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RESEARCH ARTICLE

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Gain-scheduled H_2/H_∞ filtering for linear discrete-time systems with polytopic uncertainties

Abstract The design of the gain-scheduled H_2/H_∞ filter for polytopic discrete-time systems is investigated. By introducing additional slack variables, a new mixed H_2/H_∞ performance criterion is proposed, which provides a decoupling between the Lyapunov matrix and system matrices. Based on the new performance criterion, a sufficient condition for the existence of the gain-scheduled H_2/H_∞ filter is derived. Furthermore, the filter design problem is converted into a convex optimization problem with linear matrix inequality (LMI) constraints. Simulation results show the effectiveness of the proposed approach.

Keywords mixed H_2/H_∞ filtering, gain-scheduled filtering, convex optimization, linear matrix inequality (LMI)

1 Introduction

In recent years, much attention has been directed to the mixed H_2/H_∞ filtering problem for systems with parameter uncertainties, which are inherent in physical systems due to changes in temperature, humidity or operating points (see Refs. [1–6], and the references therein). For systems with norm-bounded parameter uncertainties, there are basically two approaches to the problem. One is the Riccati equation based approach [1,2], and another is the linear matrix inequality (LMI) approach [3,4]. It is well known that polytopic model uncertainties exist extensively in aerospace systems, active suspension systems and circuit systems [7–9]. A LMI approach is presented in Refs. [5] and [9] for discrete-time systems with polytopic uncertainties to solve the mixed H_2/H_∞ filtering problem based on the concept of quadratic stability. However, its main problem is that it requires a common Lyapunov function for different performance indices and all vertices of the polytope, which renders itself as a conservative design.

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To reduce the conservativeness mentioned above, recently, Gao, et al. [6] proposed a new approach known as the parameter-dependent Lyapunov function method. The new approach handles mixed H_2/H_∞ filtering by introducing slack variables and employing some new bounding techniques, which results in a much less conservative design.

Motivated by the work of Ref. [6], in this paper, we further investigate the problem of the mixed H_2/H_∞ filter design for discrete-time systems with polytopic uncertainties, aiming to give a design with less conservativeness or equal result than results of Refs. [5,6]. To this end, a more general mixed H_2/H_∞ performance criterion is proposed with additional slack variables using LMI technique. Thereafter, based on the proposed performance criterion, a sufficient condition for the existence of gain-scheduled H_2/H_∞ filter is derived. Furthermore, the filter design problem is converted into a convex optimization problem with LMI constraints. Finally, a numerical example is illustrated to verify the effectiveness of the proposed design approach.

Throughout the paper, the superscript 'T' stands for matrix transposition; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices; and for $P \in \mathbb{R}^{n\times n}$, the notation $P > 0$ means that P is symmetric and positive definite. In symmetric block matrices or complex matrix expressions, we use an asterisk (*) to represent a term that is induced by the symmetry and diag $\{\cdots\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. In addition, $\|T\|_2$ and $\|T\|_\infty$ denote the standard H_2 and H_∞ norms for the operator T respectively.

2 Problem formulation

Consider the following uncertain discrete-time systems described by:

$$\begin{cases} \boldsymbol{x}(k+1) = \boldsymbol{A}(\alpha)\boldsymbol{x}(k) + \boldsymbol{B}_2(\alpha)\boldsymbol{w}_2(k) + \boldsymbol{B}_\infty(\alpha)\boldsymbol{w}_\infty(k) \\ \boldsymbol{y}(k) = \boldsymbol{C}(\alpha)\boldsymbol{x}(k) + \boldsymbol{D}_2(\alpha)\boldsymbol{w}_2(k) + \boldsymbol{D}_\infty(\alpha)\boldsymbol{w}_\infty(k) \\ \boldsymbol{z}(k) = \boldsymbol{L}(\alpha)\boldsymbol{x}(k) \end{cases}, \qquad (1)$$

where $\boldsymbol{x}(k)\in\mathbb{R}^n$ is the state vector, $\boldsymbol{y}(k)\in\mathbb{R}^m$ is the measured output, $\boldsymbol{z}(k)\in\mathbb{R}^m$ is the output to be estimated, $\boldsymbol{w}_2(k)\in\mathbb{R}^r$ is a Gaussian white noise with zero-mean and unit covariance, $\boldsymbol{w}_\infty(k)\in\mathbb{R}^s$ is the exogenous disturbance that belongs to $L(0,\infty)$. The parameter α is assumed to be gain scheduled, i.e., it is measured on line. Without loss of generality, it allows to vary in the unit simplex:

$$\Omega=\left\{(\alpha_1,\alpha_2,\ldots,\alpha_N):\sum_{i=1}^{N}\alpha_j=1,\alpha_i\geqslant 0\right\}.$$

The matrices $\boldsymbol{A}(\alpha)$, $\boldsymbol{B}_2(\alpha)$, $\boldsymbol{B}_\infty(\alpha)$, $\boldsymbol{C}(\alpha)$, $\boldsymbol{D}_2(\alpha)$, $\boldsymbol{D}_\infty(\alpha)$ and $\boldsymbol{L}(\alpha)$ are assumed linearly in α, i.e.,

$$\begin{bmatrix}\boldsymbol{A}(\alpha) & \boldsymbol{B}_2(\alpha) & \boldsymbol{B}_\infty(\alpha)\\ \boldsymbol{C}(\alpha) & \boldsymbol{D}_2(\alpha) & \boldsymbol{D}_\infty(\alpha)\\ \boldsymbol{L}(\alpha) & \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}=\sum_{i=1}^{N}\alpha_i\begin{bmatrix}\boldsymbol{A}_i & \boldsymbol{B}_{2i} & \boldsymbol{B}_{\infty i}\\ \boldsymbol{C}_i & \boldsymbol{D}_{2i} & \boldsymbol{D}_{\infty i}\\ \boldsymbol{L}_i & \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}. \quad (2)$$

Consider a gain-scheduled filter for system Eq. (1) of the form:

$$\begin{cases}\boldsymbol{x}_{\mathrm{F}}(k+1)=\boldsymbol{A}_{\mathrm{F}}(\alpha)\boldsymbol{x}_{\mathrm{F}}(k)+\boldsymbol{B}_{\mathrm{F}}(\alpha)\boldsymbol{y}(k)\\ \boldsymbol{z}_{\mathrm{F}}(k)=\boldsymbol{C}_{\mathrm{F}}(\alpha)\boldsymbol{x}_{\mathrm{F}}(k)\end{cases}, \quad (3)$$

where $\boldsymbol{x}_{\mathrm{F}}(k)\in\mathbb{R}^n$ is the filter state vector, and ($\boldsymbol{A}_{\mathrm{F}}(\alpha)$, $\boldsymbol{B}_{\mathrm{F}}(\alpha)$, $\boldsymbol{C}_{\mathrm{F}}(\alpha)$) are gain-scheduled filter matrices to be determined.

Combining Eqs. (1) and (3) leads to the following filtering error system:

$$\begin{cases}\bar{\boldsymbol{x}}(k+1)=\bar{\boldsymbol{A}}(\alpha)\bar{\boldsymbol{x}}(k)+\bar{\boldsymbol{B}}_2(\alpha)\boldsymbol{w}_2(k)+\bar{\boldsymbol{B}}_\infty(\alpha)\boldsymbol{w}_\infty(k)\\ \bar{\boldsymbol{z}}(k)=\bar{\boldsymbol{C}}(\alpha)\bar{\boldsymbol{x}}(k)\end{cases}, \quad (4)$$

where $\bar{\boldsymbol{x}}(k)=\left[\boldsymbol{x}^{\mathrm{T}}(k),\boldsymbol{x}_{\mathrm{F}}^{\mathrm{T}}(k)\right]^{\mathrm{T}}$, $\bar{\boldsymbol{z}}(k)=\boldsymbol{z}(k)-\boldsymbol{z}_{\mathrm{F}}(k)$, and

$$\bar{\boldsymbol{A}}(\alpha)=\begin{bmatrix}\boldsymbol{A}(\alpha) & \boldsymbol{0}\\ \boldsymbol{B}_{\mathrm{F}}(\alpha)C(\alpha) & \boldsymbol{A}_{\mathrm{F}}(\alpha)\end{bmatrix},$$
$$\bar{\boldsymbol{B}}_2(\alpha)=\begin{bmatrix}\boldsymbol{B}_2(\alpha)\\ \boldsymbol{B}_{\mathrm{F}}(\alpha)\boldsymbol{D}_2(\alpha)\end{bmatrix},$$
$$\bar{\boldsymbol{B}}_\infty(\alpha)=\begin{bmatrix}\boldsymbol{B}_\infty(\alpha)\\ \boldsymbol{B}_{\mathrm{F}}(\alpha)\boldsymbol{D}_\infty(\alpha)\end{bmatrix},$$
$$\bar{\boldsymbol{C}}(\alpha)=[\,\boldsymbol{L}(\alpha)\quad -\boldsymbol{C}_F(\alpha)\,].$$

The filter design problem under consideration in this paper is expressed as follows. Given the parameter-varying Eq. (1) with scalars $\beta>0$ and $\gamma>0$, the aim is to design an admissible filter in the form of Eq. (3), such that for all admissible parameter uncertainties, the following conditions are satisfied:

1) The filtering error Eq. (4) is asymptotically stable with $\max_\alpha\|T_{\bar{z}w_\infty}(\alpha)\|_\infty<\gamma$ and $\max_\alpha\|T_{\bar{z}w_2}(\alpha)\|_2^2<\beta$.

2) The upper bound of the H_2 norm of the transfer function $T_{\bar{z}w_2}(\alpha)$, i.e., $\max_\alpha\|T_{\bar{z}w_2}(\alpha)\|_2^2$ is minimized. The matrices $T_{\bar{z}w_2}(\alpha)$ and $T_{\bar{z}w_\infty}(\alpha)$ are the closed-loop transfer matrices from $\boldsymbol{w}_2(k)$ to $\bar{\boldsymbol{z}}(k)$ and from $\boldsymbol{w}_\infty(k)$ to $\bar{\boldsymbol{z}}(k)$ respectively. Filters satisfying the above conditions are called gain-scheduled H_2/H_∞ filters.

Mathematically, the filtering problem above can be expressed by:

$$\begin{cases}\min\limits_{\boldsymbol{A}_{\mathrm{F}}(\alpha),\,\boldsymbol{B}_{\mathrm{F}}(\alpha),\,\boldsymbol{C}_{\mathrm{F}}(\alpha)}\ \max_\alpha\|T_{\bar{z}w_2}(\alpha)\|_2^2,\\ \text{s.t.}\quad \max_\alpha\|T_{\bar{z}w_\infty}(\alpha)\|_\infty<\gamma,\\ \qquad\ \max_\alpha\|T_{\bar{z}w_2}(\alpha)\|_2^2<\beta.\end{cases} \quad (5)$$

3 Main result

3.1 Extended H_2 performance criterion

The following lemma can be found in Ref. [10], which plays an important role in subsequent derivations.

Lemma 1 Let $\boldsymbol{H}=\boldsymbol{H}^{\mathrm{T}}\in\mathbb{R}^{n\times n}$, $\boldsymbol{U}\in\mathbb{R}^{n\times m}$, and $\boldsymbol{V}\in\mathbb{R}^{k\times n}$ be given matrices. Consider finding some matrix $\boldsymbol{X}\in\mathbb{R}^{m\times k}$ satisfying

$$\boldsymbol{H}+\boldsymbol{U}\boldsymbol{X}\boldsymbol{V}+(\boldsymbol{U}\boldsymbol{X}\boldsymbol{V})^{\mathrm{T}}<\boldsymbol{0}. \quad (6)$$

Then, Eq. (6) is solvable for $\boldsymbol{X}$ if and only if

$$\boldsymbol{U}^{\perp}\boldsymbol{H}\boldsymbol{U}^{\perp\mathrm{T}}<\boldsymbol{0},\ \boldsymbol{V}^{\mathrm{T}\perp}\boldsymbol{H}\boldsymbol{V}^{\perp}<\boldsymbol{0}, \quad (7)$$

where $\boldsymbol{U}^{\perp}$ and $\boldsymbol{V}^{\perp}$ are orthogonal complements of $\boldsymbol{U}$ and $\boldsymbol{V}$ respectively.

The following lemma characterizes the H_2 performance for discrete-time systems.

Lemma 2 Given Eqs. (1) and (3) with the scalar $\beta>0$, the filtering error Eq. (4) is asymptotically stable with $\|T_{\bar{z}w_2}(\alpha)\|_2^2<\beta$ if and only if there exist matrix functions $\boldsymbol{0}<\boldsymbol{Q}(\alpha)\in\mathbb{R}^{r\times r}$, $\boldsymbol{0}<\boldsymbol{P}_2(\alpha)=\boldsymbol{P}_2^{\mathrm{T}}(\alpha)\in\mathbb{R}^{2n\times 2n}$, such that

$$\mathrm{trace}(\boldsymbol{Q}(\alpha))<\beta, \quad (8)$$

$$\bar{\boldsymbol{B}}_2^{\mathrm{T}}(\alpha)\boldsymbol{P}_2(\alpha)\bar{\boldsymbol{B}}_2(\alpha)<\boldsymbol{Q}(\alpha), \quad (9)$$

$$-\boldsymbol{P}_2(\alpha)+\bar{\boldsymbol{A}}^{\mathrm{T}}(\alpha)\boldsymbol{P}_2(\alpha)\bar{\boldsymbol{A}}(\alpha)+\bar{\boldsymbol{C}}^{\mathrm{T}}(\alpha)\bar{\boldsymbol{C}}(\alpha)<\boldsymbol{0}. \quad (10)$$

In the following theorem, a new extended H_2 performance criterion is established for linear discrete-time systems by using Lemma 1 and Lemma 2.

Theorem 1 Given Eqs. (1) and (3) with the scalar $\beta>0$, the filtering error systems Eq. (4) is asymptotically stable with $\|T_{\bar{z}w_2}(\alpha)\|_2^2<\beta$ if and only if there exist matrix functions $\boldsymbol{0}<\boldsymbol{P}_2(\alpha)=\boldsymbol{P}_2^{\mathrm{T}}(\alpha)\in\mathbb{R}^{2n\times 2n}$, $\boldsymbol{0}<\boldsymbol{Q}(\alpha)\in\mathbb{R}^{r\times r}$, $\boldsymbol{E}(\alpha)\in\mathbb{R}^{2n\times 2n}$, $\boldsymbol{G}_2(\alpha)\in\mathbb{R}^{2n\times 2n}$ and $\boldsymbol{F}_2(\alpha)\in\mathbb{R}^{2n\times 2n}$, satisfying Eq. (8) and

$$\begin{bmatrix}\boldsymbol{P}_2(\alpha)-\boldsymbol{E}(\alpha)-\boldsymbol{E}^{\mathrm{T}}(\alpha) & *\\ \bar{\boldsymbol{B}}_2^{\mathrm{T}}(\alpha)\boldsymbol{E}(\alpha) & -\boldsymbol{Q}(\alpha)\end{bmatrix}<\boldsymbol{0}, \quad (11)$$

$$\begin{bmatrix}\boldsymbol{P}_2(\alpha)-\boldsymbol{G}_2(\alpha)-\boldsymbol{G}_2^{\mathrm{T}}(\alpha) & * & *\\ -\boldsymbol{F}_2^{\mathrm{T}}(\alpha)+\bar{\boldsymbol{A}}^{\mathrm{T}}(\alpha)\boldsymbol{G}_2(\alpha) & \Gamma_1 & *\\ \boldsymbol{0} & \bar{\boldsymbol{C}}(\alpha) & -\boldsymbol{I}\end{bmatrix}<\boldsymbol{0}, \quad (12)$$

where $\Gamma_1=-\boldsymbol{P}_2(\alpha)+\boldsymbol{F}_2^{\mathrm{T}}(\alpha)\bar{\boldsymbol{A}}(\alpha)+\bar{\boldsymbol{A}}^{\mathrm{T}}(\alpha)\boldsymbol{F}_2(\alpha)$.

Proof In the light of Lemma 2, the problem is reduced to the proof that Eq. (11) holds if and only if Eq. (9) holds, and Eq. (12) holds if and only if Eq. (10) holds. Therefore, rewrite Eq. (11) in the form of Eq. (6), where

$$\begin{aligned} &\boldsymbol{H}=\begin{bmatrix}\boldsymbol{P}_2(\alpha) & \mathbf{0}\\ \mathbf{0} & -\boldsymbol{Q}(\alpha)\end{bmatrix},\ \boldsymbol{U}=\begin{bmatrix}-\boldsymbol{I}\\ \bar{\boldsymbol{B}}_2^{\mathrm{T}}(\alpha)\end{bmatrix},\\ &\boldsymbol{V}=[\boldsymbol{I}\quad \mathbf{0}],\ \boldsymbol{X}=\boldsymbol{E}(\alpha). \end{aligned} \tag{13}$$

Note that $\boldsymbol{U}^{\perp}=\begin{bmatrix}\bar{\boldsymbol{B}}_2^{\mathrm{T}}(\alpha) & \boldsymbol{I}\end{bmatrix}$ and $\boldsymbol{V}^{\mathrm{T}\perp}=[\mathbf{0}\ \boldsymbol{I}]$. It follows Lemma 1 that Eq. (6) is solvable for $\boldsymbol{X}$ if and only if Eq. (9) holds, i.e., Eqs. (11) and (9) are equivalent.

The equivalence of Eqs. (12) and (9) follows similarly from Lemma 1. This completes the proof.

3.2 Extended H_∞ performance criterion

Based on the bounded real lemma (BRL) of discrete-time systems in Ref. [6], the following theorem presents an improved version of the BRL criterion by using the LMI technique.

Theorem 2 Given Eqs. (1) and (3) with the scalar $\gamma>0$, the filtering error Eq. (4) is asymptotically stable with $||T_{\bar{z}w_\infty}(\alpha)||_\infty<\gamma$ if and only if there exist matrix functions $\mathbf{0}<\boldsymbol{P}_\infty(\alpha)=\boldsymbol{P}_\infty^{\mathrm{T}}(\alpha)\in\mathbb{R}^{2n\times 2n}$, $\boldsymbol{G}_\infty(\alpha)\in\mathbb{R}^{2n\times 2n}$ and $\boldsymbol{F}_\infty(\alpha)\in\mathbb{R}^{2n\times 2n}$, such that

$$\begin{bmatrix}\boldsymbol{P}_\infty(\alpha)-\boldsymbol{G}_\infty(\alpha)-\boldsymbol{G}_\infty^{\mathrm{T}}(\alpha) & * & * & *\\ -\boldsymbol{F}_\infty^{\mathrm{T}}(\alpha)+\bar{\boldsymbol{A}}^{\mathrm{T}}(\alpha)\boldsymbol{G}_\infty(\alpha) & \Gamma_2 & * & *\\ \bar{\boldsymbol{B}}_\infty^{\mathrm{T}}(\alpha)\boldsymbol{G}_\infty(\alpha) & \bar{\boldsymbol{B}}_\infty^{\mathrm{T}}(\alpha)\boldsymbol{F}_\infty(\alpha) & -\gamma^2\boldsymbol{I} & *\\ \mathbf{0} & \bar{\boldsymbol{C}}(\alpha) & \mathbf{0} & -\boldsymbol{I}\end{bmatrix}<\mathbf{0}, \tag{14}$$

where $\Gamma_2=-\boldsymbol{P}_\infty(\alpha)+\boldsymbol{F}_\infty^{\mathrm{T}}(\alpha)\bar{\boldsymbol{A}}(\alpha)+\bar{\boldsymbol{A}}^{T}(\alpha)\boldsymbol{F}_\infty(\alpha)$.

Proof The proof proceeds similar to that of Theorem 1, hence it shall be omitted here.

3.3 Mixed H_2/H_∞ performance criterion

Upon Theorem 1 and Theorem 2, the following theorem presents a more general mixed H_2/H_∞ performance criterion, which provides a decoupling between the Lyapunov matrix and system matrices.

Theorem 3 Given Eqs. (1) and (3) with scalars $\beta>0$, $\gamma>0$, the filtering error Eq. (4) is asymptotically stable with $||T_{\bar{z}w_2}(\alpha)||_2^2<\beta$ and $||T_{\bar{z}w_\infty}(\alpha)||_\infty<\gamma$ if there exist matrix functions

$$\mathbf{0}<\boldsymbol{P}_2(\alpha)=\boldsymbol{P}_2^{\mathrm{T}}(\alpha)\in\mathbb{R}^{2n\times 2n},$$

$$\mathbf{0}<\boldsymbol{P}_\infty(\alpha)=\boldsymbol{P}_\infty^{\mathrm{T}}(\alpha)\in\mathbb{R}^{2n\times 2n},$$

$$\mathbf{0}<\boldsymbol{Q}(\alpha)\in\mathbb{R}^{r\times r},\ \boldsymbol{E}(\alpha)\in\mathbb{R}^{2n\times 2n},$$

$$\boldsymbol{G}(\alpha)\in\mathbb{R}^{2n\times 2n},\ \boldsymbol{F}_2(\alpha)\in\mathbb{R}^{2n\times 2n},\ \boldsymbol{F}_\infty(\alpha)\in\mathbb{R}^{2n\times 2n}$$

satisfying Eqs. (8) and (11), and

$$\begin{bmatrix}\boldsymbol{P}_2(\alpha)-\boldsymbol{G}(\alpha)-\boldsymbol{G}^{\mathrm{T}}(\alpha) & * & *\\ -\boldsymbol{F}_2^{\mathrm{T}}(\alpha)+\bar{\boldsymbol{A}}^{\mathrm{T}}(\alpha)\boldsymbol{G}(\alpha) & \Gamma_1 & *\\ \mathbf{0} & \bar{\boldsymbol{C}}(\alpha) & -\boldsymbol{I}\end{bmatrix}<\mathbf{0}, \tag{15}$$

$$\begin{bmatrix}\boldsymbol{P}_\infty(\alpha)-\boldsymbol{G}(\alpha)-\boldsymbol{G}^{\mathrm{T}}(\alpha) & * & * & *\\ -\boldsymbol{F}_\infty^{\mathrm{T}}(\alpha)+\bar{\boldsymbol{A}}^{\mathrm{T}}(\alpha)\boldsymbol{G}(\alpha) & \Gamma_2 & * & *\\ \bar{\boldsymbol{B}}_\infty^{\mathrm{T}}(\alpha)\boldsymbol{G}(\alpha) & \bar{\boldsymbol{B}}_\infty^{\mathrm{T}}(\alpha)\boldsymbol{F}_\infty(\alpha) & -\gamma^2\boldsymbol{I} & *\\ \mathbf{0} & \bar{\boldsymbol{C}}(\alpha) & \mathbf{0} & -\boldsymbol{I}\end{bmatrix}<\mathbf{0}. \tag{16}$$

Remark 1 By introducing more matrix variables, Theorem 3 presents a more general mixed H_2/H_∞ criterion than previous results. To be specific, apart from the matrix variable $\boldsymbol{G}(\alpha)$, additional slack variables, i.e., $\boldsymbol{E}(\alpha)$, $\boldsymbol{F}_2(\alpha)$ and $\boldsymbol{F}_\infty(\alpha)$, are introduced in Theorem 3 compared with the Proposition 1 of Ref. [6]. We observe that when setting $\boldsymbol{E}(\alpha)=\boldsymbol{G}(\alpha)$ and $\boldsymbol{F}_2(\alpha)=\boldsymbol{F}_\infty(\alpha)=\mathbf{0}$, Theorem 3 reduces to the Proposition 1 of Ref. [9]. It is also worth noting that when setting $\boldsymbol{F}_2(\alpha)=\boldsymbol{F}_\infty(\alpha)=\mathbf{0}$ and $\boldsymbol{E}(\alpha)=\boldsymbol{G}(\alpha)=\boldsymbol{P}_2(\alpha)=\boldsymbol{P}_\infty(\alpha)$, Theorem 3 recovers the Lemma 3 of Ref. [6]. Theoretically speaking, Theorem 3 is less conservative than previous results, which means that Theorem 3 will be more powerful in designing H_2/H_∞ filters.

3.4 Gain-scheduled H_2/H_∞ filter design

In the sequel, we will present a sufficient condition for the existence of the gain-scheduled H_2/H_∞ filter in the form of Eq. (3), and show how to construct a filter based on Theorem 3.

Theorem 4 Given Eq. (1) with scalars $\beta>0$, $\gamma>0$, δ_{1i}, δ_{2i}, ε_{1i}, ε_{2i}, τ_{1i} and τ_{2i}, then an admissible gain-scheduled H_2/H_∞ filter in the form of Eq. (3) exists such that the filtering error Eq. (4) is asymptotically stable with $||T_{\bar{z}w_2}(\alpha)||_2^2<\beta$ and $||T_{\bar{z}w_\infty}(\alpha)||_\infty<\gamma$, if there exist matrices $\mathbf{0}<\boldsymbol{Q}_i$, $\mathbf{0}<\bar{\boldsymbol{P}}_{2i}=\begin{bmatrix}\bar{\boldsymbol{P}}_{21i} & \boldsymbol{P}_{22i}\\ \bar{\boldsymbol{P}}_{22i}^{\mathrm{T}} & \bar{\boldsymbol{P}}_{22i}\end{bmatrix}$, $\mathbf{0}<\bar{\boldsymbol{P}}_{\infty i}=\begin{bmatrix}\bar{\boldsymbol{P}}_{\infty 1i} & \bar{\boldsymbol{P}}_{\infty 2i}\\ \bar{\boldsymbol{P}}_{\infty 2i}^{\mathrm{T}} & \bar{\boldsymbol{P}}_{\infty 3i}\end{bmatrix}$, $\boldsymbol{R}_i$, $\boldsymbol{S}_i$, $\boldsymbol{T}_i$, $\bar{\mathrm{A}}_{\mathrm{F}i}$, $\bar{\boldsymbol{B}}_{\mathrm{F}i}$, $\bar{\boldsymbol{C}}_{\mathrm{F}i}$, $\boldsymbol{\Lambda}_{ij}$, $\boldsymbol{\Sigma}_{ij}$ and $\boldsymbol{\Xi}_{ij}$ such that the following LMIs hold.

$$\mathrm{trace}(\boldsymbol{Q}_i)<\beta,\qquad i=1,2,\ldots,N, \tag{17}$$

$$\Phi_{ij}+\Phi_{ji}-\Lambda_{ij}-\Lambda_{ij}^{\mathrm{T}}\leqslant\mathbf{0},\quad 1\leqslant j<i\leqslant N, \tag{18}$$

$$\Psi_{ij}+\Psi_{ji}-\Sigma_{ij}-\Sigma_{ij}^{\mathrm{T}}\leqslant\mathbf{0},\quad 1\leqslant j<i\leqslant N, \tag{19}$$

$$\Theta_{ij}+\Theta_{ji}-\Xi_{ij}-\Xi_{ij}^{\mathrm{T}}\leqslant\mathbf{0},\quad 1\leqslant j<i\leqslant N, \tag{20}$$

$$\begin{bmatrix} \Phi_{11} & * & \cdots & * \\ \Lambda_{21} & \Phi_{22} & \cdots & * \\ \vdots & \vdots & \ddots & \vdots \\ \Lambda_{N1} & \Lambda_{N2} & \cdots & \Phi_{NN} \end{bmatrix} < \mathbf{0}, \tag{21}$$

$$\begin{bmatrix} \Psi_{11} & * & \cdots & * \\ \Sigma_{21} & \Psi_{22} & \cdots & * \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{N1} & \Sigma_{N2} & \cdots & \Psi_{NN} \end{bmatrix} < \mathbf{0}, \tag{22}$$

$$\begin{bmatrix} \Theta_{11} & * & \cdots & * \\ \Xi_{21} & \Theta_{22} & \cdots & * \\ \vdots & \vdots & \ddots & \vdots \\ \Xi_{N1} & \Xi_{N2} & \cdots & \Theta_{NN} \end{bmatrix} < \mathbf{0}. \tag{23}$$

Moreover, if the above LMIs are feasible, the matrix functions for an admissible gain-scheduled H_2/H_∞ filter in the form of Eq. (3) can be constructed by

$$\begin{bmatrix} \boldsymbol{A}_{\mathrm{F}}(\alpha) & \boldsymbol{B}_{\mathrm{F}}(\alpha) \\ \boldsymbol{C}_{\mathrm{F}}(\alpha) & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N} \alpha_i^{-1} \boldsymbol{T}_i^{-1} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{N} \alpha_i \bar{\boldsymbol{A}}_{\mathrm{F}i} & \sum_{i=1}^{N} \alpha_i \bar{\boldsymbol{B}}_{\mathrm{F}i} \\ \sum_{i=1}^{N} \alpha_i \bar{\boldsymbol{C}}_{\mathrm{F}i} & \mathbf{0} \end{bmatrix}, \tag{24}$$

where

$$\Phi_{ij} = \begin{bmatrix} \bar{\boldsymbol{P}}_{21i} - \delta_{1i}\boldsymbol{R}_i - \delta_{1i}\boldsymbol{R}_i^{\mathrm{T}} & * & * \\ \bar{\boldsymbol{P}}_{22i}^{T} - \delta_{1i}\boldsymbol{S}_i^{\mathrm{T}} - \delta_{2i}\boldsymbol{T}_i^{\mathrm{T}} & \bar{\boldsymbol{P}}_{23i} - \delta_{2i}\boldsymbol{T}_i - \delta_{2i}\boldsymbol{T}_i^{\mathrm{T}} & * \\ \delta_{1i}\boldsymbol{B}_{2j}^{\mathrm{T}}\boldsymbol{R}_i + \delta_{2i}\boldsymbol{D}_{2j}^{\mathrm{T}}\bar{\boldsymbol{B}}_{\mathrm{F}i}^{\mathrm{T}} & \delta_{1i}\boldsymbol{B}_{2j}^{\mathrm{T}}\boldsymbol{S}_i + \delta_{2i}\boldsymbol{D}_{2j}^{\mathrm{T}}\bar{\boldsymbol{B}}_{\mathrm{F}i}^{\mathrm{T}} & -\boldsymbol{Q}_i \end{bmatrix},$$

$$\Sigma_{ij} = \begin{bmatrix} \bar{\boldsymbol{P}}_{21i} - \boldsymbol{R}_i - \boldsymbol{R}_i^{\mathrm{T}} & * & * & * & * \\ \bar{\boldsymbol{P}}_{22i}^{T} - \boldsymbol{S}_i^{\mathrm{T}} - \boldsymbol{T}_i^{\mathrm{T}} & \bar{\boldsymbol{P}}_{23i} - \boldsymbol{T}_i - \boldsymbol{T}_i^{\mathrm{T}} & * & * & * \\ \boldsymbol{A}_j^{\mathrm{T}}\boldsymbol{R}_i + \boldsymbol{C}_j^{\mathrm{T}}\bar{\boldsymbol{B}}_{\mathrm{F}i}^{\mathrm{T}} - \varepsilon_{1i}\boldsymbol{R}_i^{\mathrm{T}} & \boldsymbol{A}_j^{\mathrm{T}}\boldsymbol{S}_i + \boldsymbol{C}_j^{\mathrm{T}}\bar{\boldsymbol{B}}_{\mathrm{F}i}^{\mathrm{T}} - \varepsilon_{2i}\boldsymbol{T}_i & \Gamma_3 & * & * \\ \bar{\boldsymbol{A}}_{\mathrm{F}i}^{\mathrm{T}} - \varepsilon_{1i}\boldsymbol{S}_i^{\mathrm{T}} & \bar{\boldsymbol{A}}_{\mathrm{F}i}^{\mathrm{T}} - \varepsilon_{2i}\boldsymbol{T}_i & \Gamma_4 & -\bar{\boldsymbol{P}}_{23i} + \varepsilon_{2i}(\bar{\boldsymbol{A}}_{\mathrm{F}i} + \bar{\boldsymbol{A}}_{\mathrm{F}i}^{\mathrm{T}}) & * \\ \mathbf{0} & \mathbf{0} & \boldsymbol{L}_i & -\bar{\boldsymbol{C}}_{\mathrm{F}i} & -\boldsymbol{I} \end{bmatrix},$$

$$\Xi_{ij} = \begin{bmatrix} \bar{\boldsymbol{P}}_{\infty 1i} - \boldsymbol{R}_i - \boldsymbol{R}_i^{\mathrm{T}} & * & * & * & * & * \\ \bar{\boldsymbol{P}}_{\infty 2i}^{\mathrm{T}} - \boldsymbol{S}_i^{\mathrm{T}} - \boldsymbol{T}_i^{\mathrm{T}} & \bar{\boldsymbol{P}}_{\infty 3i} - \boldsymbol{T}_i - \boldsymbol{T}_i^{\mathrm{T}} & * & * & * & * \\ \boldsymbol{A}_j^{T}\boldsymbol{R}_i + \boldsymbol{C}_j^{\mathrm{T}}\bar{\boldsymbol{B}}_{\mathrm{F}i}^{\mathrm{T}} - \tau_{1i}\boldsymbol{R}_i^{\mathrm{T}} & \boldsymbol{A}_j^{\mathrm{T}}\boldsymbol{S}_i + \boldsymbol{C}_j^{\mathrm{T}}\bar{\boldsymbol{B}}_{\mathrm{F}i}^{\mathrm{T}} - \tau_{2i}\boldsymbol{T}_i & \Gamma_5 & * & * & * \\ \bar{\boldsymbol{A}}_{\mathrm{F}i}^{\mathrm{T}} - \tau_{1i}\boldsymbol{S}_i^{\mathrm{T}} & \bar{\boldsymbol{A}}_{\mathrm{F}i}^{\mathrm{T}} - \tau_{2i}\boldsymbol{T} & \Gamma_6 & -\bar{\boldsymbol{P}}_{\infty 3i} + \tau_{2i}(\bar{\boldsymbol{A}}_{\mathrm{F}i} + \bar{\boldsymbol{A}}_{\mathrm{F}i}^{\mathrm{T}}) & * & * \\ \boldsymbol{B}_{\infty j}^{\mathrm{T}}\boldsymbol{R}_i + \boldsymbol{D}_{\infty j}^{\mathrm{T}}\bar{\boldsymbol{B}}_{\mathrm{F}i}^{\mathrm{T}} & \boldsymbol{B}_{\infty j}^{\mathrm{T}}\boldsymbol{S}_i + \boldsymbol{D}_{\infty j}^{\mathrm{T}}\bar{\boldsymbol{B}}_{\mathrm{F}i}^{\mathrm{T}} & \Gamma_7 & \tau_{1i}\boldsymbol{B}_{\infty j}^{\mathrm{T}}\boldsymbol{S}_i + \tau_{2i}\boldsymbol{D}_{\infty j}^{\mathrm{T}}\bar{\boldsymbol{B}}_{\mathrm{F}i}^{\mathrm{T}} & -\gamma^2\boldsymbol{I} & * \\ \mathbf{0} & \mathbf{0} & \boldsymbol{L}_i & -\bar{\boldsymbol{C}}_{Fi} & \mathbf{0} & -\boldsymbol{I} \end{bmatrix},$$

and

$$\Gamma_3 = -\bar{\boldsymbol{P}}_{21i} + \varepsilon_{1i}(\boldsymbol{R}_i^{\mathrm{T}}\boldsymbol{A}_j + \boldsymbol{A}_j^{\mathrm{T}}\boldsymbol{R}_i) + \varepsilon_{2i}(\bar{\boldsymbol{B}}_{\mathrm{F}i}\boldsymbol{C}_j + \boldsymbol{C}_j^{\mathrm{T}}\bar{\boldsymbol{B}}_{\mathrm{F}i}^{\mathrm{T}}),$$

$$\Gamma_4 = -\bar{\boldsymbol{P}}_{22i}^{\mathrm{T}} + \varepsilon_{1i}\boldsymbol{S}_i^{\mathrm{T}}\boldsymbol{A}_j + \varepsilon_{2i}(\bar{\boldsymbol{A}}_{\mathrm{F}i}^{\mathrm{T}} + \bar{\boldsymbol{B}}_{\mathrm{F}i}\boldsymbol{C}_j),$$

$$\Gamma_5 = -\bar{\boldsymbol{P}}_{\infty 1i} + \tau_{1i}(\boldsymbol{R}_i^{\mathrm{T}}\boldsymbol{A}_j + \boldsymbol{A}_j^{\mathrm{T}}\boldsymbol{R}_i) + \tau_{2i}(\bar{\boldsymbol{B}}_{\mathrm{F}i}\boldsymbol{C}_j + \boldsymbol{C}_j^{\mathrm{T}}\bar{\boldsymbol{B}}_{\mathrm{F}i}^{\mathrm{T}}),$$

$$\Gamma_6 = -\bar{\boldsymbol{P}}_{\infty 2i}^{\mathrm{T}} + \tau_{1i}\boldsymbol{S}_i^{\mathrm{T}}\boldsymbol{A}_j + \tau_{2i}(\bar{\boldsymbol{A}}_{\mathrm{F}i}^{\mathrm{T}} + \bar{\boldsymbol{B}}_{\mathrm{F}i}\boldsymbol{C}_j),$$

$$\Gamma_7 = \tau_{1i}\boldsymbol{B}_{\infty j}^{\mathrm{T}}\boldsymbol{R}_i + \tau_{2i}\boldsymbol{D}_{\infty j}^{\mathrm{T}}\bar{\boldsymbol{B}}_{\mathrm{F}i}^{\mathrm{T}}.$$

Proof The proof of Theorem 4 can be carried out by the following similar lines as in the proof of Theorem 1 of Ref. [6], hence it is omitted for brevity. Additional details can be found in Refs. [11,12].

Remark 2 It is worth pointing out that when setting $\delta_{1i}=\delta_{2i}=1$ and $\varepsilon_{1i}=\varepsilon_{2i}=\tau_{1i}=\tau_{2i}=0$, Theorem 4 reduces to the Theorem 1 of Ref. [6]. It is clear that the result in Theorem 4 is guaranteed to be less conservative in general than the Theorem 1 of Ref. [6] due to the extra $6N$ degrees of freedom provided by the free parameters $\delta_{1i}, \delta_{2i}, \varepsilon_{1i}, \varepsilon_{2i}, \tau_{1i}$ and τ_{2i} $(1 \leqslant i \leqslant N)$.

Corollary 1 Given Eq. (1) with the scalar $\gamma > 0$, consider the following convex optimization problem:

$$\begin{cases} \min\limits_{Q_i,\bar{P}_{21i},\bar{P}_{22i},\bar{P}_{23i},\bar{P}_{\infty 1i},\bar{P}_{\infty 2i},\bar{P}_{\infty 3i},R_i,S_i,T_i,\bar{A}_{Fi},\bar{B}_{Fi},\bar{C}_{Fi},\delta_{1i},\delta_{2i},\varepsilon_{1i},\varepsilon_{2i},\tau_{1i},\tau_{2i}} \quad \beta \\ \text{s.t.} \quad (17)-(23). \end{cases} \tag{25}$$

If there exists a feasible solution to Eq. (25), then by Theorem 4, the optimal gain-scheduled filter with minimized H_2 performance β and prescribed H_∞ performance γ can be derived by Eq. (24).

Remark 3 It should be mentioned that the optimal solution to Eq. (25) can be obtained offline by using the LMI toolbox of Matlab. And by Theorem 4, the computation that needs to be carried out online is only Eq. (24) when designing a gain-scheduled H_2/H_∞ filter, which leads to an easy implementation in a realistic design.

4 Illustrative example

Let us consider the uncertain discrete-time system in Eq. (1) with

$$\boldsymbol{A}=\begin{bmatrix} 0 & -0.8 \\ 1.2+0.1d & -0.5 \end{bmatrix}, \boldsymbol{B}_2=\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \boldsymbol{B}_\infty=\begin{bmatrix} -0.45 \\ 0.35 \end{bmatrix},$$
$$\boldsymbol{C}=[0.35 \quad -0.65], D_2=1.3, D_\infty=0.4,$$
$$\boldsymbol{L}=[0.2 \quad 0], |d| \leqslant \bar{d},$$

where $\bar{d} \geqslant 0$ is a prescribed non-negative value.

As stated in Refs. [5,6], the above system can be modeled with a two-vertex polytope.

Assume $\bar{d}=0.1$ and fix the H_∞ performance $\gamma=0.2$. The obtained minimum H_2 performance of admissible H_2/H_∞ filters is $\beta=0.0324$ by Corollary 1 and the associated filter matrices are as follows:

$$\begin{aligned} &\boldsymbol{T}_1=\begin{bmatrix} 0.0573 & 0.0070 \\ 0.0075 & 0.0308 \end{bmatrix}, \boldsymbol{T}_2=\begin{bmatrix} 0.0584 & 0.0068 \\ 0.0103 & 0.0305 \end{bmatrix}, \\ &\bar{\boldsymbol{A}}_{F1}=\begin{bmatrix} -0.0037 & -0.0277 \\ 0.0203 & 0.0077 \end{bmatrix}, \\ &\bar{\boldsymbol{A}}_{F2}=\begin{bmatrix} -0.0040 & -0.0275 \\ 0.0207 & 0.0068 \end{bmatrix}, \\ &\bar{\boldsymbol{B}}_{F1}=\begin{bmatrix} -0.0377 \\ -0.0447 \end{bmatrix}, \bar{\boldsymbol{B}}_{F2}=\begin{bmatrix} -0.0353 \\ -0.0463 \end{bmatrix}, \\ &\bar{\boldsymbol{C}}_{F1}=[-0.2012 \quad 0.0006], \bar{\boldsymbol{C}}_{F2}=[-0.1988 \quad 0.0010]. \end{aligned} \tag{26}$$

With the above matrices, the matrix functions for an admissible gain-scheduled H_2/H_∞ filter are given by Eq. (24). Moreover, by connecting the designed gain-scheduled H_2/H_∞ filter to the original system, it is found that the maximum H_∞ norm of the filtering error system for different d is 0.1985, and the maximum H_2 norm is 0.0271. It is obvious that both maximum H_∞ norm and H_2 norm are below their proscribed values, which demonstrate the feasibility of the proposed approach.

To verify the effectiveness of the proposed method, Fig. 1 presents a comparison between Corollary 1 of this paper, Corollary 1 of Ref. [5], and Theorem 1 of Ref. [6] for different prescribed values of $\bar{d}$.

As shown in Fig. 1, it is clear that the proposed approach can yield less conservative results compared with previous methods.

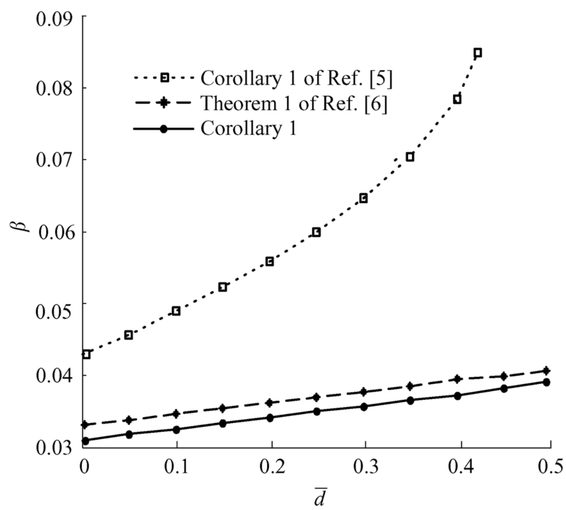

Fig. 1 Comparison of obtained minimum H_2 performance of H_2/H_∞ filter for different values of $\bar{d}$

5 Conclusions

The design problem of gain-scheduled filter has been investigated for linear discrete-time systems with polytopic uncertainties. To reduce design conservatism, a new extended H_2/H_∞ performance criterion is proposed by introducing additional slack variables, including the present results as special cases. Then, upon the new performance criterion, a sufficient condition for the existence of the gain-scheduled H_2/H_∞ filter is presented in the form of LMIs, which can be solved efficiently with the aid of the LMI toolbox of Matlab. Finally, a numerical example is given to demonstrate the effectiveness and advantages of the proposed design approach.

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