of complex number.

Substituting Eq. (27) into Eq. (14), we obtain

$$\left(\boldsymbol{w}(n-1)+\boldsymbol{K}(n)\boldsymbol{X}(n)e^{*}(n)+\boldsymbol{K}(n)\bar{\boldsymbol{a}}\lambda_{\mathrm{r}}\right)^{\mathrm{H}}\bar{\boldsymbol{a}}=1.\tag{28}$$

Solving Eq. (28), we obtain the Lagrange multiplier

$$\lambda_{\mathrm{r}}=\frac{1-\bar{\boldsymbol{a}}^{\mathrm{H}}\boldsymbol{w}(n-1)}{\bar{\boldsymbol{a}}^{\mathrm{H}}\boldsymbol{K}(n)\bar{\boldsymbol{a}}}-\frac{\bar{\boldsymbol{a}}^{\mathrm{H}}\boldsymbol{K}(n)\boldsymbol{X}(n)e^{*}(n)}{\bar{\boldsymbol{a}}^{\mathrm{H}}\boldsymbol{K}(n)\bar{\boldsymbol{a}}}. \tag{29}$$

Inserting Eq. (29) into Eq. (27), the updating weight vector is rewritten as

$$\boldsymbol{w}(n)=\boldsymbol{Q}(n)[\boldsymbol{w}(n-1)+\boldsymbol{K}(n)\boldsymbol{X}(n)e^{*}(n)]+\boldsymbol{F}(n), \tag{30}$$

where

$$\boldsymbol{Q}(n)=\boldsymbol{I}-\frac{\boldsymbol{K}(n)\bar{\boldsymbol{a}}\bar{\boldsymbol{a}}^{\mathrm{H}}}{\bar{\boldsymbol{a}}^{\mathrm{H}}\boldsymbol{K}(n)\bar{\boldsymbol{a}}}, \tag{31}$$

$$\boldsymbol{F}(n)=\frac{\boldsymbol{K}(n)\bar{\boldsymbol{a}}}{\bar{\boldsymbol{a}}^{\mathrm{H}}\boldsymbol{K}(n)\bar{\boldsymbol{a}}}. \tag{32}$$

4 Simulation results

Simulations demonstrate the performance of the proposed algorithm. We assume a uniform linear array with $M=10$ omni-directional sensors spaced half a wavelength apart. For each scenario, 100 simulation runs are used to obtain each simulated point. The a priori uncertainty in the DOA is over the region $u=\sin\theta\in[-0.3,0.3]$. The set Θ is composed of $P=13$ evenly spaced points within the interval $[-0.3,0.3]$. We assume that the desired signal spatial signature is a plane wave impinging from the DOA $u_{\mathrm{s}}=0.223$. Two interfering sources are assumed to impinge on the array from the DOAs $u_{\mathrm{I}}=\{-0.6,0.6\}$. For the LCMV beamformer, five distortionless constraints are used at the points $\{-0.3,-0.15,0,0.15,0.3\}$.

Example 1 Comparison of the beampatterns

Figure 1 shows beampatterns of the three methods tested for the fixed SNR = 0 dB. Figure 2 displays beampatterns of the three methods tested for the fixed SNR = −20 dB. The vertical lines in the two figures denote the direction of arrival of the desired signal $u_{\mathrm{s}}=0.223$. In the example, the MVDR algorithm treats the desired signal as a main beam interferer and tries to place a null on it and the LCMV algorithm does not suppress noise and interferences sufficiently. Note that when SNR is high, the proposed algorithm can adapt the radiation pattern of the antenna to direct narrow beam to the desired signal and place nulls on interfering sources. When SNR is low, the array beampattern of the proposed algorithm is over 0 dB, which is robust against uncertain steering vectors.

Example 2 Comparison of output SINR

Figure 3 displays performance of the three methods tested versus the number of snapshots for the fixed SNR = 10 dB. The performance of the three methods tested versus the number of snapshots for the fixed SNR = −10 dB is shown in Fig. 4. In this example, the MVDR algorithm is sensitive to DOA uncertainty that easily occurs in practice. The LCMV algorithm

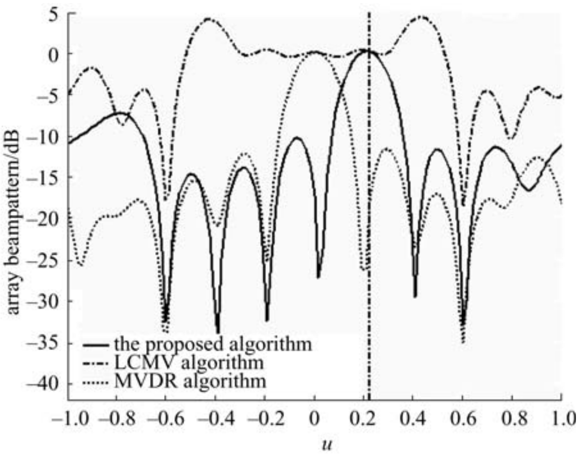

Fig. 1 Comparison of beampatterns (SNR = 0 dB)

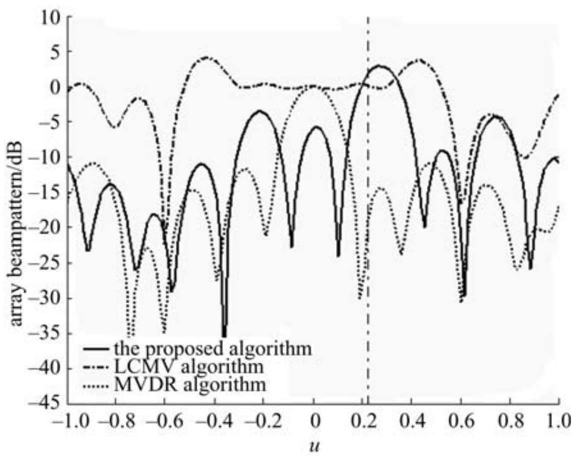

Fig. 2 Comparison of beampatterns (SNR = −20 dB)

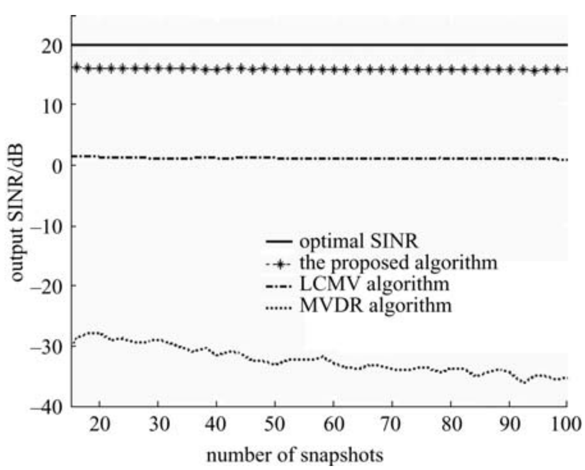

Fig. 3 Comparison of output SINR (SNR = 10 dB)

can improve robustness to pointing errors. The proposed robust adaptive beamforming algorithm provides a significantly improved robustness against

DOA uncertainty. Moreover, the proposed algorithm shows excellent performance at all values of N.

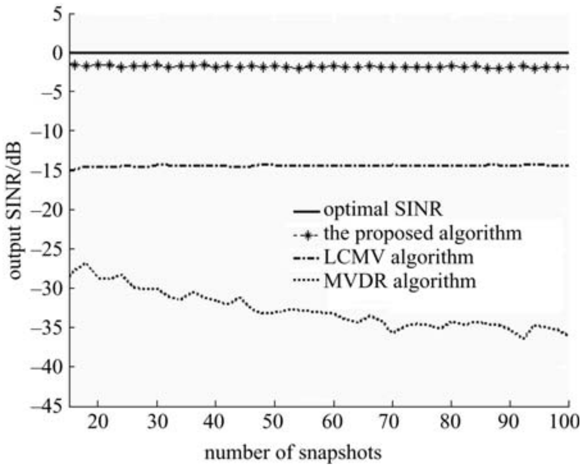

Fig. 4 Comparison of output SINR (SNR = −10 dB)

5 Conclusions

The robust adaptive beamforming algorithm based on a Bayesian approach is proposed to address uncertainty in source DOA in practice. The proposed algorithm balances the use of observations and a priori knowledge about the source DOA. The weight vector can be derived by implementing additional constraints of uncertainty of the array steering vector. The algorithm provides a significantly improved robustness against DOA uncertainty. Its mean output SINR is better than that of conventional algorithms in a wide range of N and is close to the optimum. Moreover, the proposed algorithm can be efficiently computed at a comparable cost with other algorithms by using the recursive method to obtain inverse matrix. Simulation results are presented to demonstrate the feasibility and superiority of the proposed algorithm.

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