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RESEARCH ARTICLE

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Tracking control for first-order multi-agent systems

Abstract In this paper, the conventional tracking control problem is expanded to first-order multi-agent systems, which can be solved by directly guiding any agent in the group. The following three kinds of desired motions are considered for all agents to track: 1) stillness in space, 2) variable motion with known acceleration, 3) variable motion with partly unknown acceleration. Specifically, fixed networks with time delays and switching networks without delays are both considered in case 1). Switching networks with and without time delays are both studied in case 2), while for 3), switching networks without delays are mainly investigated. A numerical simulation example is included to illustrate the results.

Keywords multi-agent systems, tracking control, switching topologies, time delays

1 Introduction

A multi-agent system is composed of multiple agents that are autonomous enough to operate independently, yet can function collectively as a group through communication. Cooperative control of such a system has received considerable attention due to its broad applications in many areas, including cooperative control of unmanned air and underwater vehicles, formation control, flocking of mobile vehicles, distributed optimization of multiple mobile robotic systems, and scheduling of automated highway systems. In cooperative control, it is important to design appropriate distributed protocols using just local information, such that the group of agents can move along the desired trajectory.

Recently, cooperative control of multi-agent systems has been extensively studied [1–9]. For example, the tracking control problem was considered for first-order multi-agent systems without communication delays by using the leader-follower approach [4]. In Ref. [5], Ferrari-Trecate G et al. first introduced the framework of partial difference equations (PdEs) over graphs for analyzing the behavior of multi-agent systems equipped with decentralized control schemes. Based on this framework, average consensus problem was studied for undirected networks of first-order dynamic agents having communication delays [6].

Motivated by Ref. [4], we solve the tracking control problem for multi-agent systems using a new approach similar to the leader-follower method. By guiding any agent in the system, defined as the controlled agent, we can control all agents to move along a desired trajectory, without an additional leader introduced. Though the controlled agent's behavior does not necessarily obey local rules as ordinary agents do, other agents still treat it as an ordinary agent. Therefore, the original communication networks among agents will not change. For the convenience of analysis, the network of agents is modeled in the framework of PdEs over graphs introduced in Ref. [5], and the tracking problems for different desired motions in various networks are considered. Simulations illustrate the effectiveness of the obtained results.

2 Preliminaries

In this section, we will introduce the algebraic graph theory, partial difference equations over graphs and the problem to be considered. Throughout this paper, let $\mathbf{1}$ and $\mathbf{0}$ respectively denote the column vectors of appropriate dimensions, whose elements are all ones and all zeros, $\boldsymbol{I}_r$ denote an $r \times r$ identity matrix, and $\mathbf{0}_r$ denote an $r \times r$ zero matrix. Given a symmetric matrix $\boldsymbol{A}$, let $\boldsymbol{A}>0$ represent that $\boldsymbol{A}$ is positive definite.

2.1 Algebraic graph theory

Undirected graphs are applied to modeling the interaction topologies among agents. Let $G=(N,\varepsilon)$ be a weighted undirected graph of order n with a set of nodes $N=\{1,2,\ldots,n\}$ and a set of unordered pairs of nodes $\varepsilon \subseteq N \times N$. In graph G, node $x \in N$ represents the agent

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A_x, and $x \sim y$ denotes an undirected edge between agents A_x and A_y. Then the set of neighbors of node x is denoted by $N_x = \{y \in N: x \sim y\}$. To describe the variable interconnection topology, we define a switching signal $sw(t)$:[0, ∞)→{1,2,…,m}, where $m \in \mathbb{Z}^+$ denotes the total number of all possible graphs. If $sw(t)$ is constant, the corresponding networks are fixed. Graph G is called connected if there is an undirected path between any pair of distinct nodes x and y, $\forall x,y \in N$.

Weights on the communication links are defined by a function $\omega : N \times N \to \mathbb{R}^+$ with the properties:

$$\omega(x,y) = \omega(y,x),$$

$$\omega(x,y) > 0 \Leftrightarrow (x \sim y) \in G.$$

Let S be a nonempty subgraph of the connected graph G, and s be the set of nodes of S. Then the boundary of S is defined as $\partial S = \{y \in G/S : \exists x \in s, s.t., x \sim y\}$. By properly choosing the subgraph S, we can guarantee $N = s \cup \partial S$.

2.2 Partial difference equations over graphs

Consider vector functions $\boldsymbol{f} : N \to \mathbb{R}^q$ defined over a graph G, where $\boldsymbol{f}(x)$, $x \in N$ may represent the position or the velocity vectors of agent A_x. According to Ref. [5], the partial derivative of $\boldsymbol{f}$ is defined by

$$\partial_y \boldsymbol{f}(x) \doteq \boldsymbol{f}(y) - \boldsymbol{f}(x)$$

with the following elementary properties:

$$\partial_y \boldsymbol{f}(x) = -\partial_x \boldsymbol{f}(y),\ \partial_x \boldsymbol{f}(x) = \boldsymbol{0},\ \partial_y^2 \boldsymbol{f}(x) = -\partial_y \boldsymbol{f}(x).$$

The Laplacian of $\boldsymbol{f}$ is given by

$$\Delta \boldsymbol{f}(x) \doteq -\sum_{y \sim x} \omega(x,y)\partial_y^2 \boldsymbol{f}(x) = \sum_{y \sim x} \omega(x,y)\partial_y \boldsymbol{f}(x)$$

and the integral and the average of $\boldsymbol{f}$ are respectively defined as:

$$\int_G \boldsymbol{f} \doteq \sum_{x \in N} \boldsymbol{f}(x),\ \langle \boldsymbol{f} \rangle \doteq \frac{1}{n}\int_G \boldsymbol{f}.$$

Denote $L^2(G|\mathbb{R}^q)$ as the Hilbert space composed by all functions $\boldsymbol{f} : N \to \mathbb{R}^q$ equipped with the scalar product and the norm:

$$(\boldsymbol{f}, \boldsymbol{g})_{L^2} = \int_G \boldsymbol{f} \cdot \boldsymbol{g},\ \|\boldsymbol{f}\|_{L^2}^2 = \int_G \|\boldsymbol{f}\|^2,$$

where $\cdot$ and $\|\cdot\|$ represent the scalar product and the Euclidean norm on $\mathbb{R}^q$. Then define two subspaces of $L^2(G|\mathbb{R}^q)$ as:

$$H^1(G|\mathbb{R}^q) \doteq \{\boldsymbol{f} \in L^2(G|\mathbb{R}^q) : \langle \boldsymbol{f} \rangle = \boldsymbol{0}\},$$
$$H_0^1(G|\mathbb{R}^q) \doteq \{\boldsymbol{f} \in L^2(G|\mathbb{R}^q) : \boldsymbol{f}(x) = \boldsymbol{0}, \forall x \in \partial S\}.$$

We will use the shorthand notation L^2,H^1 and H_0^1 when there is no ambiguity. By Ref. [10], if the graph G is connected, H^1 and H_0^1 are both Hilbert spaces endowed with scalar product:

$$(\boldsymbol{f}, \boldsymbol{g})_H = \int_G \int_G \omega(x,y)\partial_y \boldsymbol{f}(x) \cdot \partial_y \boldsymbol{g}(x)\mathrm{d}x\mathrm{d}y.$$

Let $H_\perp^1$ denote the space orthogonal to H^1. Apparently, $H_\perp^1$ is the space of constant functions on G with $\dim(H_\perp^1) = q$. Moreover, the decomposition $L^2 = H^1 \oplus H_\perp^1$ is direct, i.e., $\int_G \boldsymbol{f}^\mathrm{T}\boldsymbol{g} = 0$, $\forall \boldsymbol{f} \in H^1$ and $\forall \boldsymbol{g} \in H_\perp^1$.

Lemma 1 [5] Let G be a connected graph. Then,

1) The operator $\Delta: H^1 \to H^1$ has $(n-1)q$ strictly negative eigenvalues and the corresponding eigenvectors form a basis for H^1;

2) For $\boldsymbol{f} \in L^2$, $\Delta \boldsymbol{f} = \boldsymbol{0}$ if and only if $\boldsymbol{f} \in H_\perp^1$;

3) The operator $\Delta : H_0^1 \to H^1$ has $|S|q$ strictly negative eigenvalues, where $|S|$ denotes the number of nodes of S. Moreover, the corresponding eigenvectors form a basis for H_0^1.

2.3 Problem statement and protocol design

In this paper, we consider the agents moving in a q (= 1,2,3) dimensional space. Let $\boldsymbol{r}(x,t)$, $\boldsymbol{u}(x,t)$ and $\boldsymbol{y}(x,t)$ be the position, input and measured output of the agent A_x ($x \in N$), respectively. The dynamical equation of agent A_x is given by

$$\begin{cases} \dot{\boldsymbol{r}}(x,t) = \boldsymbol{u}(x,t), \\ \boldsymbol{y}(x,t) = \boldsymbol{r}(x,t). \end{cases} \tag{1}$$

Let the controlled agent be A_c ($c \in N$), and its underlying dynamics be expressed as follows:

$$\begin{cases} \dot{\boldsymbol{r}}_c(t) = \boldsymbol{v}_c(t), \\ \dot{\boldsymbol{v}}_c(t) = \boldsymbol{a}(t) = \boldsymbol{a}_c(t) + \boldsymbol{\delta}(t), \\ \boldsymbol{y}_c(t) = \boldsymbol{r}_c(t), \end{cases} \tag{2}$$

where $\boldsymbol{r}_c(t)$, $\boldsymbol{v}_c(t)$ and $\boldsymbol{y}_c(t)$ respectively represent its position, velocity and measured output, and $\boldsymbol{a}(t) = \boldsymbol{a}_c(t) + \boldsymbol{\delta}(t)$ is the acceleration. It is assumed that $\boldsymbol{a}_c(t)$ is known and $\boldsymbol{\delta}(t)$ is unknown but bounded with a given upper bound $\overline{\delta}$, that is $\|\boldsymbol{\delta}(t)\| \leqslant \overline{\delta}$. Our objective is to design a decentralized protocol for each agent to track the desired movement as given in Eq. (2). Note that no additional leader is introduced. Therefore, the original communication networks among agents will not change.

Since $\boldsymbol{v}_c(t)$ cannot be measured even when the agents are connected to A_c, we have to estimate $\boldsymbol{v}_c(t)$ using local information during the evolution, and the estimate of $\boldsymbol{v}_c(t)$ by agent A_x is denoted by $\boldsymbol{v}(x,t)$. Design the protocol as

$$\boldsymbol{u}(x,t) = k\Delta \boldsymbol{r}(x,t) + \boldsymbol{v}(x,t),\ k > 0 \tag{3}$$

with $\boldsymbol{v}(x,t)$ estimated by

$$\dot{\boldsymbol{v}}(x,t)=\boldsymbol{a}_c(t)+\alpha k\Delta\boldsymbol{r}(x,t),\ 0<\alpha<1. \tag{4}$$

Substituting Eqs. (3) and (4) into Eq. (1) yields a closed-loop system:

$$\begin{cases}\dot{\boldsymbol{r}}(x,t)=k\Delta\boldsymbol{r}(x,t)+\boldsymbol{v}(x,t),\\ \dot{\boldsymbol{v}}(x,t)=\boldsymbol{a}_c(t)+\alpha k\Delta\boldsymbol{r}(x,t).\end{cases} \tag{5}$$

Let $s=N\backslash c$ for connected graph G, then it can be derived that $\partial S=\{c\}$ and $N=s\cup c=s\cup\partial S$. Introduce the decomposition of $\boldsymbol{r}(x,t)$ and $\boldsymbol{v}(x,t)$ as follows:

$$\begin{aligned}\boldsymbol{r}(x,t)&=\boldsymbol{r}_0(x,t)+\boldsymbol{r}_c(x,t),\quad x\in N,\boldsymbol{r}_0\in H_0^1,\\ \boldsymbol{v}(x,t)&=\boldsymbol{v}_0(x,t)+\boldsymbol{v}_c(x,t),\quad x\in N,\boldsymbol{v}_0\in H_0^1,\end{aligned} \tag{6}$$

where $\boldsymbol{r}_c(x,t)=\boldsymbol{r}_c(t)$ and $\boldsymbol{v}_c(x,t)=\boldsymbol{v}_c(t)$, and $\Delta\boldsymbol{r}_c(x,t)=\boldsymbol{0}$ holds. Denote $\boldsymbol{r}(x,t)$, $\boldsymbol{r}_0(x,t)$, $\boldsymbol{v}(x,t)$, $\boldsymbol{v}_0(x,t)$ as $\boldsymbol{r}$, $\boldsymbol{r}_0$, $\boldsymbol{v}$, $\boldsymbol{v}_0$ for short. Substituting Eq. (6) into Eq. (5) and combining $\Delta\boldsymbol{r}_c(x,t)=\boldsymbol{0}$ result in:

$$\begin{cases}\dot{\boldsymbol{r}}_0+\dot{\boldsymbol{r}}_c=k\Delta\boldsymbol{r}_0+\boldsymbol{v}_0+\boldsymbol{v}_c,\\ \dot{\boldsymbol{v}}_0+\dot{\boldsymbol{v}}_c=\boldsymbol{a}_c(t)+\alpha k\Delta\boldsymbol{r}_0,\end{cases}$$

which can be decomposed into the following two independent subsystems:

$$\Sigma_0:\begin{cases}\dot{\boldsymbol{r}}_0=k\Delta\boldsymbol{r}_0+\boldsymbol{v}_0,\\ \dot{\boldsymbol{v}}_0=\alpha k\Delta\boldsymbol{r}_0-\boldsymbol{\delta}(t),\ x\in s\end{cases} \tag{7}$$

and

$$\Sigma_c:\begin{cases}\dot{\boldsymbol{r}}_c=\boldsymbol{v}_c,\\ \dot{\boldsymbol{v}}_c=\boldsymbol{a}_c(t)+\boldsymbol{\delta}(t)=\boldsymbol{a}(t).\end{cases} \tag{8}$$

Note that Eq. (8) is just the dynamics of agent A_c, therefore, we only need to analyze the subsystem Σ_0.

3 Main results

In this paper, three kinds of desired motions are respectively considered as follows: stillness in space, variable motion with known acceleration, and variable motion with partly unknown acceleration.

3.1 Stillness in space

Assume that the desired motion is stillness at the position $\boldsymbol{r}_c$, that is, $\boldsymbol{v}_c(t)=\boldsymbol{a}(t)=\boldsymbol{0}$ in Eq. (2), then protocol Eq. (3) becomes

$$\boldsymbol{u}(x,t)=k\Delta\boldsymbol{r}(x,t). \tag{9}$$

The corresponding subsystems Eqs. (7) and (8) can be simplified as

$$\Sigma_0^1:\dot{\boldsymbol{r}}_0=k\Delta\boldsymbol{r}_0,\ x\in s \tag{10}$$

and

$$\Sigma_c^1:\boldsymbol{r}_c(t)=\boldsymbol{r}_c. \tag{11}$$

3.1.1 Networks with fixed topology and time delays

In practice, time-delay effects may arise naturally. Here, we consider networks with a fixed topology and communication time-varying delay $\tau(t)$ verifying $0\leqslant\tau(t)\leqslant\bar{\tau}$, then the protocol is:

$$\boldsymbol{u}(x,t)=k\Delta\boldsymbol{r}(x,t-\tau), \tag{12}$$

which leads to a closed-loop system:

$$\dot{\boldsymbol{r}}(x,t)=k\Delta\boldsymbol{r}(x,t-\tau). \tag{13}$$

Theorem 1 Consider fixed networks of agents with time delay $\tau(t)$, and assume that the interaction graph G is connected. Then under the protocol Eq. (12), all agents asymptotically arrive at $\boldsymbol{r}_c$ if $\bar{\tau}<\pi/(2k|\lambda_{\min}|)$, where $\lambda_{\min}$ is the smallest eigenvalue of Δ.

Proof By the above analysis, we only need to demonstrate that the zero solution to the following system $\Sigma_0^{1\tau}$ is asymptotically stable:

$$\Sigma_0^{1\tau}:\dot{\boldsymbol{r}}_0(x,t)=k\Delta\boldsymbol{r}_0(x,t-\tau),\quad x\in s. \tag{14}$$

By Lemma 1, there exists $\{\boldsymbol{\psi}_i\}_{i=1}^{(n-1)q}$ that is an orthonormal set of eigenvectors of $\Delta: H_0^1\to H^1$ forming a basis for H_0^1 and is associated with negative eigenvalues $\{\lambda_i\}_{i=1}^{(n-1)q}$. Then, $\boldsymbol{r}_0(x,t)=\sum_{i=1}^{(n-1)q}\beta_i(t)\boldsymbol{\psi}_i(x)$ holds for a suitable function $\beta_i(t):\mathbb{R}^+\to\mathbb{R}$. By substituting it into Eq. (14) and testing each side of the obtained equation against $\boldsymbol{\psi}_m$, we get $\dot{\beta}_m(t)=k\lambda_m\beta_m(t-\tau)$, which is exponentially stable if and only if $\tau<\pi/(2k|\lambda_m|)$ according to Ref. [11]. Then $\lim\limits_{t\to\infty}\beta_m(t)=0$ holds for all $m=1,2,\ldots,(n-1)q$ if $\bar{\tau}<\pi/(2k|\lambda_{\min}|)$, from which it is derived that $\lim\limits_{t\to\infty}\boldsymbol{r}_0(x,t)=\boldsymbol{0}$ for $\forall x\in s$. Accordingly, we have $\lim\limits_{t\to\infty}\boldsymbol{r}(x,t)=\lim\limits_{t\to\infty}(\boldsymbol{r}_0(x,t)+\boldsymbol{r}_c)=\boldsymbol{r}_c$ for $\forall x\in s$, and this completes the proof.

Regarding networks without delays as a special case of those with communication delays by setting $\bar{\tau}=0$, we can directly obtain the following corollary:

Corollary 1 Consider fixed networks of agents with no delays, and assume that the interaction graph G is connected. Then under the protocol Eq. (9), all agents asymptotically arrive at $\boldsymbol{r}_c$.

3.1.2 Networks with switching topologies and no delays

Denote Δ_{sw} as the Laplacian operator associated with graph G_{sw}, where $sw(t)$ is the switching signal.

Theorem 2 Assume that the graphs G_{sw} are always connected. Then under the protocol $\boldsymbol{u}(x,t)=k\Delta_{sw}\boldsymbol{r}(x,t)$, all agents asymptotically arrive at $\boldsymbol{r}_c$.

Proof For networks with switching topologies, rewrite the subsystem Eq. (10) as $\Sigma_0^1: \dot{\boldsymbol{r}}_0 = k\Delta_{sw}\boldsymbol{r}_0$, $x \in s$. Consider the Lyapunov function $V = (1/2)\boldsymbol{r}_0{}^{\mathrm{T}}\boldsymbol{r}_0$. Then its derivative is $\dot{V} = \int_G \boldsymbol{r}_0^{\mathrm{T}} k\Delta_{sw}\boldsymbol{r}_0 \leqslant k\bar{\lambda}\int_G \|\boldsymbol{r}_0\|^2$, where $\bar{\lambda}$ is the largest eigenvalue of the operators Δ_{sw} associated with all possible graphs. By incorporating the condition that graphs G_{sw} are always connected into Lemma 1, we know $\bar{\lambda} < 0$, which implies that $\dot{V}$ is negative definite. Thus, $\lim\limits_{t\to\infty} \boldsymbol{r}_0(x,t) = \mathbf{0}$ holds for $\forall x \in s$, resulting that $\lim\limits_{t\to\infty} \boldsymbol{r}(x,t) = \lim\limits_{t\to\infty}(\boldsymbol{r}_0(x,t) + \boldsymbol{r}_c) = \boldsymbol{r}_c$. This completes the proof.

3.2 Variable motion with known acceleration

Assume that the acceleration input of the controlled agent is known, then the corresponding dynamics of A_c is

$$\begin{cases} \dot{\boldsymbol{r}}_c(t) = \boldsymbol{v}_c(t), \\ \dot{\boldsymbol{v}}_c(t) = \boldsymbol{a}_c(t), \\ \boldsymbol{y}_c(t) = \boldsymbol{r}_c(t). \end{cases} \tag{15}$$

Consider networks of agents with switching topologies, and the subsystems Eqs. (7) and (8) become

$$\Sigma_0: \begin{cases} \dot{\boldsymbol{r}}_0 = k\Delta_{sw}\boldsymbol{r}_0 + \boldsymbol{v}_0, \\ \dot{\boldsymbol{v}}_0 = \alpha k\Delta_{sw}\boldsymbol{r}_0, \quad x \in s \end{cases} \tag{16}$$

and

$$\Sigma_c: \begin{cases} \dot{\boldsymbol{r}}_c = \boldsymbol{v}_c, \\ \dot{\boldsymbol{v}}_c = \boldsymbol{a}_c(t). \end{cases} \tag{17}$$

3.2.1 Networks with switching topologies and no delays

Lemma 2 (Schur complements [12]) For a given symmetric matrix $\boldsymbol{S}$ with the form $\boldsymbol{S} = [\boldsymbol{S}_{ij}]$, $\boldsymbol{S}_{11} \in \mathbb{R}^{r\times r}$, $\boldsymbol{S}_{12} \in \mathbb{R}^{r\times(n-r)}$, $\boldsymbol{S}_{22} \in \mathbb{R}^{(n-r)\times(n-r)}$, then $\boldsymbol{S} < 0$ if and only if $\boldsymbol{S}_{11} < 0$, $\boldsymbol{S}_{22} - \boldsymbol{S}_{21}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12} < 0$ or $\boldsymbol{S}_{22} < 0$, $\boldsymbol{S}_{11} - \boldsymbol{S}_{12}\boldsymbol{S}_{22}^{-1}\boldsymbol{S}_{21} < 0$.

Theorem 3 Consider networks with switching topologies. Then all agents in the system asymptotically track the desired motion, if the interaction graphs G_{sw} are always connected, and parameters k and α in protocol Eqs. (3) and (4) satisfy $k > 1/\left[4\alpha|\bar{\lambda}|\left(1-\alpha^2\right)\right] > 0$, where $\bar{\lambda}$ (and afterwards) is the largest eigenvalue of the operators Δ_{sw} associated with all possible graphs.

Proof Denote

$$\bar{\boldsymbol{r}}_0(t) = \left[\boldsymbol{r}_0^{\mathrm{T}}(1,t) \quad \cdots \quad \boldsymbol{r}_0^{\mathrm{T}}(c-1,t) \quad \boldsymbol{r}_0^{\mathrm{T}}(c+1,t) \quad \cdots \quad \boldsymbol{r}_0^{\mathrm{T}}(n,t)\right]^{\mathrm{T}}$$

and

$$\bar{\boldsymbol{v}}_0(t) = \left[\boldsymbol{v}_0^{\mathrm{T}}(1,t) \quad \cdots \quad \boldsymbol{v}_0^{\mathrm{T}}(c-1,t) \quad \boldsymbol{v}_0^{\mathrm{T}}(c+1,t) \quad \cdots \quad \boldsymbol{v}_0^{\mathrm{T}}(n,t)\right]^{\mathrm{T}}.$$

From $\boldsymbol{r}_0 \in H_0^1$, it is derived that

$$\begin{aligned} \Delta_{sw}\boldsymbol{r}_0(x,t) &= \sum_{y\sim x, y\neq c} \omega(x,y)[\boldsymbol{r}_0(y,t) - \boldsymbol{r}_0(x,t)] \\ &\quad + \omega(x,c)[-\boldsymbol{r}_0(x,t)] \\ &\triangleq \boldsymbol{L}_x(sw)\otimes\boldsymbol{I}_q\bar{\boldsymbol{r}}_0, \end{aligned}$$

which can be rewritten in the matrix form

$$\Delta_{sw}\bar{\boldsymbol{r}}_0 = \boldsymbol{L}^*(sw)\otimes\boldsymbol{I}_q\bar{\boldsymbol{r}}_0,$$

where

$$\boldsymbol{L}^*(sw) = [\boldsymbol{L}_1{}^{\mathrm{T}}(sw) \ \cdots \ \boldsymbol{L}_{c-1}{}^{\mathrm{T}}(sw) \ \boldsymbol{L}_{c+1}{}^{\mathrm{T}}(sw) \ \cdots \ \boldsymbol{L}_n{}^{\mathrm{T}}(sw)]^{\mathrm{T}}$$

is the symmetry matrix associated with the operator Δ_{sw}, and $\bar{\lambda}$ is the largest eigenvalue of $\boldsymbol{L}^*(sw)$.

By Lemma 1, the eigenvalues of $\Delta: H_0^1 \to H^1$ are all negative, therefore, the eigenvalues of matrix $\boldsymbol{L}^*(sw)$ are all negative, i.e., $\boldsymbol{L}^*(sw)$ is a negative definite matrix. Rewrite Eq. (16) in the matrix form:

$$\begin{cases} \dot{\bar{\boldsymbol{r}}}_0 = k\left(\boldsymbol{L}^*(sw)\otimes\boldsymbol{I}_q\right)\bar{\boldsymbol{r}}_0 + \bar{\boldsymbol{v}}_0, \\ \dot{\bar{\boldsymbol{v}}}_0 = \alpha k\left(\boldsymbol{L}^*(sw)\otimes\boldsymbol{I}_q\right)\bar{\boldsymbol{r}}_0. \end{cases}$$

Let $\boldsymbol{\eta}(t) = \left[\bar{\boldsymbol{r}}_0^{\mathrm{T}}(t) \quad \bar{\boldsymbol{v}}_0^{\mathrm{T}}(t)\right]^{\mathrm{T}} \in \mathbb{R}^{2(n-1)q}$, then we have:

$$\dot{\boldsymbol{\eta}} = \begin{bmatrix} k\boldsymbol{L}^*(sw)\otimes\boldsymbol{I}_q & \boldsymbol{I}_{n-1}\otimes\boldsymbol{I}_q \\ \alpha k\boldsymbol{L}^*(sw)\otimes\boldsymbol{I}_q & \mathbf{0}_{n-1}\otimes\boldsymbol{I}_q \end{bmatrix}\boldsymbol{\eta} \triangleq \boldsymbol{F}(sw)\otimes\boldsymbol{I}_q\boldsymbol{\eta},$$

where

$$\boldsymbol{F}(sw) = \begin{bmatrix} k\boldsymbol{L}^*(sw) & \boldsymbol{I}_{n-1} \\ \alpha k\boldsymbol{L}^*(sw) & \mathbf{0}_{n-1} \end{bmatrix}.$$

Take the Lyapunov function as $V(\boldsymbol{\eta}) = \boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{P}\otimes\boldsymbol{I}_q\boldsymbol{\eta}$, where the symmetry matrix

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{I}_{n-1} & -\alpha\boldsymbol{I}_{n-1} \\ -\alpha\boldsymbol{I}_{n-1} & \boldsymbol{I}_{n-1} \end{bmatrix} \tag{18}$$

is positive definite under the condition $0 < \alpha < 1$. Then, computing the derivative of $V(\boldsymbol{\eta})$ yields:

$$\begin{aligned} \dot{V}(\boldsymbol{\eta}) &= \boldsymbol{\eta}^{\mathrm{T}}\left[\boldsymbol{F}(sw)^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{F}(sw)\right]\otimes\boldsymbol{I}_q\boldsymbol{\eta} \\ &\triangleq \boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{Q}(sw)\otimes\boldsymbol{I}_q\boldsymbol{\eta}, \end{aligned}$$

where

$$\begin{aligned} \boldsymbol{Q}(sw) &= \boldsymbol{F}(sw)^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{F}(sw) \\ &= -\begin{bmatrix} 2(\alpha^2-1)k\boldsymbol{L}^*(sw) & -\boldsymbol{I}_{n-1} \\ -\boldsymbol{I}_{n-1} & 2\alpha\boldsymbol{I}_{n-1} \end{bmatrix}. \end{aligned} \tag{19}$$

By Schur complements, matrix $\boldsymbol{Q}(sw)$ is negative definite if $k > 1/\left[-4\alpha\bar{\lambda}\left(1-\alpha^2\right)\right] > 0$. Consequently, $\boldsymbol{\eta}(t) \to \mathbf{0}$ as $t \to \infty$, i.e., $\boldsymbol{r}_0 \to \mathbf{0}$ and $\boldsymbol{v}_0 \to \mathbf{0}$, which leads to $\lim\limits_{t\to\infty}\boldsymbol{r}(x,t) = \lim\limits_{t\to\infty}(\boldsymbol{r}_0(x,t) + \boldsymbol{r}_c) = \boldsymbol{r}_c$ for all $x \in s$. This completes the proof.

3.2.2 Networks with switching topologies and time delays

Due to coupling communication delays in switching networks, each agent cannot instantly obtain information from others. Thus protocol Eqs. (3) and (4) can be expressed as follows:

$$\boldsymbol{u}(x,t)=k\Delta_{sw}\boldsymbol{r}(x,t-\tau)+\boldsymbol{v}(x,t),\quad k>0, \tag{20}$$

$$\dot{\boldsymbol{v}}(x,t)=\boldsymbol{a}_c(t)+\alpha k\Delta_{sw}\boldsymbol{r}(x,t-\tau),\quad 0<\alpha<1, \tag{21}$$

where $\tau>0$ is a constant delay, and the corresponding subsystems are

$$\Sigma_0:\begin{cases}\dot{\boldsymbol{r}}_0(x,t)=k\Delta_{sw}\boldsymbol{r}_0(x,t-\tau)+\boldsymbol{v}_0(x,t),\\ \dot{\boldsymbol{v}}_0(x,t)=\alpha k\Delta_{sw}\boldsymbol{r}_0(x,t-\tau),\quad x\in s\end{cases} \tag{22}$$

and Eq. (17). By Theorem 3, system Eq. (22) can be rewritten in the matrix form:

$$\begin{aligned}\dot{\boldsymbol{\eta}}(t)&=\begin{bmatrix}k\boldsymbol{L}^*(sw) & \boldsymbol{0}_{n-1}\\ \alpha k\boldsymbol{L}^*(sw) & \boldsymbol{0}_{n-1}\end{bmatrix}\otimes\boldsymbol{I}_q\boldsymbol{\eta}(t-\tau)\\ &\quad+\begin{bmatrix}\boldsymbol{0}_{n-1} & \boldsymbol{I}_{n-1}\\ \boldsymbol{0}_{n-1} & \boldsymbol{0}_{n-1}\end{bmatrix}\otimes\boldsymbol{I}_q\boldsymbol{\eta}(t)\\ &\triangleq\boldsymbol{C}(sw)\otimes\boldsymbol{I}_q\boldsymbol{\eta}(t-\tau)+\boldsymbol{E}\otimes\boldsymbol{I}_q\boldsymbol{\eta}(t),\end{aligned} \tag{23}$$

where $\boldsymbol{\eta}(t)$ has been defined in Theorem 3. Note that $\boldsymbol{C}(sw)+\boldsymbol{E}=\boldsymbol{F}(sw)$ holds.

Theorem 4 Consider networks with switching topologies and constant time delay $\tau>0$, and assume that the interaction graphs G_{sw} are always connected. Protocol Eqs. (20) and (21) asymptotically solve the tracking problem, if there exist positive definite matrices $\boldsymbol{T},\boldsymbol{R}\in\mathbb{R}^{(n-1)\times(n-1)}$ satisfying:

$$\begin{bmatrix}\boldsymbol{TF}(sw)+\boldsymbol{F}^{\mathrm{T}}(sw)\boldsymbol{T}+2\tau\boldsymbol{R} & \boldsymbol{TC}^2(sw) & \boldsymbol{TC}(sw)\boldsymbol{E}\\ (\boldsymbol{C}^{\mathrm{T}}(sw))^2\boldsymbol{T} & -\boldsymbol{R}/\tau & \boldsymbol{0}_{n-1}\\ \boldsymbol{E}^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}}(sw)\boldsymbol{T} & \boldsymbol{0}_{n-1} & -\boldsymbol{R}/\tau\end{bmatrix}<0. \tag{24}$$

Proof From Eq. (23), we have:

$$\begin{aligned}\boldsymbol{\eta}(t-\tau)&=\boldsymbol{\eta}(t)-\int_{-\tau}^{0}\dot{\boldsymbol{\eta}}(t+s)\mathrm{d}s\\ &=\boldsymbol{\eta}(t)-\int_{-2\tau}^{-\tau}\big(\boldsymbol{C}(sw)\otimes\boldsymbol{I}_q\big)\boldsymbol{\eta}(t+s)\mathrm{d}s\\ &\quad-\int_{-\tau}^{0}\big(\boldsymbol{E}\otimes\boldsymbol{I}_q\big)\boldsymbol{\eta}(t+s)\mathrm{d}s.\end{aligned}$$

Thus, the delayed differential Eq. (23) can be rewritten as

$$\begin{aligned}\dot{\boldsymbol{\eta}}(t)&=\big(\boldsymbol{F}(sw)\otimes\boldsymbol{I}_q\big)\boldsymbol{\eta}(t)\\ &\quad-\int_{-2\tau}^{-\tau}\big(\boldsymbol{C}^2(sw)\otimes\boldsymbol{I}_q\big)\boldsymbol{\eta}(t+s)\mathrm{d}s\\ &\quad-\int_{-\tau}^{0}\big(\boldsymbol{C}(sw)\boldsymbol{E}\otimes\boldsymbol{I}_q\big)\boldsymbol{\eta}(t+s)\mathrm{d}s,\end{aligned} \tag{25}$$

where $\boldsymbol{F}(sw)=\boldsymbol{C}(sw)+\boldsymbol{E}$.

Define a common Lyapunov-Krasovskii function for system Eq. (25)

$$\begin{aligned}V&=\boldsymbol{\eta}^{\mathrm{T}}(t)\big(\boldsymbol{T}\otimes\boldsymbol{I}_q\big)\boldsymbol{\eta}(t)\\ &\quad+\int_{-\tau}^{0}\int_{t+\theta}^{t}\boldsymbol{\eta}^{\mathrm{T}}(s)\big(\boldsymbol{R}\otimes\boldsymbol{I}_q\big)\boldsymbol{\eta}(s)\mathrm{d}s\mathrm{d}\theta\\ &\quad+\int_{-2\tau}^{-\tau}\int_{t+\theta}^{t}\boldsymbol{\eta}^{\mathrm{T}}(s)\big(\boldsymbol{R}\otimes\boldsymbol{I}_q\big)\boldsymbol{\eta}(s)\mathrm{d}s\mathrm{d}\theta,\end{aligned}$$

where $\boldsymbol{T},\boldsymbol{R}\in\mathbb{R}^{(n-1)\times(n-1)}$ are positive definite matrices. Then computing the derivative of V yields:

$$\begin{aligned}\dot{V}&=\boldsymbol{\eta}^{\mathrm{T}}(t)\big(\boldsymbol{TF}(sw)+\boldsymbol{F}^{\mathrm{T}}(sw)\boldsymbol{T}\big)\otimes\boldsymbol{I}_q\boldsymbol{\eta}(t)\\ &\quad-2\boldsymbol{\eta}^{\mathrm{T}}(t)\big(\boldsymbol{TC}^2(sw)\otimes\boldsymbol{I}_q\big)\int_{-2\tau}^{-\tau}\boldsymbol{\eta}(t+s)\mathrm{d}s\\ &\quad-2\boldsymbol{\eta}^{\mathrm{T}}(t)\big(\boldsymbol{TC}(sw)\boldsymbol{E}\otimes\boldsymbol{I}_q\big)\int_{-\tau}^{0}\boldsymbol{\eta}(t+s)\mathrm{d}s\\ &\quad+\tau\boldsymbol{\eta}^{\mathrm{T}}(t)\big(\boldsymbol{R}\otimes\boldsymbol{I}_q\big)\boldsymbol{\eta}(t)-\int_{-\tau}^{0}\boldsymbol{\eta}^{\mathrm{T}}(t+\theta)\big(\boldsymbol{R}\otimes\boldsymbol{I}_q\big)\boldsymbol{\eta}(t+\theta)\mathrm{d}\theta\\ &\quad+\tau\boldsymbol{\eta}^{\mathrm{T}}(t)\big(\boldsymbol{R}\otimes\boldsymbol{I}_q\big)\boldsymbol{\eta}(t)-\int_{-2\tau}^{-\tau}\boldsymbol{\eta}^{\mathrm{T}}(t+\theta)\big(\boldsymbol{R}\otimes\boldsymbol{I}_q\big)\boldsymbol{\eta}(t+\theta)\mathrm{d}\theta\\ &\leqslant\boldsymbol{\eta}^{\mathrm{T}}(t)\big(\boldsymbol{TF}(sw)+\boldsymbol{F}^{\mathrm{T}}(sw)\boldsymbol{T}\big)\otimes\boldsymbol{I}_q\boldsymbol{\eta}(t)\\ &\quad+\tau\boldsymbol{\eta}^{\mathrm{T}}(t)\big(\boldsymbol{TC}^2(sw)\boldsymbol{R}^{-1}\boldsymbol{C}^{\mathrm{T}2}(sw)\boldsymbol{T}\big)\otimes\boldsymbol{I}_q\boldsymbol{\eta}(t)\\ &\quad+\int_{-2\tau}^{-\tau}\boldsymbol{\eta}^{\mathrm{T}}(t+s)\big(\boldsymbol{R}\otimes\boldsymbol{I}_q\big)\boldsymbol{\eta}(t+s)\mathrm{d}s\\ &\quad+\tau\boldsymbol{\eta}^{\mathrm{T}}(t)\big(\boldsymbol{TC}(sw)\boldsymbol{E}\boldsymbol{R}^{-1}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}}(sw)\boldsymbol{T}\big)\otimes\boldsymbol{I}_q\boldsymbol{\eta}(t)\\ &\quad+\int_{-\tau}^{0}\boldsymbol{\eta}^{\mathrm{T}}(t+s)\big(\boldsymbol{R}\otimes\boldsymbol{I}_q\big)\boldsymbol{\eta}(t+s)\mathrm{d}s\\ &\quad+2\tau\boldsymbol{\eta}^{\mathrm{T}}(t)\big(\boldsymbol{R}\otimes\boldsymbol{I}_q\big)\boldsymbol{\eta}(t)-\int_{-\tau}^{0}\boldsymbol{\eta}^{\mathrm{T}}(t+\theta)\big(\boldsymbol{R}\otimes\boldsymbol{I}_q\big)\boldsymbol{\eta}(t+\theta)\mathrm{d}\theta\\ &\quad-\int_{-2\tau}^{-\tau}\boldsymbol{\eta}^{\mathrm{T}}(t+\theta)\big(\boldsymbol{R}\otimes\boldsymbol{I}_q\big)\boldsymbol{\eta}(t+\theta)\mathrm{d}\theta\\ &=\boldsymbol{\eta}^{\mathrm{T}}(t)\big(\boldsymbol{TF}(sw)+\boldsymbol{F}^{\mathrm{T}}(sw)\boldsymbol{T}+\tau\boldsymbol{TC}^2(sw)\boldsymbol{R}^{-1}\boldsymbol{C}^{\mathrm{T}2}(sw)\boldsymbol{T}\\ &\quad+\tau\boldsymbol{TC}(sw)\boldsymbol{E}\boldsymbol{R}^{-1}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}}(sw)\boldsymbol{T}+2\tau\boldsymbol{R}\big)\otimes\boldsymbol{I}_q\boldsymbol{\eta}(t),\end{aligned} \tag{26}$$

where in the second step, we have used the facts that:

$$-2\boldsymbol{\eta}^{\mathrm{T}}(t)\big(\boldsymbol{T}\boldsymbol{C}^2(sw)\otimes\boldsymbol{I}_q\big)\int_{-2\tau}^{-\tau}\boldsymbol{\eta}(t+s)\mathrm{d}s$$
$$\leqslant\tau\boldsymbol{\eta}^{\mathrm{T}}(t)\big(\boldsymbol{T}\boldsymbol{C}^2(sw)\boldsymbol{R}^{-1}\boldsymbol{C}^{\mathrm{T}2}(sw)\boldsymbol{T}\big)\otimes\boldsymbol{I}_q\boldsymbol{\eta}(t)$$
$$+\int_{-2\tau}^{-\tau}\boldsymbol{\eta}^{\mathrm{T}}(t+s)\big(\boldsymbol{R}\otimes\boldsymbol{I}_q\big)\boldsymbol{\eta}(t+s)\mathrm{d}s,$$
$$-2\boldsymbol{\eta}^{\mathrm{T}}(t)\big(\boldsymbol{T}\boldsymbol{C}(sw)\boldsymbol{E}\otimes\boldsymbol{I}_q\big)\int_{-\tau}^{0}\boldsymbol{\eta}(t+s)\mathrm{d}s$$
$$\leqslant\tau\boldsymbol{\eta}^{\mathrm{T}}(t)\big(\boldsymbol{T}\boldsymbol{C}(sw)\boldsymbol{E}\boldsymbol{R}^{-1}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}}(sw)\boldsymbol{T}\big)\otimes\boldsymbol{I}_q\boldsymbol{\eta}(t)$$
$$+\int_{-\tau}^{0}\boldsymbol{\eta}^{\mathrm{T}}(t+s)\big(\boldsymbol{R}\otimes\boldsymbol{I}_q\big)\boldsymbol{\eta}(t+s)\mathrm{d}s.$$

Then, by Lemma 2, $\dot{V}<0$ is equivalent to

$$\begin{bmatrix} \boldsymbol{T}\boldsymbol{F}(sw)+\boldsymbol{F}^{\mathrm{T}}(sw)\boldsymbol{T}+2\tau\boldsymbol{R} & \boldsymbol{T}\boldsymbol{C}^2(sw) & \boldsymbol{T}\boldsymbol{C}(sw)\boldsymbol{E} \\ \big(\boldsymbol{C}^{\mathrm{T}}(sw)\big)^2\boldsymbol{T} & -\boldsymbol{R}/\tau & \boldsymbol{0}_{n-1} \\ \boldsymbol{E}^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}}(sw)\boldsymbol{T} & \boldsymbol{0}_{n-1} & -\boldsymbol{R}/\tau \end{bmatrix}<0.$$

This completes the proof.

Remark 1 If parameters α and k in protocol Eqs. (20) and (21) have been selected, then for given τ, the matrix Inequality (24) is a linear matrix inequality (LMI), which can be easily solved by using available numerical software. Since matrix $\boldsymbol{C}(sw)$ is time-varying, the matrix Inequality (24) should be satisfied for all possible graphs.

3.3 Variable motion with partly unknown acceleration

Theorem 5 Consider networks with switching topologies. If the interaction graphs G_{sw} are always connected, and parameters k and α in protocol Eqs. (3) and (4) satisfy $k>1/\big[4\alpha|\bar{\lambda}|\big(1-\alpha^2\big)\big]>0$, then there exists $T\geqslant 0$ such that $\|\boldsymbol{r}_0(x,t)\|\leqslant C\bar{\delta}$ whenever $t\geqslant t_0+T$, which implies that $\lim_{t\to\infty}\|\boldsymbol{r}(x,t)-\boldsymbol{r}_c(t)\|\leqslant C\bar{\delta}$, where the constant $C>0$ relies on the topologies of networks.

Proof Take the Lyapunov function $V(\boldsymbol{\eta})=\boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{P}\otimes\boldsymbol{I}_q\boldsymbol{\eta}$, where matrix $\boldsymbol{P}$ has been defined in Eq. (18) and its largest and smallest eigenvalues are $1+\alpha$ and $1-\alpha$ respectively, resulting in $(1-\alpha)\|\boldsymbol{\eta}\|^2\leqslant V(\boldsymbol{\eta})\leqslant(1+\alpha)\|\boldsymbol{\eta}\|^2$. Calculating the derivative of $V(\boldsymbol{\eta})$ yields

$$\dot{V}(\boldsymbol{\eta})=\boldsymbol{\eta}^{\mathrm{T}}\big[\boldsymbol{F}(sw)^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}\boldsymbol{F}(sw)\big]\otimes\boldsymbol{I}_q\boldsymbol{\eta}$$
$$+2\boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{P}\otimes\boldsymbol{I}_q\begin{bmatrix}0\\-1\otimes\boldsymbol{\delta}(t)\end{bmatrix}$$
$$\leqslant\bar{\lambda}_Q\|\boldsymbol{\eta}\|^2+2(1+\alpha)\sqrt{n-1}\bar{\delta}\|\boldsymbol{\eta}\|$$
$$=(1-\theta)\bar{\lambda}_{\boldsymbol{Q}}\|\boldsymbol{\eta}\|^2+\theta\bar{\lambda}_{\boldsymbol{Q}}\|\boldsymbol{\eta}\|^2$$
$$+2(1+\alpha)\sqrt{n-1}\bar{\delta}\|\boldsymbol{\eta}\|,\quad 0<\theta<1$$
$$\leqslant(1-\theta)\bar{\lambda}_{\boldsymbol{Q}}\|\boldsymbol{\eta}\|^2,\quad\forall\|\boldsymbol{\eta}\|\geqslant2(1+\alpha)\sqrt{n-1}\bar{\delta}/(\theta|\bar{\lambda}_{\boldsymbol{Q}}|)\triangleq\mu,$$

where $\bar{\lambda}_{\boldsymbol{Q}}$ is the largest eigenvalue of all possible matrices $\boldsymbol{Q}(sw)$ defined in Eq. (19). By Schur complements, $\bar{\lambda}_{\boldsymbol{Q}}<0$ can be derived from the condition $k>1/\big[4\alpha|\bar{\lambda}|\big(1-\alpha^2\big)\big]>0$.

According to the conclusion of Exercise 4.51 in Ref. [13], we have:

$$\|\boldsymbol{\eta}(t)\|\leqslant k\mu=\left[\frac{1+\alpha}{1-\alpha}\right]^{1/2}\left[\frac{2(1+\alpha)\sqrt{n-1}\bar{\delta}}{\theta|\bar{\lambda}_{\boldsymbol{Q}}|}\right],\quad\text{whenever } t\geqslant t_0+T. \tag{27}$$

To proceed, the upper bound of $\bar{\lambda}_{\boldsymbol{Q}}$ will be estimated. Calculate the eigenpolynomial of matrix $\boldsymbol{Q}(sw)$ as

$$\upsilon^2+\big[2\alpha-2k(1-\alpha^2)\lambda_i(\boldsymbol{L}^*)\big]\upsilon-1-4\alpha k\big(1-\alpha^2\big)\lambda_i(\boldsymbol{L}^*)=0,$$

where $\lambda_i(\boldsymbol{L}^*)$ is the ith eigenvalue of matrix $\boldsymbol{L}^*(sw)$. Then we get the eigenvalues of matrix $\boldsymbol{Q}(sw)$:

$$\upsilon=k\big(1-\alpha^2\big)\lambda_i(\boldsymbol{L}^*)-\alpha\pm\sqrt{[\alpha+k(1-\alpha^2)\lambda_i(\boldsymbol{L}^*)]^2+1},$$

which leads to

$$\bar{\lambda}_{\boldsymbol{Q}}\leqslant k\big(1-\alpha^2\big)\bar{\lambda}-\alpha+\sqrt{\big[\alpha+k(1-\alpha^2)\bar{\lambda}\big]^2+1}\triangleq\bar{\omega}.$$

Deduced from the condition in the theorem, $-4\alpha k\big(1-\alpha^2\big)\bar{\lambda}>1$, then $\big[\alpha+k\big(1-\alpha^2\big)\bar{\lambda}\big]^2+1<[\alpha-k\big(1-\alpha^2\big)\bar{\lambda}]^2$ holds, which implies $\bar{\omega}<0$. Substituting $\bar{\lambda}_{\boldsymbol{Q}}\leqslant\bar{\omega}$ and $\bar{\omega}<0$ into Eq. (27), we derive that:

$$\|\boldsymbol{\eta}(t)\|\leqslant\frac{2(1+\alpha)^{3/2}\sqrt{n-1}}{\theta(-\bar{\omega})(1-\alpha)^{1/2}}\cdot\bar{\delta}\triangleq C\bar{\delta},\quad\text{whenever } t\geqslant t_0+T.$$

Therefore, for $\forall x\in s$, $\|\boldsymbol{r}(x,t)-\boldsymbol{r}_c(t)\|=\|\boldsymbol{r}_0(x,t)\|\leqslant\|\bar{\boldsymbol{r}}_0\|\leqslant\|\boldsymbol{\eta}\|\leqslant C\bar{\delta}$ holds whenever $t\geqslant t_0+T$, from which it follows that $\lim_{t\to\infty}\|\boldsymbol{r}(x,t)-\boldsymbol{r}_c(t)\|\leqslant C\bar{\delta}$. This completes the proof.

4 Simulation results

In this section, we simulate the tracking performance of four agents under dynamically changing interaction topologies using protocol Eqs. (3) and (4). For simplicity, the possible interaction connected graphs are constrained to be within the set $G_s=\{G_1,G_2,G_3\}$ as shown in Fig. 1, with weight functions $\omega\equiv1$. Assume that the interaction graph switches randomly in G_s at each random time $t=t_k$, $k=0,1,2,\ldots$.

Let A_1 be the controlled agent and q=1. If $a(t)=a_1(t)=0.02$, Fig. 2 shows that the other three agents can track A_1 in a few seconds, where the behavior of A_1 is presented by dash-dot line. When $a_1(t)=0.02$ and $\boldsymbol{\delta}(t)=\sin t$, the simulation result is in Fig. 3, from which it can

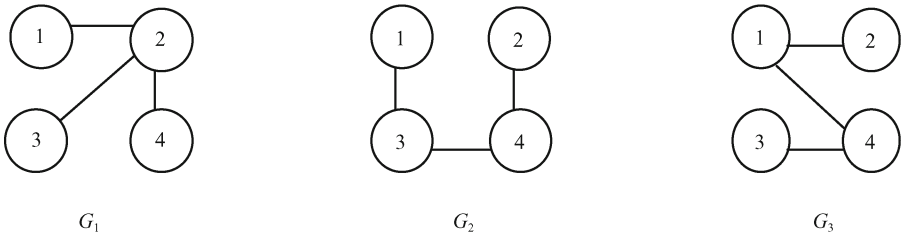

Fig. 1 Set of possible graphs G_s

be seen that the other three agents move around A_1. This means that all other agents can follow the desired motion within some range of tracking errors. The above simulations illustrate the correctness of the obtained results.

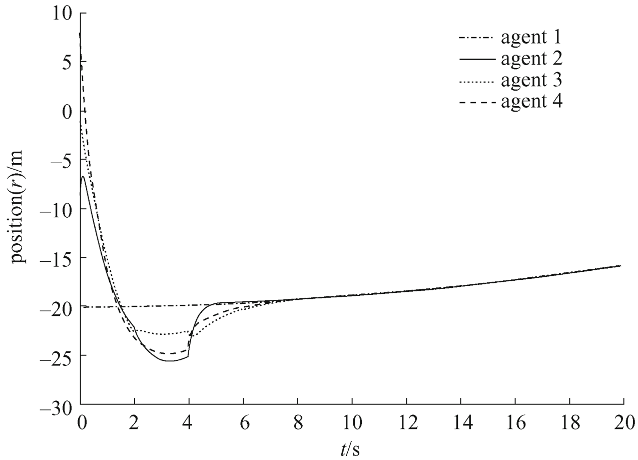

Fig. 2 A_1 tracked by the other three agents

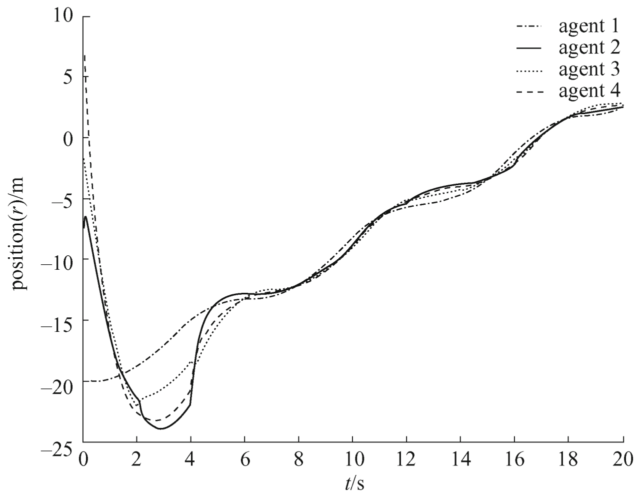

Fig. 3 A_1 with the other three agents moving around

5 Conclusions

This paper solved the tracking control problem for first-order multi-agent systems, by guiding any agent in the system, and designing a neighbor-based distributed protocol. In the framework of partial difference equations over graphs, the tracking control problem for various desired motions is discussed in fixed/switching networks with and without time delays. Simulations illustrate the correctness of the obtained results.

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