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RESEARCH ARTICLE

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Diversity analysis of space-time-frequency coded broadband MIMO-OFDM system with correlation across space time and frequency

Abstract For the frequency selective and time variant multiple-input multiple-output (MIMO) channel model taking into account transmitting and receiving antenna correlation, the diversity of space-time-frequency coded broadband orthogonal frequency division multiplexing (MIMO-OFDM) system is analyzed. Based on the average pairwise error probability (PEP), the design criterion of space-time-frequency code (STFC) is expanded. For a given STFC, it is found that the achievable diversity order is related to the transmitter and the receiver correlation matrix as well as the time correlation and frequency correlation matrix. The maximum available diversity of STFC over the correlation channel is $L\mathrm{rank}(\boldsymbol{P})\mathrm{rank}(\boldsymbol{Q})\mathrm{rank}(\boldsymbol{R}_{\mathrm{T}})$. The space-time code and space-frequency code are special cases in our approach. Simulation results validate the findings.

Keywords diversity, multiple-input multiple-output (MIMO), orthogonal frequency division multiplexing, pairwise error probability, space-time-frequency coding

1 Introduction

The next generation wireless communication systems are required to provide mobile and stationary users with wireless multimedia services. The rapidly rising demand for high data rates, along with high mobility, quality and services are driving recent developments in communication technologies for broadband wireless systems. Recently, space-time coding (STC) [1,2] has been proposed as one of the most attractive techniques for multiple-input multiple-output (MIMO) wireless communications. In this case, coding is performed across spatial (antenna) dimension as well as time dimension. However, in broadband wireless communications, the duration of ST-encoded symbols may become smaller than the delay spread of multipath fading, which consequently causes severe intersymbol interference (ISI) [3]. Diversity is one of the effective methods to combat fading, which depends on signals that are transmitted across multiple independent fading paths. Clearly, the probability of multiple paths fading at the same time is very low, which can greatly reduce the possibility of the error decision at the receiver. To support higher-speed ST coded applications, space-frequency coded (SFC) orthogonal frequency division multiplexing (OFDM) systems have been proposed for frequency selective fading channels [4–8]. How to maximize exploitation of the diversity of the MIMO-OFDM systems is becoming the target of research.

The first SF coding scheme was proposed in Ref. [2], in which previously existing space-time (ST) codes were used by replacing time domain with frequency domain (OFDM tones). Later works [4,5] also described similar schemes, i.e., using ST codes directly as SF codes. The resulting SF codes achieved only spatial diversity, and they were not guaranteed to achieve full spatial and frequency diversities. Later in Refs. [7,9,10] systematic SF codes designing methods were proposed, which are guaranteed to achieve full diversity. This coding approach can exploit spatial and frequency diversity.

To exploit full spatial, temporal and frequency diversity available in frequency selective MIMO broadband channels, a new space-time-frequency coding (STFC) approach is proposed. The STF coding strategy, by coding across multiple OFDM blocks, was firstly proposed in Ref. [11] for two transmitting antennas and further developed in Refs. [12–14] for multiple transmitting antennas. In Refs. [11,14], it was assumed that the MIMO channels stay constant over multiple OFDM blocks. In Ref. [13], an intuitive explanation of the equivalence between antennas and OFDM tones was presented in terms of capacity. In Ref. [12], the performance criteria for STF codes were derived,

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and an upper bound of the maximum achievable diversity order was established. In Ref. [15], the diversity of STF coded MIMO-OFDM system was analyzed, considering temporal and frequency correlation. However, this conclusion was based on the MIMO channel that was spatially uncorrelated. In Ref. [16], using a physically motivated broadband MIMO channel model with transmitting and receiving antenna correlation, the performance of SF coded OFDM was studied. However, the performance of STFC over frequency selective and time variant fading channel is not reported when MIMO channels are correlated.

Combining the advantages of Refs. [15,16], we extend the results reported in Ref. [15] to incorporate spatial correlation. We expand the conclusion of performance criteria for designing STF coded MIMO-OFDM systems. For a given STF code, we quantify the achievable diversity order and coding gains as a function of transmitting and receiving correlation matrices. Our model incorporates the ST and SF coding approaches as special cases.

2 System model

We consider a STF-coded MIMO-OFDM system with N_t transmitting antennas, N_r receiving antennas and N_c subcarriers. Suppose that the frequency selective fading channels between each pair of transmitting and receiving antennas have L independent delay paths and the same power delay profiles. The MIMO channel is assumed to be constant over each OFDM block period, but it may vary from one OFDM block to another. At the kth OFDM block, the channel impulse response from transmitting antenna i to receiving antenna j at time τ can be modeled as

$$h_{m,n}^k(\tau)=\sum_{l=0}^{L-1}\alpha_{m,n}^k(l)\delta(\tau-\tau_l), \tag{1}$$

where τ_l is delay of the lth path, and $\alpha_{m,n}^k(l)$ is complex amplitude of the lth path between transmitting antenna m and receiving antenna n at the kth OFDM block. The $\alpha_{m,n}^k(l)$ are modeled as zero-mean, complex Gaussian random variables with variances $E|\alpha_{m,n}^k(l)|^2=\delta_l^2$, where E stands for expectation. The powers of the L paths are normalized such that $\sum_{l=0}^{L-1}\delta_l^2=1$. From Eq. (1), the frequency response of the channel is given by

$$\mathrm{H}_{m,n}^k(f)=\sum_{l=0}^{L-1}\alpha_{m,n}^k(l)\mathrm{e}^{-\mathrm{j}2\pi f\tau_l}, \tag{2}$$

where $\mathrm{j}=\sqrt{-1}$. we assume that the MIMO channel is spatially correlated, i.e., the channel taps $\alpha_{m,n}^k(l)$ are correlated with the pairwise difference of indices (m,n).

We consider STF coding across N_t transmitting antennas, N_c OFDM sub-carries and K OFDM blocks. Each STF codeword can be expressed as a $KN_\mathrm{c}\times N_\mathrm{t}$ matrix:

$$\boldsymbol{C}=\left[\boldsymbol{C}_1^\mathrm{T}\ \boldsymbol{C}_2^\mathrm{T}\ \cdots\ \boldsymbol{C}_K^\mathrm{T}\right]^\mathrm{T}, \tag{3}$$

where

$$\boldsymbol{C}_k=\begin{bmatrix} c_1^k(0) & c_2^k(0) & \cdots & c_{N_\mathrm{t}}^k(0)\\ c_1^k(1) & c_2^k(1) & \cdots & c_{N_\mathrm{t}}^k(1)\\ \vdots & \vdots & \ddots & \vdots\\ c_1^k(N_\mathrm{c}-1) & c_2^k(N_\mathrm{c}-1) & \cdots & c_{N_\mathrm{t}}^k(N_\mathrm{c}-1)\end{bmatrix} \tag{4}$$

is the channel symbol matrix transmitted in the kth OFDM block, and $c_m^k(p)$ is the channel symbol transmitted over the pth sub-carrier by transmitting antenna m at the kth OFDM block. It is assumed that the STF code's energy is constrained to $E||\boldsymbol{C}||_\mathrm{F}^2=KN_\mathrm{c}N_\mathrm{t}$, where $\|\boldsymbol{C}\|_\mathrm{F}$ is the Frobenius norm of $\boldsymbol{C}$. At the kth OFDM block, the OFDM transmitter applies an N_c-point inverse fast Fourier transform (IFFT) to each column of matrix $\boldsymbol{C}_k$. After appending a cyclic prefix, the OFDM symbol corresponding to the mth ($i=1,2,\ldots,N_\mathrm{t}$) column of $\boldsymbol{C}_k$ is transmitted by the transmitting antenna m.

At the receiver, after matched filtering, removing the cyclic prefix and applying fast Fourier transform (FFT), the received signal at the nth subcarrier at the receiving antenna n in the kth OFDM block is given by

$$y_n^k(p)=\sqrt{\frac{\rho}{N_\mathrm{t}}}\sum_{m=1}^{N_\mathrm{t}}c_m^k(p)\boldsymbol{H}_{m,n}^k(p)+z_n^k(p), \tag{5}$$

where

$$\boldsymbol{H}_{m,n}^k(p)=\sum_{l=0}^{L-1}\alpha_{m,n}^k(l)\mathrm{e}^{-\mathrm{j}2\pi p\Delta f\tau_l} \tag{6}$$

is the channel frequency response at the pth sub-carrier between the transmitting antenna m and the receiving antenna n, $\Delta f=1/T$ is the sub-carrier separation in the frequency domain, and T is the OFDM symbol period. We assume that the channel state information (CSI) $\boldsymbol{H}_{m,n}^k(p)$ is known at the receiver, but not at the transmitter. In Eq. (5), $z_n^k(p)$ denotes additive complex Gaussian noise with zero mean and unit variance at the pth subcarrier at the receiving antenna n in the kth OFDM block. The factor $\sqrt{\rho/N_\mathrm{t}}$ in Eq. (5) ensures that ρ is the average signal to noise ratio (SNR) at each receiving antenna, and it is independent of the number of transmitting antennas.

3 Performance criteria

In this section, we derive the performance criteria for STF coded MIMO-OFDM systems. We shall first obtain the average pairwise error probability (PEP) for STF codes and then quantify the maximum achievable diversity order and coding gain.

Let

$$\begin{aligned}\boldsymbol{Y}=\big[&y_1^1(0),\dots,y_1^1(N_c-1),\ y_1^2(0),\dots,y_1^K(N_c-1),\\ &y_2^1(0),\dots,y_{N_r}^K(N_c-1)\big]^{\mathrm{T}},\end{aligned}\tag{7}$$

$$\begin{aligned}\boldsymbol{H}_{m,n}=\Big[&\boldsymbol{H}_{m,n}^1(0),\dots,\boldsymbol{H}_{m,n}^1(N_c-1),\boldsymbol{H}_{m,n}^2(0),\dots,\\ &\boldsymbol{H}_{m,n}^K(N_c-1)\Big]^{\mathrm{T}},\end{aligned}\tag{8}$$

$$\boldsymbol{Z}=\Big[z_1^1(0),\dots,z_1^1(N_c-1),\dots,z_1^K(N_c-1),\dots,z_{N_r}^K(N_c-1)\Big]^{\mathrm{T}}.$$

We rewrite Eq. (5) in vector form as

$$\boldsymbol{Y}=\sqrt{\frac{\rho}{N_t}}\boldsymbol{D}\boldsymbol{H}+\boldsymbol{Z},\tag{9}$$

where $\boldsymbol{D}$ is a $KN_cN_r \times KN_cN_tN_r$ matrix constructed from STF codeword $\boldsymbol{C}$ in Eq. (3) as follows:

$$\boldsymbol{D}=\boldsymbol{I}_{N_r}\otimes[\boldsymbol{D}_1,\boldsymbol{D}_2,\dots,\boldsymbol{D}_{N_t}],\tag{10}$$

where $\otimes$ denotes the Kronecker product, $\boldsymbol{I}_{N_r}$ is the identity matrix of size $N_r \times N_r$, and

$$\begin{aligned}\boldsymbol{D}_m=\mathrm{diag}\big\{&c_m^1(0),\dots,c_m^1(N_c-1),\dots,\\ &c_m^2(0),\dots,c_m^k(N_c-1)\big\}.\end{aligned}\tag{11}$$

Each $\boldsymbol{D}_m$ in Eq. (11) is related to the mth column of the STF codeword $\boldsymbol{C}$. The channel vector $\boldsymbol{H}$ of size $KN_cN_tN_r \times 1$ is formatted as:

$$\boldsymbol{H}=\Big[\boldsymbol{H}_{1,1}^{\mathrm{T}},\dots,\boldsymbol{H}_{N_t,1}^{\mathrm{T}},\boldsymbol{H}_{1,2}^{\mathrm{T}},\dots,\boldsymbol{H}_{N_t,N_r}^{\mathrm{T}}\Big]^{\mathrm{T}}.\tag{12}$$

Suppose that $\boldsymbol{D}$ and $\tilde{\boldsymbol{D}}$ are two different matrices related to two different STF codewords $\boldsymbol{C}$ and $\tilde{\boldsymbol{C}}$, respectively. Then, the pairwise error probability between $\boldsymbol{D}$ and $\tilde{\boldsymbol{D}}$ can be upper bounded as in Ref. [14]:

$$P\big(\boldsymbol{D}\to\tilde{\boldsymbol{D}}\big)\leqslant\binom{2r-1}{r}\left(\prod_{i=1}^{r}\gamma_i\right)^{-1}\left(\frac{\rho}{N_t}\right)^{-r},\tag{13}$$

where, r is the rank of $\big(\boldsymbol{D}-\tilde{\boldsymbol{D}}\big)\boldsymbol{R}\big(\boldsymbol{D}-\tilde{\boldsymbol{D}}\big)^{\mathrm{H}}$, $\gamma_1,\gamma_2,\dots,\gamma_r$ are the non-zero eigenvalues of $\big(\boldsymbol{D}-\tilde{\boldsymbol{D}}\big)\boldsymbol{R}\big(\boldsymbol{D}-\tilde{\boldsymbol{D}}\big)^{\mathrm{H}}$, and $\boldsymbol{R}=E\{\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}\}$ is the correlation matrix of $\boldsymbol{H}$. The superscript H stands for the complex conjugate and transpose of a matrix. Based on the upper bound on the PEP in Eq. (13), two general STF code performance criteria can be proposed as follows:

1) Diversity (rank) criterion: the minimum rank of $\big(\boldsymbol{D}-\tilde{\boldsymbol{D}}\big)\boldsymbol{R}\big(\boldsymbol{D}-\tilde{\boldsymbol{D}}\big)^{\mathrm{H}}$ over all pairs of different codewords $\boldsymbol{C}$ and $\tilde{\boldsymbol{C}}$ should be as large as possible.

2) Product criterion: the minimum value of the product $\prod_{i=1}^{r}\gamma_i$ over all pairs of different codewords $\boldsymbol{C}$ and $\tilde{\boldsymbol{C}}$ should be maximized.

4 Diversity analysis

In this section, we will explicitly demonstrate the achievable diversity of the given STF coded MIMO-OFDM systems when correlated across space, time and frequency, and we will revise the performance criteria of STF codes.

Suppose the correlation matrices at the transmitter and the receiver are independent, if antennas at the transmitter and the receiver are distributed at an equal interval, the correlation matrix at the transmitter and the receiver is related to the interval distance [17]. Here we denote the transmitter correlation matrix and the receiver correlation matrix separately as below:

$$\boldsymbol{P}=\begin{bmatrix}1 & \alpha & \cdots & \alpha^{N_t-1}\\ \alpha & 1 & \cdots & \alpha^{N_t-2}\\ \vdots & \vdots & \ddots & \vdots\\ \alpha^{N_t-1} & \alpha^{N_t-2} & \cdots & 1\end{bmatrix},$$

$$\boldsymbol{Q}=\begin{bmatrix}1 & \beta & \cdots & \beta^{N_r-1}\\ \beta & 1 & \cdots & \beta^{N_r-2}\\ \vdots & \vdots & \ddots & \vdots\\ \beta^{N_r-1} & \beta^{N_r-2} & \cdots & 1\end{bmatrix}.$$

We model the spatial correlation according to the pairwise difference of indices (m,n) of the transmitter antenna and the receiver antenna. For example, the cross correlation matrix of channel gain of the transmitter antenna m to the receiver antenna n and the transmitter antenna u to the receiver antenna v is shown as below:

$$E\Big\{\boldsymbol{H}_{m,n}\boldsymbol{H}_{u,v}^{\mathrm{H}}\Big\}=p(|u-m|)\boldsymbol{\Sigma}q(|v-n|),\tag{14}$$

where

$$\boldsymbol{\Sigma}=E\Big\{\boldsymbol{H}_{m,n}\boldsymbol{H}_{m,n}^{\mathrm{H}}\Big\}\tag{15}$$

is the correlation matrix of the channel time and frequency response from the transmitting antenna m to the receiving antenna n, and Eq. (14) is the cross correlation expression of the transmitting antenna m to the receiving antenna n and the transmitting antenna u to the receiving antenna v. Using the notion $w=\mathrm{e}^{-\mathrm{j}2\pi\Delta f}$, from Eqs. (6) and (8), we obtain:

$$\boldsymbol{H}_{m,n}=(\boldsymbol{I}_K\otimes\boldsymbol{W})\boldsymbol{A}_{m,n},\tag{16}$$

where

$$\boldsymbol{A}_{m,n}=\left[\alpha^1_{m,n}(0),\ldots,\alpha^1_{m,n}(L-1),\alpha^2_{m,n}(0),\ldots,\alpha^K_{m,n}(L-1)\right]^{\mathrm{T}}$$

and

$$\boldsymbol{W}=\begin{bmatrix} 1 & 1 & \cdots & 1 \\ w^{\tau_0} & w^{\tau_1} & \cdots & w^{\tau_{L-1}} \\ \vdots & \vdots & \ddots & \vdots \\ w^{(N-1)\tau_0} & w^{(N-1)\tau_1} & \cdots & w^{(N-1)\tau_{L-1}} \end{bmatrix}.$$

Substitute Eq. (16) into Eq. (15) and we get:

$$\begin{aligned}\boldsymbol{\Sigma}&=E\left\{(\boldsymbol{I}_K\otimes\boldsymbol{W})\boldsymbol{A}_{m,n}\boldsymbol{A}^{\mathrm{H}}_{m,n}(\boldsymbol{I}_K\otimes\boldsymbol{W})^{\mathrm{H}}\right\}\\&=(\boldsymbol{I}_K\otimes\boldsymbol{W})E\left\{\boldsymbol{A}_{m,n}\boldsymbol{A}^{\mathrm{H}}_{m,n}\right\}(\boldsymbol{I}_K\otimes\boldsymbol{W})^{\mathrm{H}}.\end{aligned}$$

According to the results reported in Ref. [15], the correlation matrix $E\left\{\boldsymbol{A}_{m,n}\boldsymbol{A}^{\mathrm{H}}_{m,n}\right\}$ can be expressed as:

$$E\left\{\boldsymbol{A}_{m,n}\boldsymbol{A}^{\mathrm{H}}_{m,n}\right\}=\boldsymbol{R}_{\mathrm{T}}\otimes\boldsymbol{\Lambda},\qquad(17)$$

where $\boldsymbol{\Lambda}=\mathrm{diag}\left\{\delta_0^2,\delta_1^2,\ldots,\delta_{L-1}^2\right\}$ and $\boldsymbol{R}_{\mathrm{T}}$ is the temporal correlation matrix of size $K\times K$. Then we can also define the frequency correlation matrix $\boldsymbol{R}_{\mathrm{F}}$ as $\boldsymbol{R}_{\mathrm{F}}=E\left\{\boldsymbol{H}^k_{m,n}\boldsymbol{H}^{k\ \mathrm{H}}_{m,n}\right\}$. Thus $\boldsymbol{R}_{\mathrm{F}}=\boldsymbol{W}\boldsymbol{\Lambda}\boldsymbol{W}^{\mathrm{H}}$. As a result, we arrive at:

$$\begin{aligned}\boldsymbol{\Sigma}&=(\boldsymbol{I}_K\otimes\boldsymbol{W})(\boldsymbol{R}_{\mathrm{T}}\otimes\boldsymbol{\Lambda})\left(\boldsymbol{I}_K\otimes\boldsymbol{W}^{\mathrm{H}}\right)\\&=\boldsymbol{R}_{\mathrm{T}}\otimes\left(\boldsymbol{W}\boldsymbol{\Lambda}\boldsymbol{W}^{\mathrm{H}}\right)=\boldsymbol{R}_{\mathrm{T}}\otimes\boldsymbol{R}_{\mathrm{F}}.\end{aligned}\qquad(18)$$

It is found that the correlation matrix of the channel frequency response does not depend on the transmitting antenna m and the receiving antenna n. Therefore, we can rewrite Eq. (14) as

$$E\left\{\boldsymbol{H}_{m,n}\boldsymbol{H}^{\mathrm{H}}_{u,v}\right\}=p(|u-m|)E\left\{\boldsymbol{H}_{1,1}\boldsymbol{H}^{\mathrm{H}}_{1,1}\right\}q(|v-n|),$$

where $p(0)=q(0)=1$, which is a fractional ratio to a based antenna pair. Then with demonstration, we get the correlation matrix $\boldsymbol{R}$ of size $KN_{\mathrm{c}}N_{\mathrm{t}}N_{\mathrm{r}}\times KN_{\mathrm{c}}N_{\mathrm{t}}N_{\mathrm{r}}$ as

$$\boldsymbol{R}=(\boldsymbol{Q}\otimes\boldsymbol{P})\otimes\boldsymbol{\Sigma},\qquad(19)$$

where $\boldsymbol{Q}$ and $\boldsymbol{P}$ are the receiving antenna correlation matrix of size $N_{\mathrm{r}}\times N_{\mathrm{r}}$ and the transmitting antenna correlation matrix of size $N_{\mathrm{t}}\times N_{\mathrm{t}}$, whose entry in the mth row and the nth column is given by $q(n-m)$ and $p(n-m)$ respectively for $1\leqslant m,n\leqslant N_{\mathrm{r}}(N_{\mathrm{t}})$.

Finally, by combining Eqs. (4), (10), (11) and (19), the expression for $(\boldsymbol{D}-\tilde{\boldsymbol{D}})\boldsymbol{R}(\boldsymbol{D}-\tilde{\boldsymbol{D}})^{\mathrm{H}}$ in Eq. (13) can be rewritten into Eq. (20) as below, where $\circ$ denotes the Hadamard product:

$$\begin{aligned}&(\boldsymbol{D}-\tilde{\boldsymbol{D}})\boldsymbol{R}(\boldsymbol{D}-\tilde{\boldsymbol{D}})^{\mathrm{H}}\\&=\left(\boldsymbol{I}_{N_{\mathrm{r}}}\otimes\left[\boldsymbol{D}_1-\tilde{\boldsymbol{D}}_1,\ldots,\boldsymbol{D}_{N_{\mathrm{t}}}-\tilde{\boldsymbol{D}}_{N_{\mathrm{t}}}\right]\right)((\boldsymbol{Q}\otimes\boldsymbol{P})\\&\quad\otimes(\boldsymbol{R}_{\mathrm{T}}\otimes\boldsymbol{R}_{\mathrm{F}}))\left(\boldsymbol{I}_{N_{\mathrm{r}}}\otimes\left[\boldsymbol{D}_1-\tilde{\boldsymbol{D}}_1,\ldots,\boldsymbol{D}_{N_t}-\tilde{\boldsymbol{D}}_{N_t}\right]^{\mathrm{H}}\right)\\&=\boldsymbol{Q}\otimes\left\{\left[\left(\boldsymbol{C}-\tilde{\boldsymbol{C}}\right)\boldsymbol{P}\left(\boldsymbol{C}-\tilde{\boldsymbol{C}}\right)^{\mathrm{H}}\right]\circ(\boldsymbol{R}_{\mathrm{T}}\otimes\boldsymbol{R}_{\mathrm{F}})\right\}.\end{aligned}\qquad(20)$$

Denote $\boldsymbol{\Delta}=(\boldsymbol{C}-\tilde{\boldsymbol{C}})\boldsymbol{P}(\boldsymbol{C}-\tilde{\boldsymbol{C}})^{\mathrm{H}}$ and substitute Eq. (20) into Eq. (13). The PEP between $\boldsymbol{C}$ and $\tilde{\boldsymbol{C}}$ can be upper bounded as

$$P(\boldsymbol{C}-\tilde{\boldsymbol{C}})\leqslant\begin{pmatrix}2\upsilon\mathrm{rank}(\boldsymbol{Q})-1\\ \upsilon\mathrm{rank}(\boldsymbol{Q})\end{pmatrix}\left(\prod_{i=1}^{\upsilon}\lambda_i\right)^{-\mathrm{rank}(\boldsymbol{Q})}\left(\frac{\rho}{N_{\mathrm{t}}}\right)^{-\upsilon\mathrm{rank}(\boldsymbol{Q})},$$

where υ is the rank of $\boldsymbol{\Delta}\circ\boldsymbol{\Sigma}$, and $\lambda_1,\lambda_2,\ldots,\lambda_\upsilon$ are the non-zero eigenvalues of $\boldsymbol{\Delta}\circ\boldsymbol{\Sigma}$. As a consequence, we can formulate the performance criteria for STF codes as follows:

1) Diversity (rank) criterion: the minimum rank of $\boldsymbol{\Delta}\circ\boldsymbol{\Sigma}$ over all pairs of different code words $\boldsymbol{C}$ and $\tilde{\boldsymbol{C}}$ should be as large as possible.

2) Product criterion: the minimum value of the product $\prod_{i=1}^{\upsilon}\lambda_i$ over all pairs of different code words $\boldsymbol{C}$ and $\tilde{\boldsymbol{C}}$ should be maximized.

If the minimum rank of $\boldsymbol{\Delta}\circ\boldsymbol{\Sigma}$ is υ for any pairs of distinct STF code words $\boldsymbol{C}$ and $\tilde{\boldsymbol{C}}$ and the rank of receiving antenna correlation matrix $\boldsymbol{Q}$ is rank($\boldsymbol{Q}$), we say that the STF code achieves a diversity order of υrank($\boldsymbol{Q}$).

Theorem 1 For a fixed number of OFDM block K, a transmitting antenna correlation matrix $\boldsymbol{P}$ and a receiving antenna correlation matrix $\boldsymbol{Q}$, and correlation matrices $\boldsymbol{R}_{\mathrm{T}}$ and $\boldsymbol{R}_{\mathrm{F}}$, the maximum achievable diversity or full diversity is defined as the maximum diversity order that can be achieved by STF codes of size $KN_{\mathrm{c}}\times N_{\mathrm{t}}$.

Proof According to the rank inequalities on Hadamard products and Kronecker products in Ref. [18], we have: $\mathrm{rank}(\boldsymbol{\Delta}\circ\boldsymbol{\Sigma})\leqslant\mathrm{rank}(\boldsymbol{\Delta})\mathrm{rank}(\boldsymbol{R}_{\mathrm{T}})\mathrm{rank}(\boldsymbol{R}_{\mathrm{F}})$. Since the rank of $\boldsymbol{\Delta}$ is at most rank($\boldsymbol{P}$), the rank of $\boldsymbol{R}_{\mathrm{F}}$ is at most L, and the rank of $\boldsymbol{\Delta}\circ\boldsymbol{\Sigma}$ is at most KN, we obtain:

$$\mathrm{rank}(\boldsymbol{\Delta}\circ\boldsymbol{\Sigma})\leqslant\min\{L\mathrm{rank}(\boldsymbol{P})\mathrm{rank}(\boldsymbol{R}_{\mathrm{T}}),KN\}.\qquad(21)$$

Thus, the maximum achievable diversity is at most $\{L\mathrm{rank}(\boldsymbol{P})\mathrm{rank}(\boldsymbol{Q})\mathrm{rank}(\boldsymbol{R}_{\mathrm{T}}),KN_{\mathrm{c}}\mathrm{rank}(\boldsymbol{Q})\}$, in agreement with the results in Refs. [10,15,16]. Without loss of generality, we assume that the number of sub-carriers, N_{c}, is far larger than LN_{t}, therefore, the maximum achievable diversity order is $L\mathrm{rank}(\boldsymbol{P})\mathrm{rank}(\boldsymbol{Q})\mathrm{rank}(\boldsymbol{R}_{\mathrm{T}})$.

Our results are general conclusions on diversity analysis when correlated across space, time and frequency, and our analyses incorporate the spatial correlation, temporal correlation and frequency correlation as special cases.

Special case 1 If the fading channels are constant during K OFDM blocks, which means $\text{rank}(\boldsymbol{R}_\text{T}) = 1$, then the maximum achievable diversity order for STF codes is $L\text{rank}(\boldsymbol{P})\text{rank}(\boldsymbol{Q})$. This is also the case for SF codes, which coincide with the conclusions in Ref. [16].

Special case 2 When the transmitter correlation matrix and the receiver correlation matrix are full rank, i.e.,

$$\text{rank}(\boldsymbol{Q}) = N_\text{r},\ \text{rank}(\boldsymbol{P}) = N_\text{t},$$

then the maximum achievable diversity order for STF codes is $\min\{LN_\text{t}N_\text{r}\text{rank}(\boldsymbol{R}_\text{T}), KN_\text{c}N_\text{r}\}$.When the number of sub-carriers, N_c, is far larger than LN_t, the maximum achievable diversity order is $LN_\text{t}N_\text{r}\text{rank}(\boldsymbol{R}_\text{T})$, which also coincides with the conclusions in Ref. [15].

5 Simulation results

Each channel is generated based on the time-variant Jakes model. In this paper, it is based on a simple two equal-path delay power channel, and the durations between the two paths are 5 μs (as shown in Fig. 1) and 20 μs (as shown in Fig. 2). Antenna correlation matrixes are considered at the transmitter and the receiver, and the fractional ratio to a based antenna pair of the correlation matrix are both 0.3, that is, $\alpha = \beta = 0.3$.

The repeated mapping method reported in Ref. [15] is adopted to construct the space-time-frequency codes. Repeated mapping is made to form a space-frequency coding across sub-carriers from a known space-time coding. Then repeated mapping is made to form a space-time-frequency coding across the K OFDM symbol blocks.

For fairness of comparison (for same rate), we consider binary phase shift keying (BPSK) modulated space-frequency coding and quadrature phase shift keying (QPSK) modulated space-time-frequency coding in the two transmitter antennas and one receiver antenna systems. The initial space-time coding was used by Alamouti space-time codes. The performance comparison is shown in Fig. 1. The performance of the space-time-frequency codes is superior to the space-frequency codes because it considers the space, time and frequency diversity as a whole to obtain more time diversity.

The performance of full diversity space-frequency coding and space-time-frequency coding are considered in the four transmitter antennas and one receiver antenna systems when the vehicle speed is 120 km/h. As the system can effectively get the time diversity brought from time-variant channels, the maximum available diversity of space-time-frequency coding is higher than that of space-frequency coding. We can find in Fig. 2 that the performance curve slope of space-time-frequency code is obviously steeper than that of space-frequency code, i.e., the available diversity is greater that that of space-frequency code (see Fig. 2). However, as the time-variant channel, at about 12 dB, the performance of space-time-frequency codes has an error platform.

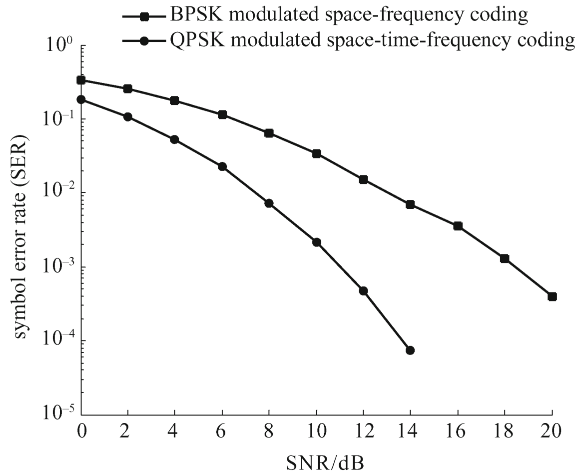

Fig. 1 Performance comparison of SFC and STFC for 2*1 systems

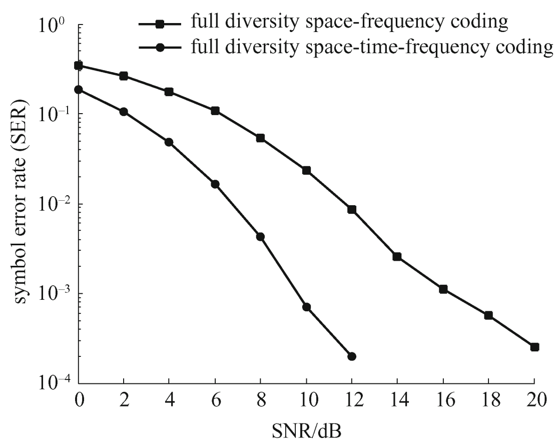

Fig. 2 Full diversity performance comparison of space-frequency coding and space-time-frequency coding

6 Conclusions

This paper proposes a general framework of the diversity analysis of STF coded MIMO-OFDM system when correlated across space, time and frequency. The design criterion of STF code words is revised and the maximum achievable diversity order of STF codes is $L\text{rank}(\boldsymbol{P})\text{rank}(\boldsymbol{Q})\text{rank}(\boldsymbol{R}_\text{T})$. Our analyses incorporate the spatial correlation, temporal correlation and frequency correlation as special cases.

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References

1. Alamouti S M. A simple transmit diversity technique for wireless communication. IEEE Journal on Selected Areas in Communications, 1998, 16(8): 1451–1458
2. Agrawal D, Tarokh V, Naguib A, et al. Space-time coded OFDM for high data-rate wireless communication over wideband channels. In: Proceedings of the 48th IEEE Vehicular Technology Conference, Ottawa, Canada.1998, 3: 2232–2236
3. Gong Y, Letaief K B. Performance evaluation and analysis of space-time coding in unequalized multi-path fading links. IEEE Transactions on communications, 2000, 48(11): 1778–1782
4. Lee K F, Williams D B. A space-frequency transmitter diversity technique for OFDM systems. In: Proceedings of IEEE Global Telecommunications Conference, San Francisco, USA. 2000, 3: 1473–1477
5. Gong Y, Letaief K B. An efficient space frequency coded wideband OFDM system for wireless communications. In: Proceedings of IEEE International Conference on Communications, NY, USA. 2002, 1: 475–479
6. Bölcskei H, Paulraj A J. Space-frequency coded broadband OFDM systems. In: Proceedings of IEEE Wireless Communications and Networking Conference, Chicago, USA. 2000, 1: 1–6
7. Bölcskei H, Paulraj A J. Space-frequency codes for broadband fading channels. In: Proceedings of IEEE International Symposium on Information Theory, Washington DC, USA. 2001, 219
8. Lu B, Wang X. Space-time code design in OFDM systems. In: Proceedings of IEEE Global Telecommunications Conference, San Francisco, USA. 2000, 2: 1000–1004
9. Su W, Safar Z, Olfat M, et al. A full diversity space-frequency code construction method for broadband OFDM systems. In: Proceedings of Conference on Information Sciences and Systems (CISS'03), Baltimore, MD. 2003, 12–14
10. Su W, Safar Z, Olfat M, et al. Obtaining full-diversity space-frequency codes from space time codes via mapping. IEEE Transactions on Signal Processing, 2003, 51(11): 2905–2916
11. Gong Y, Letaief K B. Space-frequency time coded OFDM for broadband wireless communications. In: Proceedings of IEEE Global Telecommunications Conference, San Antonio, USA: 2001, 1: 519–523
12. Lu B, Wang X, Narayanan K R. LDPC based space-time coded OFDM systems over correlated fading channels: performance analysis and receiver design. IEEE Transactions on Communications. 2002, 50(1): 74–88
13. Molisch A F, Win M Z, Winters J H. Space-time-frequency (STF) coding for MIMO OFDM systems. IEEE Communications Letters, 2002, 6(9): 370–372
14. Liu Z, Xin Y, Giannakis G B. Space-time frequency coded OFDM over frequency selective fading channels. IEEE Transactions on Signal Processing, 2002, 50(10): 2465–2476
15. Su W, Safar Z, Liu K J R. Diversity analysis of space-time-frequency coded broadband OFDM systems. In: Proceedings of European Wireless, 2004, 2(2): 1–5
16. Bölcskei H, Borgmann M, Paulraj A J. Performance of space-frequency coded broadband OFDM under real-world propagation conditions. XI European Signal Processing Conference (EUSIPCO), Toulouse, France. 2002, 413–416
17. Abdi A, Kaveh M. A Space-time correlation modeling of multielement antenna system in mobile fading channels. IEEE Journal on Selected Areas in Communications, 2002, 20(3): 550–560
18. Horn R A, Johnson C R. Topics in Matrix Analysis. Cambridge University Press, 1991