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RESEARCH ARTICLE

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Resource allocation algorithm for multi-user MIMO-OFDM downlink with correlated channels

Abstract To minimize transmitting power, an adaptive resource allocation algorithm is proposed for multi-user multiple input multiple output-orthogonal frequency division multiplexing (MIMO-OFDM) downlink with correlated channels, which, based on the user's grouping according to their spatial correlations, combines the shared manner and the exclusive manner to allocate sub-carriers. Between different groups the shared manner with a null steering method based on group marginal users is applied, whereas within a group the exclusive manner is applied. The simulations show that the power efficiency and spectral efficiency are improved; the base station transmitting antenna number and the computational complexity is decreased.

Keywords MIMO-OFDM, multi-user, adaptive resource allocation, co-channel interference (CCI), null steering

1 Introduction

Adaptive resource allocation for multi-user multiple input multiple output-orthogonal frequancy division multiplexing (MIMO-OFDM) system can improve its power efficiency and spectral efficiency [1]. The allocation includes two classes, i.e., the exclusive manner and the shared manner to sub-carrier. The exclusive manner is easily realized with low spectral efficiency. While the shared manner has high spectral efficiency, the co-channel interference (CCI) is introduced by the spatial multiplexing of different users on the same sub-carrier. Moreover, its performance is affected by user spatial correlation, antenna number and computation complexity [2,3]. Reference [4] proposes a zero forcing (ZF) algorithm at the base station (BS) to mitigate CCI, but it does not consider the impact of user spatial correlation and computation complexity.

This research proposes a resource allocation algorithm for MIMO-OFDM downlink in correlated channels, which combines the exclusive manner and the shared manner based on the grouping of users according to their spatial correlations. The algorithm improves the system performance and reduces both the transmitting antenna number and the computational complexity.

2 System model

In this paper, an outdoor downlink adaptive multi-user MIMO-OFDM system equipped with N_c sub-carriers, n_T transmitting antennas at the BS and n_R receiving antennas for each of K users is considered. Each sub-carrier can be considered as a Rayleigh flat fading channel and the fading is correlated at the transmitter side, but it is uncorrelated at the receiver side [5]. In this case, the channel for user k ($k = 1,2,\ldots,K$) in the sub-carrier m ($m = 1,2,\ldots,N_\mathrm{c}$) can be modeled as

$$\boldsymbol{H}_m^k = \boldsymbol{G}_m^k \boldsymbol{A}_m^k = \begin{bmatrix} h_{1,1}^{k,m} & h_{1,2}^{k,m} & \cdots & h_{1,n_\mathrm{T}}^{k,m} \\ h_{2,1}^{k,m} & h_{2,2}^{k,m} & \cdots & h_{2,n_\mathrm{T}}^{k,m} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_\mathrm{R},1}^{k,m} & h_{n_\mathrm{R},2}^{k,m} & \cdots & h_{n_\mathrm{R},n_\mathrm{T}}^{k,m} \end{bmatrix}, \tag{1}$$

where $\boldsymbol{G}_m^k$ is an $n_\mathrm{R} \times D_k$ matrix with zero-mean unit-variance i.i.d complex Gaussian entries; $\boldsymbol{A}_m^k$ is the steering matrix of size $D_k \times n_\mathrm{T}$ that contains D_k steering vectors of the transmitting antenna array, i.e., D_k directions of departure (DOD); $h_{i,j}^{k,m}$ is the channel gain from the jth transmitting antenna to the ith receiving antenna. We assume that the perfect channel state information (CSI) is known to the BS and the receiver. For a uniform linear array, the steering matrix $\boldsymbol{A}_m^k$ is given by

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$$\begin{aligned}\boldsymbol{A}_m^k&=\frac{1}{\sqrt{D_k}}\left[\boldsymbol{a}^{\mathrm{T}}(\theta_{k1}),\dots,\boldsymbol{a}^{\mathrm{T}}(\theta_{kD_k})\right]^{\mathrm{T}},\\ \boldsymbol{a}(\theta)&=\left[1,\mathrm{e}^{\mathrm{j}2\pi\frac{d\sin\theta}{\lambda}},\dots,\mathrm{e}^{\mathrm{j}2\pi(n_{\mathrm{T}}-1)\frac{d\sin\theta}{\lambda}}\right],\end{aligned}\tag{2}$$

where T represents the transpose operation, d is the equidistant antenna spacing, λ is the carrier wavelength, and θ is the DOD. $\hat{\theta}_k$ denotes the main angle of DOD for the user k, and $\Delta\psi_k$ is the angle spread, then $\theta_{kD_k}=\hat{\theta}_k+\rho\Delta\psi_k$, $\rho\in[-1,1]$. We define $\boldsymbol{d}_m^k$, $\boldsymbol{W}_m^k$, $\boldsymbol{R}_m^k$, and $\boldsymbol{Y}_m^k$ as the transmitting complex symbol, the transmitting beamforming vector, the weighted combining vector, and the receiving complex symbol vector for the kth user on the mth sub-carrier, respectively. Then, $\boldsymbol{Y}_m^k$ can be formulated as follows:

$$\begin{aligned}\boldsymbol{Y}_m^k&=\boldsymbol{H}_m^k\boldsymbol{W}_m^k\boldsymbol{d}_m^k+\sum_{j=1,j\neq k}^{K}\boldsymbol{H}_m^k\boldsymbol{W}_m^j\boldsymbol{d}_m^j+\boldsymbol{n}_m^k\\ &=\boldsymbol{G}_m^k\boldsymbol{A}_m^k\boldsymbol{W}_m^k\boldsymbol{d}_m^k+\sum_{j=1,j\neq k}^{K}\boldsymbol{G}_m^k\boldsymbol{A}_m^k\boldsymbol{W}_m^j\boldsymbol{d}_m^j+\boldsymbol{n}_m^k,\end{aligned}\tag{3}$$

where the first item on the right side of Eq. (3) represents the received symbols from the desired user, the second represents the CCI from the interference users, and $\boldsymbol{n}_m^k$ is a noise vector.

After weighted combining, the received symbol is given by

$$\hat{\boldsymbol{d}}_m^k=\boldsymbol{R}_m^k\boldsymbol{H}_m^k\boldsymbol{W}_m^k\boldsymbol{d}_m^k+\sum_{j=1,j\neq k}^{K}\boldsymbol{R}_m^k\boldsymbol{H}_m^k\boldsymbol{W}_m^j\boldsymbol{d}_m^j+\boldsymbol{R}_m^k\boldsymbol{n}_m^k.\tag{4}$$

The key of demodulation is that the CCI is mitigated and the linear processing is achieved at both the transmitter and the receiver by properly choosing $\boldsymbol{R}_m^k$ and $\boldsymbol{W}_m^k$. The optimal problem of minimizing the overall transmitting power is formulated as follows:

$$\begin{aligned}&\arg\min_{P_m^k,b_m^k}\sum_{k=1}^{K}\sum_{m=1}^{N_{\mathrm{c}}}P_m^k,\\ &\text{subject to}:\ \sum_{m=1}^{N_{\mathrm{c}}}b_m^k=R_k,\\ &\qquad\qquad\quad \mathrm{BER}_m^k\leqslant\mathrm{BER}_{\mathrm{target}}.\end{aligned}\tag{5}$$

The transmitting power of the shared user can be described as

$$\begin{aligned}P_m^k=g\Bigg(&\mathrm{BER}_{\mathrm{target}},b_m^k,\left\{\boldsymbol{H}_m^{k'}\right\}_{k'=1,\cdots,K},\\ &\left\{P_m^{k'}\right\}_{k'=1,\cdots,k-1,k+1,\dots,K}\Bigg).\end{aligned}\tag{6}$$

We can see that the transmitting power of different users is interrelated owing to the CCI. This makes the optimal problem very complex. The CCI can be removed by the exclusive manner or the ZF shared manner. The transmitting power of the shared user turns into

$$P_m^k=g\left(\mathrm{BER}_{\mathrm{target}},b_m^k,\left\{\boldsymbol{H}_m^{k'}\right\}_{k'=1,\cdots,K}\right).\tag{7}$$

Thus, the complexity of multi-user resource allocation is reduced. However, the two manners have different performances for different user spatial correlations.

3 System performance and user spatial correlation

As the angle spread of DOD is usually slight, the spatial correlation coefficient between the users can be denoted by $\hat{\theta}$ as follows:

$$\rho_{k1,k2}^m=\begin{cases}\dfrac{1}{n_T}\left|\dfrac{1-\exp\left(\mathrm{j}2\pi\frac{d}{\lambda}n_{\mathrm{T}}\left(\sin\hat{\theta}_{k1}-\sin\hat{\theta}_{k2}\right)\right)}{1-\exp\left(\mathrm{j}2\pi\frac{d}{\lambda}\left(\sin\hat{\theta}_{k1}-\sin\hat{\theta}_{k2}\right)\right)}\right|, & \hat{\theta}_{k1}\neq\hat{\theta}_{k2},\\ 1, & \hat{\theta}_{k1}=\hat{\theta}_{k2}.\end{cases}\tag{8}$$

Thus, it can be adjusted by varying $\hat{\theta}$ of the users. Two users are highly correlated when the angle between them is less than 5°.

Two allocation manners are simulated for a two-user system with $d=\lambda/2$, $n_{\mathrm{T}}=4$, $n_{\mathrm{R}}=2$, $\hat{\theta}_{k1}=0^{\circ}$, $\mathrm{BER}=10^{-3}$, 64 sub-carrier, on average 3 bits per sub-carrier for each user. Figure 1 plots $\mathrm{E_b/N_0}$ versus $\hat{\theta}_{k2}$ curves. It demonstrates that the power required of the exclusive manner is not affected by $\hat{\theta}_{k2}$, but the performance is not optimal with low user spatial correlation. The power of the ZF shared

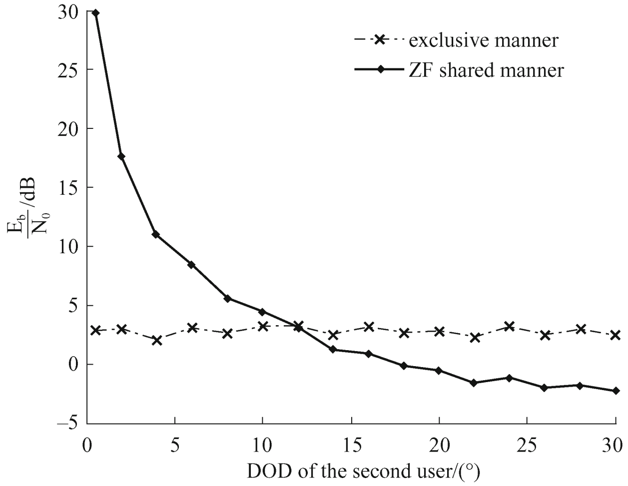

Fig. 1 Performance versus user spatial correlation for two manners

manner decreases with an increase of $\hat{\theta}_{k2}$, but the performance is the worst with high user spatial correlation.

4 Proposed resource allocation algorithm

4.1 User grouping and resource allocation principle

The highly correlated users whose difference of $\hat{\theta}$ is less than 5° are put in the same group. Thus, there are no highly correlated users between the user groups. Assume that there are in total G groups and the gth group has K_g users ($K_1 + K_2 + \cdots + K_G = K$). We allocate a sub-carrier to each user group for only one user. Thus, there are G shared users in each sub-carrier. As there are no highly correlated shared users in each sub-carrier, the performance of the ZF shared method can be improved. At the same time, the exclusive manner, such as Greedy algorithm, is applied to the users in the same group.

4.2 Group ZF method based on group marginal users

$X_{g,k}$ denotes the kth user of the gth user group. $\hat{\theta}_{g,1}$ and $\hat{\theta}_{g,K_g}$ are two main angles of two marginal users in the gth group ($\hat{\theta}_{g,1} \leqslant \hat{\theta}_{g,k} \leqslant \hat{\theta}_{g,K_g}$).

By Eq. (3), applying the ZF method to the shared users' A_m^k, we can absolutely mitigate the CCI. Furthermore, for the small angle spread of $\hat{\theta}$, we can only apply the ZF method to the shared users' $\boldsymbol{a}(\hat{\theta})$ to mitigate the CCI for reducing both the antenna number and the computation complexity. For the small angle difference between two marginal users, we can only apply the ZF method to the interference group marginal users' $\boldsymbol{a}(\hat{\theta})$ to mitigate the CCI caused by any user of the interference user group. Thus, to whomever in the interference user group one sub-carrier is allocated, the desired user's ZF matrix in the sub-carrier is invariant. Moreover, each user of the desired user group has the equal ZF matrix in the sub-carrier. Therefore, the computation complexity decreases.

To sum up, for any sub-carrier, the improved group ZF method based on the group marginal users is described as follows:

$$\begin{aligned} \boldsymbol{W}_{g,k(\mathrm{opt})} = \; & \underset{W_{g,k} \in C^{n_{\mathrm{T}} \times 1}}{\arg\max} \left(\left\| H_{g,k} W_{g,k} \right\|_F^2 \right), \\ & \text{subject to} : \boldsymbol{a}\left(\hat{\theta}_{i,1}\right) \boldsymbol{W}_{g,k} = 0, \\ & \qquad \boldsymbol{a}\left(\hat{\theta}_{i,K_i}\right) \boldsymbol{W}_{g,k} = 0, \\ & \qquad i,g = 1,2,\ldots,G,\; i \neq g,\; k = 1,2,\ldots,K_g. \end{aligned} \tag{9}$$

The process is described as follows:

1) Let

$$\boldsymbol{W}_{g,k} = \boldsymbol{W}_g^1 \boldsymbol{W}_{g,k}^2. \tag{10}$$

$\boldsymbol{W}_g^1$ is used to mitigate the CCI caused by the interference user groups for the gth desired user group, and $\boldsymbol{W}_{g,k}^2$ is used to maximize the received signal to noise ratio (SNR).

2) Let

$$\bar{\boldsymbol{A}}_{g,1} = \left[\boldsymbol{a}\left(\hat{\theta}_{1,1}\right)^{\mathrm{H}} \ldots \boldsymbol{a}\left(\hat{\theta}_{(g-1),1}\right)^{\mathrm{H}} \boldsymbol{a}\left(\hat{\theta}_{(g+1),1}\right)^{\mathrm{H}} \ldots \boldsymbol{a}\left(\hat{\theta}_{G,1}\right)^{\mathrm{H}} \right]^{\mathrm{H}},$$

$$\begin{aligned} \bar{\boldsymbol{A}}_{g,K_g} = \Big[& \boldsymbol{a}\left(\hat{\theta}_{1,K_1}\right)^{\mathrm{H}} \ldots \boldsymbol{a}\left(\hat{\theta}_{(g-1),K_{g-1}}\right)^{\mathrm{H}} \\ & \boldsymbol{a}\left(\hat{\theta}_{(g+1),K_{g+1}}\right)^{\mathrm{H}} \ldots \boldsymbol{a}\left(\hat{\theta}_{G,K_G}\right)^{\mathrm{H}} \Big]^{\mathrm{H}}. \end{aligned}$$

Apply singular value decomposition (SVD) to the following matrix:

$$\begin{bmatrix} \bar{\boldsymbol{A}}_{g,1} \\ \bar{\boldsymbol{A}}_{g,K_g} \end{bmatrix} = \bar{\boldsymbol{U}}_g \bar{\Lambda}_g \left[\bar{\boldsymbol{V}}_g^{(1)}, \bar{\boldsymbol{V}}_g^{(0)} \right]^{\mathrm{H}}, \tag{11}$$

where $\bar{\boldsymbol{V}}_g^{(0)}$ is an $n_{\mathrm{T}} \times [n_{\mathrm{T}} - (G-1) \times 2]$ matrix, the columns of which are singular vectors corresponding to null singular values. Let

$$\boldsymbol{W}_g^1 = \bar{\boldsymbol{V}}_g^{(0)}. \tag{12}$$

3) Apply SVD to the following matrix:

$$\boldsymbol{H}_{g,k} \boldsymbol{W}_g^1 = \boldsymbol{U}_{g,k} \Lambda_{g,k} \boldsymbol{V}_{g,k}^{\mathrm{H}}. \tag{13}$$

Let $\{\lambda_{g,k}\}_1$ denote the maximum singular value, and $\{\boldsymbol{V}_{g,k}\}_1$ denote the corresponding singular vector. Let $\boldsymbol{W}_{g,k}^2 = \left\{ \boldsymbol{V}_{g,k} \right\}_1$.

4) Calculate the beamforming vector and the combining vector as follows:

$$\boldsymbol{W}_{g,k} = \boldsymbol{W}_g^1 \boldsymbol{W}_{g,k}^2 = \bar{\boldsymbol{V}}_{\boldsymbol{g}}^{(0)} \left\{ \boldsymbol{V}_{g,k} \right\}_1,\; \boldsymbol{R}_{g,k} = \left\{ \boldsymbol{U}_{g,k}^{\mathrm{H}} \right\}_1. \tag{14}$$

The CCI mitigation performance of the improved group ZF method is simulated. A desired user lies in 0°, and the marginal users of three interference groups lie in −30°, −35°; 15°, 18°; 35°, 39°; respectively. Figure 2 plots the desired user's transmitting beam gain versus the direction curve for one sub-carrier. We can see that the transmitting beam gains in the marginal user directions of the three interference groups are nearly null. At the same time, the transmitting beam gains between two marginal user directions of each interference user group are also very low. Thus, the improved group ZF method is efficient to mitigate CCI caused by the interference user groups.

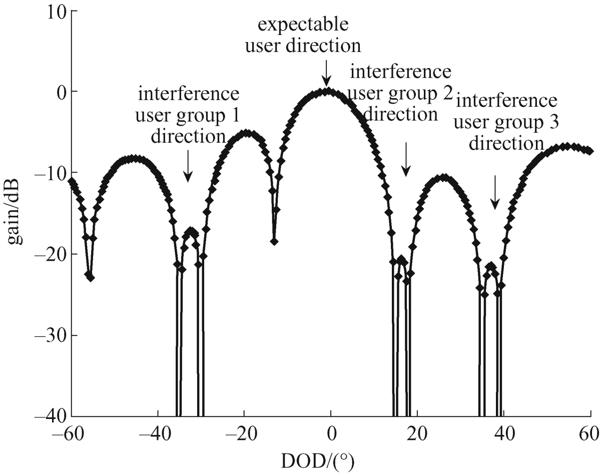

Fig. 2 Transmitting beam gain with group ZF method

4.3 Resource allocation in same group

With the above user grouping, CCI mitigating by the group ZF method, the complex multi-user resource allocation turns into the simple exclusive manner resource allocation to the users of the same group. To guarantee the user's QoS, the exclusive manner proposed in Ref. [6] is applied in each group. The sum of all K users' power is the suboptimal solution.

5 Simulation results

The performance of the proposed resource allocation algorithm is simulated with the Monte-Carlo method. The simulation parameters are shown in Table 1.

Table 1 Simulation parameters

items	parameter
simulation channel	Rayleigh fading channel
number of steering vectors(D_k)	3
angle spread of DOD (°)	3
receiving antennas(n_R)	2
sub-carrier(N_c)	64
guard interval(G_i)	16
bit number of MQAM	0, 1, 2, 4, 6, 8

In all simulations, without special description, twelve users are divided into four groups and each group has three highly correlated users. $\hat{\theta}$ is 5°, 7°, 9°; 20°, 22°, 24°; 35°, 37°, 39°; −20°, −22°, −24°; respectively. E_b/N_0 is defined as the average ratio of energy per bit to the spectral noise density for all users. It corresponds to the efficiency of the BS transmitter.

We simulate three schemes, namely, the exclusive manner ($n_T = 2$), the classic ZF shared manner ($n_T = 24$), and the proposed algorithm ($n_T = 9$) for a twelve-user system with BER = 10^{-3}. The curves are plotted for average bits per sub-carrier per user versus E_b/N_0 in Fig. 3. From the curves, we can see that the spectral efficiency is very low in the exclusive manner; the power required in classic ZF manner is extremely high for existing highly correlated shared users; the power required in the proposed algorithm is lower as there are no highly correlated shared users by user grouping. Furthermore, it also proves that the proposed algorithm reduces the number of the required BS transmitting antennas.

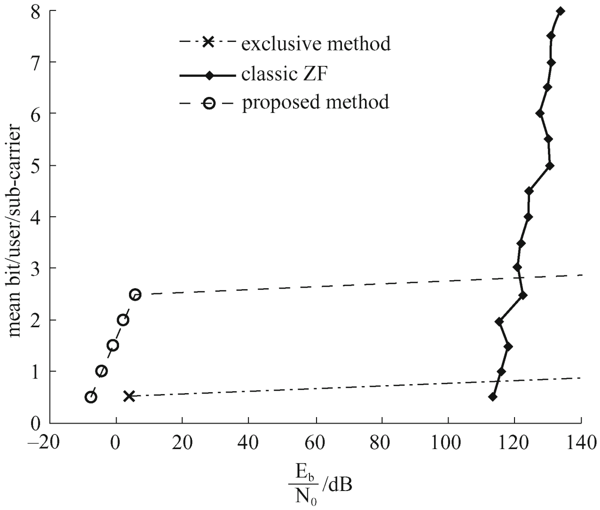

Fig. 3 Performance of different algorithms with different data rates

The spectral efficiency of each user in the proposed algorithm ($n_T = 6$) is simulated for a five-user system with BER = 10^{-3}. $\hat{\theta}$ is 5°, 7°, 9°, 20°, 35°, respectively. Figure 4 plots the average bits per sub-carrier versus E_b/N_0 curves for each user. It can be observed that only the highly correlated users' spectral efficiency decreases, and in each user's spectrum range, the required power is low.

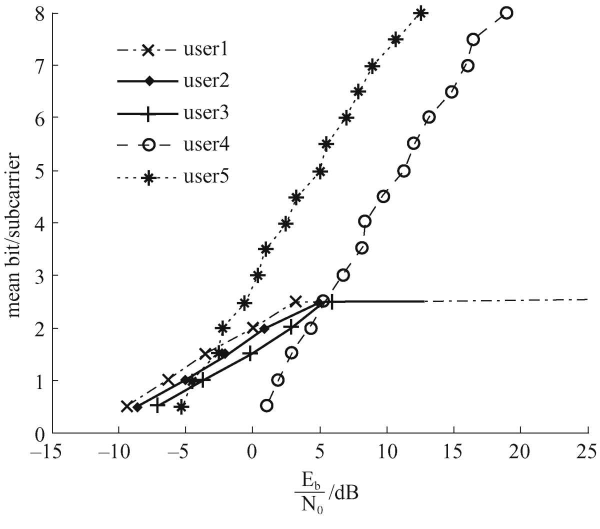

Fig. 4 Performance of proposed algorithm with different data rate for each user

Figure 5 illustrates the BS transmitting antenna number versus E_b/N_0 plot for a twelve-user system. The data rate of each user is the average 2 bits per sub-carrier and BER is limited to 10^{-3}. It demonstrates that with the increase of the BS antenna number, the E_b/N_0 of the proposed algorithm decreases. Thus, the performance and the BS transmitting antenna number can be a flexible tradeoff.

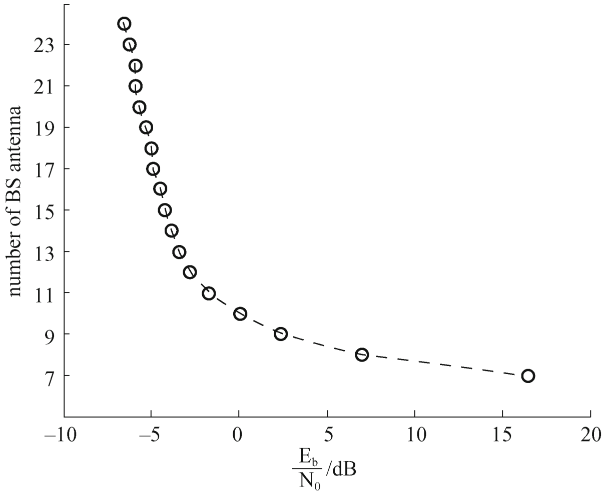

Fig. 5 Performance versus BS antenna number for proposed algorithm

6 Conclusions

An adaptive resource allocation algorithm for multi-user MIMO-OFDM downlink in correlated channels is proposed to minimize the overall transmitting power while maintaining a constant data rate and target BER. The algorithm allocates sub-carriers combining the exclusive manner and the shared manner based on the user grouping according to their spatial correlations. An improved null steering method based on the group marginal users is used to mitigate the CCI caused by the shared interference user groups. The algorithm improves the power efficiency and the spectral efficiency when the system has highly correlated users, and reduces the BS transmitting antenna number as well as computation complexity.

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