Front. Electr. Electron. Eng. China 2008, 3(2): 240–245
DOI 10.1007/s11460-008-0041-3

RESEARCH ARTICLE

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Analytical design of PI controller for AQM with robustness adjustability

Abstract Based on a linearized TCP/AQM model, a new proportional integral (PI) controller design approach is proposed. This analytical approach applies H_∞ optimization and internal model control (IMC) theory to design active queue management (AQM) routers that support transmission control protocol (TCP) flows. The most important feature of the proposed scheme lies in that it can be explicitly tuned with a single parameter for the trade-off between performance and stability of the AQM control system. It is thus flexible and easy to use in design. The proposed method and the designed PI controller are verified and compared with other existing AQM schemes using ns-2 simulator. The results show the advantages of the new PI controller design approach for AQM routers supporting TCP flows.

Keywords active queue management (AQM), proportional integral (PI) controller, transmission control protocol (TCP), time delay, robustness

1 Introduction

The Internet research community is promoting active queue management (AQM) in routers as a means of addressing congestion control and avoiding congestion. AQM can maintain shorter queuing delay and higher throughput by dropping packets at intermediate nodes. It has therefore attracted attention in the research for transmission control protocol (TCP) of end-to-end congestion control. Development of new AQM routers is required and will play a key role in meeting the increasing demand for performance in Internet applications, such as voice over IP (VoIP), class of service (CoS), and streaming video [1].

Translated from *Journal of Shanghai Jiaotong University*, 2007, 41(5): 788–791 [译自: 上海交通大学学报]

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One of the most prevalent AQM algorithms is random early detection (RED) [2]. RED can prevent global synchronization, reduce packet loss rates, and minimize bias against burst sources. However, many subsequent studies show that RED is unstable and sensitive to parametric configurations. Furthermore, it is difficult to obtain adequate values of RED parameters that would achieve good performance in different network scenarios [3–6]. Numerous modified versions of RED have been proposed to solve problems existing in RED, such as SRED [7], FRED [8], BLUE [9] and self-configuring gateway [4]. Most of them, including RED, are based on heuristic algorithms while lacking theoretical analysis and systematic evaluation [10]. Since intuitional and partial simulations are not always reliable, precise and analytical approaches need to be developed in the design and evaluation of AQM algorithms for TCP flow and congestion control. From underlying operation mechanism of AQM, it is reasonable to consider TCP flow control as a typical regulation system design in control engineering. Recently, a dynamic model of TCP behavior was developed in Ref. [11] using the fluid-flow and stochastic differential equation analysis. This non-linear and time-varying model has subsequently been linearized by using the small-signal linearization theory at an operating point [12]. Based on this linearized model, Hollot [1,12] analyzed an AQM system implementing RED and designed standard proportional (P) and proportional integral (PI) controllers for AQM using the classical control theory. The PI controller reveals that the queue length and dropping/marking probability are decoupled and the system has relatively high stability margin. However, the tuning of the P/PI controller is still a difficult task. Moreover, the mismatches in the simplified TCP flow model inevitably deteriorate the system performance. Therefore, more robust controllers are required which hopefully would adapt themselves to complex network environments and cope with plant/model mismatches. In this paper, the H_∞ control theory is applied to design such a PI controller analytically. The stability and performance of the AQM control system can be guaranteed with the proposed design method. The scheme features simple and transparent design, performance tuning and robust stability trade-off. Simulation examples illustrate the effectiveness of the proposed PI control scheme. The well-known ns-2 simulator (http://www.isi.edu/nsnam/ns) is employed to support our analysis and verify the new PI controller design approach.

The paper is organized as follows: in Sect. 2, we introduce the linearized TCP model for the AQM control system. The analytical design procedure for the PI controller is developed on the basis of H_∞ control theory in Sect. 3. In Sect. 4, the proposed H_∞ PI controller is compared with RED scheme and the PI controller in Ref. [12] using ns-2 simulator. Finally, conclusions are given in Sect. 5.

2 TCP flow-control model

A non-linear dynamic model for TCP flow control was developed in Ref. [11] using the fluid-flow and stochastic differential equation analysis. Simulation results demonstrate that this model accurately captures the dynamics of TCP. The model can be described by the following non-linear differential equations:

$$\dot{W}(t)=\frac{1}{R(t)}-\frac{\mathrm{W(t)}}{2}\frac{W(t-R(t))}{R(t-R(t))}p(t-R(t)),\qquad\qquad (1)$$
$$\dot{q}(t)=\frac{N(t)}{R(t)}W(t)-C,$$

where $\dot{W}(t)=\mathrm{d}W(t)/\mathrm{d}t$; $\dot{q}(t)=\mathrm{d}q(t)/\mathrm{d}t$ and W: average TCP window size (packet); q: average queue length (packet); $R(t)$: round-trip time $=[q(t)/C+T_p]$ (s); C: link capacity (packet/s);T_p: propagation delay (s); N: load factor (number of TCP sessions); p: probability of packet mark.

The first differential equation describes the TCP window control dynamic. The second equation models the bottleneck queue length. The queue length q and window-size W are positive, bounded quantities: $q\in[0,\bar{q}]$ and $W\in[0,\bar{W}]$ where $\bar{q}$ and $\bar{W}$ denote buffer capacity and maximum window size respectively. Also, the marking probability p assigns value only in [0,1]. This non-linear and time-varying model was approximated as a linear constant system by small-signal linearization theory about an operating point [12]. The details about modeling and linearization can be seen in Refs. [11,12]. The linearized TCP/AQM dynamics are illustrated and simplified in a block diagram in Fig. 1.

It can be seen from Fig. 1 that the TCP/AQM model, denoted by the transfer function $P_{\mathrm{AQM}}(s)$, can be expressed as:

$$P_{\mathrm{AQM}}(s)P(s)\mathrm{e}^{-sR_0}P_{\mathrm{queue}}(s)P_{\mathrm{TCP}}(s)\mathrm{e}^{-sR_0}$$
$$=\frac{K}{(s+T_1)(s+T_2)}\mathrm{e}^{-sR_0},\qquad (2)$$

where $K=C^2/2N$; $T_1=1/R_0$; $T_2=2N/R_0{}^2C$. Here $P_{\mathrm{TCP}}(s)$

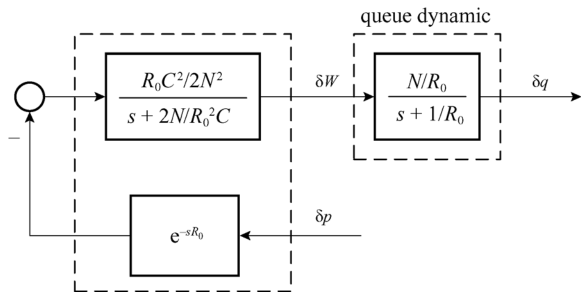

Fig. 1 Block diagram of the linearized TCP flow-control model

denotes the transfer function from loss probability δp to window size δW and $P_{\mathrm{queue}}(s)$ relates δW to queue length δq. The term e^{-sR_0} is the Laplace transform of the time delay in the delayed loss probability $\delta p(\mathrm{t}-R_0)$.

As a numerical illustration considering the case when $q_0=175$ packet, $T_p=0.2$ s and $C=3750$ packet/s. Then, for a load of $N=60$ TCP sessions, we have $W_0=15$ packet, $p_0=0.008$ and $R_0=0.246$, and

$$P_{\mathrm{AQM}}(s)=P(s)\mathrm{e}^{-sR_0}=\frac{1.17126\times10^5}{(s+4.1)(s+0.53)}\mathrm{e}^{-0.246s}.\quad (3)$$

For a load of $N=120$ TCP sessions, we have $W_0=7.7$ packet, $p_0=0.034$ and

$$P_{\mathrm{AQM}}(s)=P(s)\mathrm{e}^{-sR_0}=\frac{5.8320\times10^4}{(s+4.1)(s+1.05)}\mathrm{e}^{-0.246s}.\quad (4)$$

It is shown that the linearized TCP/AQM model is a linear second-order plant with time delay. There are two left-half-plane poles: $-2N/R_0{}^2C$ and $-1/R_0$. The plant relates how this packet-marking probability p dynamically affects the queue length q. This linear constant model is useful and helpful to analyze and explain the instability of RED under some network parameter configuration [1,12]. Furthermore, this model allows us to analyze and design new AQM control schemes from a control engineering standpoint, which shall be our work in this paper.

3 AQM control system design

In this section, the conditions that guarantee the internal stability and asymptotical property of the AQM closed-loop system will be investigated first. Then, we design a robust PI controller based on H_∞ control theory.

3.1 Internal stability and asymptotical property of the AQM control system

In Fig. 2 we give a closed-loop feedback control system depiction of AQM. The action of an AQM controller is to mark packets (with probability p) as a function of measured queue length q. Here $P(s)\mathrm{e}^{-sR_0}$ is the controlled

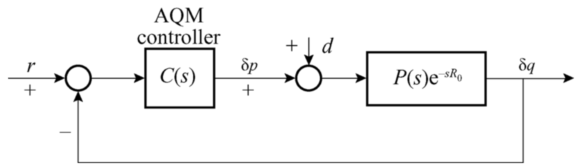

Fig. 2 Block diagram of AQM control system

plant, which denotes the previously derived small-signal linearization of TCP-queue dynamics. The reference input r is the expected queue length. The disturbance signal d includes the variations in TCP window size. The transfer function $C(s)$ denotes an AQM control strategy. In the AQM control system, PI controller is more appropriate for use than PID controller because the queue length signal is typically quite noisy. Hence, in this paper we choose $C(s)$ as a PI controller.

Assume that the model is a perfect representation of the real dynamics of TCP, i.e. in the nominal case. The unity feedback loop can be equivalently described in an internal model control (IMC) structure (see Fig. 3) through [13]

$$C(s)=\frac{Q(s)}{1-P_{\mathrm{AQM}}(s)Q(s)}, \tag{5}$$

where $Q(s)$ represents the IMC controller.

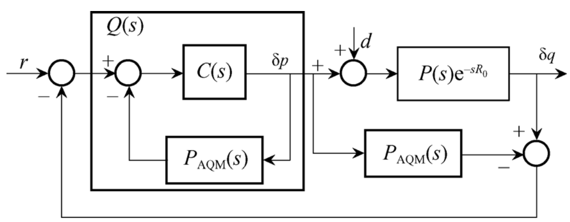

Fig. 3 Internal model control structure of AQM

Denote with $\boldsymbol{H}(s)$ the transfer function matrix from r and d to δq and δp. It follows that

$$\boldsymbol{H}(s)=\begin{bmatrix} P_{\mathrm{AQM}}(s)Q(s) & P_{\mathrm{AQM}}(s)\left(1-P_{\mathrm{AQM}}(s)Q(s)\right) \\ Q(s) & -P_{\mathrm{AQM}}(s)Q(s) \end{bmatrix}. \tag{6}$$

The closed-loop system is internal stable if all the transfer functions in $\boldsymbol{H}(s)$ are stable. Since $P_{\mathrm{AQM}}(s)$ is a stable plant, the sufficient and necessary condition that guarantees the internal stability of closed-loop system with $P_{\mathrm{AQM}}(s)$ is that $Q(s)$ is stable.

The disturbances encountered in the AQM include the variations in TCP window size, which can be approximated by a combination of many steps or ramps. As a basic closed loop performance specification, the AQM control system should satisfy the requirement of asymptotical disturbance rejection property when the closed-loop system is internal stable. The necessary and sufficient condition that guarantees the asymptotical property is [13]

$$\lim_{s\to 0}\left(1-P_{\mathrm{AQM}}(s)Q(s)\right)=0. \tag{7}$$

3.2 Design procedure of H_∞ PI controller

It is clear that the previous TCP/AQM model is a stable plant with time delay. The classical PI controller design procedures for this kind of plants are based on optimization techniques and rational approximations. The first step is to expand the time delay by rational approximations, and then the optimal controllers are derived. In this section, we will develop an analytical design procedure of PI controller in which the time delay remains in the design, but will be approximated optimally in the realization of the controller. The difference between the practically implemented PI controller and the optimal controller will be made as small as possible.

Consider the second-order model of TCP/AQM plant given in Eq. (2). Inspired by H_∞ optimization and IMC [13] theory, we propose an expected closed-loop transfer function $T(s) = P_{\mathrm{AQM}}(s)Q(s)$. Here, the H_∞ optimal performance criterion $\min\|W(s)(1 - P_{\mathrm{AQM}}(s)Q(s))\|_\infty$ is utilized to obtain the controller, where W is the weighting function and can be chosen as $1/s$ for the step change of the disturbances in the TCP/AQM control system. Then the expected closed-loop transfer function can be designed as:

$$T(s)=\frac{\mathrm{e}^{-sR_0}}{(\lambda s+1)^n}, \tag{8}$$

where n is chosen so that the controller is bi-proper, that is, both $Q(s)$ and $1/Q(s)$ are proper. Here λ is a positive constant parameter. By tuning λ, one can adjust the nominal performance and robust performance monotonically. In the case of plant and model being perfectly matched, i.e. no dynamic perturbation, the nominal performance can arbitrarily approach the optimality simply by decreasing λ.

For the nominal AQM model $P_{\mathrm{AQM}}(s)$ in Eq. (2) we have

$$Q(s)=\frac{(s+T_1)(s+T_2)}{K(\lambda s+1)^n}. \tag{9}$$

It is seen that $Q(s)$ is stable, which guarantees the internal stability of the closed-loop control system. We choose the minimum value of n, i.e. $n = 2$ and it yields

$$Q(s)=\frac{(s+T_1)(s+T_2)}{K(\lambda s+1)^2}. \tag{10}$$

Hence, the closed-loop transfer function becomes $T(s) = \mathrm{e}^{-sR_0}/(\lambda s + 1)^2$. The corresponding AQM controller of unity feedback system is:

$$C(s)=\frac{1}{K}\frac{(s+T_1)(s+T_2)}{(\lambda s+1)^2-\mathrm{e}^{-sR_0}}. \tag{11}$$

Since the condition of asymptotical disturbances rejection results in

$$\lim_{s\to 0}\left((\lambda s+1)^2-e^{-sR_0}\right)=0. \quad (12)$$

It is clear that $C(s)$ possesses one pole at the origin. So $C(s)$ can be realized as a PI controller in discrete form or approximated by a rational transfer function. Therefore, the mathematical Maclaurin expansion formula is employed to reproduce the PI controller in a simple way [14,15]. $C(s)$ can be written as:

$$C(s)=\frac{f(s)}{s}, \quad (13)$$

where

$$f(s)=\frac{s}{K}\frac{(s+T_1)(s+T_2)}{(\lambda s+1)^2-e^{-sR_0}}. \quad (14)$$

Here we expand it in Maclaurin series as:

$$C(s)=\frac{1}{s}\left[f(0)+f'(0)s+\frac{f''(0)}{2!}s^2+\cdots\right]. \quad (15)$$

The first two terms of the above expansion can be interpreted as the standard PI controller given by

$$C_{\mathrm{PI}}(s)=K_{\mathrm{C}}\left(1+\frac{1}{T_{\mathrm{I}}s}\right), \quad (16)$$

where

$$T_{\mathrm{I}}=T_1+T_2-\frac{2\lambda^2-R_0^2}{2(2\lambda+R_0)} \quad (17)$$

and

$$K_{\mathrm{C}}=\frac{T_{\mathrm{I}}}{K(2\lambda+R_0)}. \quad (18)$$

Since all the parameters are known, the computation is in fact very simple. It is obvious that the proposed PI controller is tuned by a single adjustable parameter λ. In fact, the tuning of λ corresponds to the trade-off between the performance and robust stability of AQM closed-loop control system. As λ increases, the system can tolerate larger perturbations but with a worse performance. In other words, the larger plant/model mismatch can be addressed by a larger λ. As λ decreases, the system tends to have better performance but with a narrower robust stability margin. In other words, it can obtain faster transient response and less oscillation, which is translated in the context of network as higher link utilization, a lower packet loss rate and small queue fluctuations. In this way, the proposed PI controller can be practically and conveniently tuned by λ to obtain the required design performance and robustness specifications.

4 Simulations and performance evaluation

We evaluate the effectiveness and performance of the proposed PI controller by simulations using the ns-2 simulator. Although the preceding analysis was carried out using the linearized model in Eq. (2), the ns-2 simulator captures the stochastic, nonlinear nature of the dynamic network. The network dumbbell topology is shown in Fig. 4. The only bottleneck link lies in between node R_1 and node R_2. The buffer size of all nodes is 600 packets, and the default size of the packet is 500 B. The queue in node R_1 is controlled by PI and new PI schemes respectively. The others are drop tails.

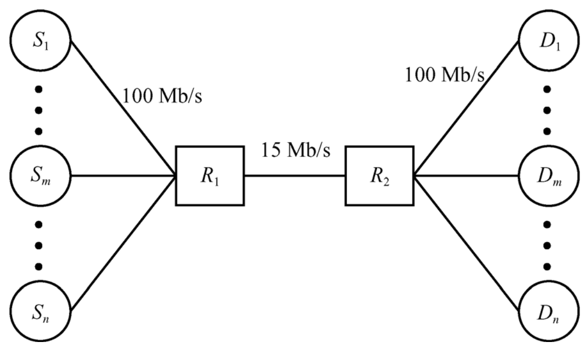

Fig. 4 Simulation of network dumbbell topology

4.1 Experiment 1

In the first simulation, we will compare the dynamic responses of the new PI control scheme with that of Hollot's PI controller in Ref. [12]. We introduce 60 TCP flows and the simulation time is 25 s. The capacity C is 15 Mb/s and the propagation delay T_{p} ranges uniformly between 160 ms and 240 ms. The TCP flows transmit data with an average packet size of 500 B. In Hollot's PI control scheme, the sample frequency is 160 Hz. Let the expected queue length be equal to 200 packets. In the proposed PI controller, we take $\lambda = 0.4R_0$. Derived from Eqs. (17) and (18), the PI controller parameters are $K_c = 8.49621 \times 10^{-5}$ and $T_{\mathrm{I}} = 2.30674$. The instantaneous queue length is shown in Fig. 5. It is obvious that the new PI controller shows a better response than the PI controller in Ref. [12]. The settling time of the new PI controller is obviously shorter than RED. The new PI controller show less oscillation than Hollot's PI. Furthermore, the average queue size of the proposed PI controller is smaller than that of the PI controller in Ref. [12].

4.2 Experiment 2

We now increase the number of TCP flows to 90. According to theoretical analysis, the response should be slower for this higher load level N. The queue lengths

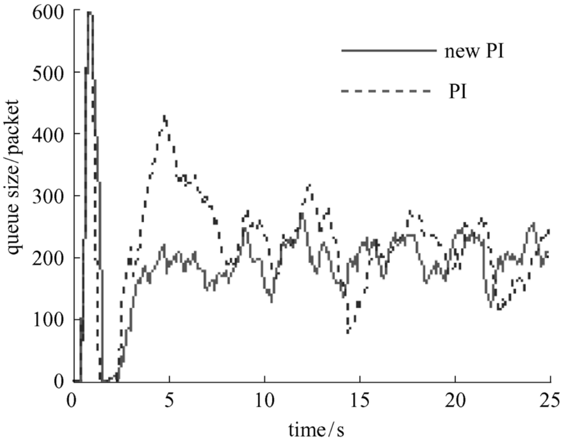

Fig. 5 Simulation results of Experiment 1

are plotted in Fig. 6. Compared with the PI controller in Ref. [12], the new PI controller has better nominal performance. On the other hand, the new PI controller still can regulate the queue length to 200 packets quickly, which shows better robustness.

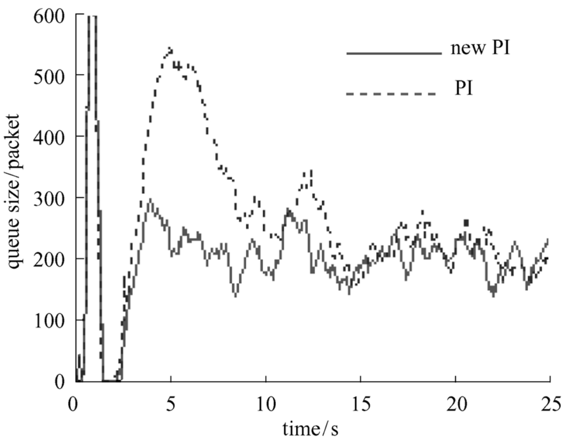

Fig. 6 Simulation results of Experiment 2

4.3 Experiment 3

On the basis of Experiment 2, to achieve a better performance, we tune the parameter λ from $0.4R_0$ to $0.2R_0$. The corresponding test results are displayed in Fig. 7. We can see clearly from Figs. 6 and 7 that the proposed PI controller can be adjusted easily when the network environment is changed.

4.4 Experiment 4

The new PI controller has an important feature that it can be tuned using a single adjustable parameter λ. To provide a more comprehensive understanding of the proposed control scheme, in this simulation, the tuning rule of λ is studied. By increasing λ in the proposed PI controller, the robust stability

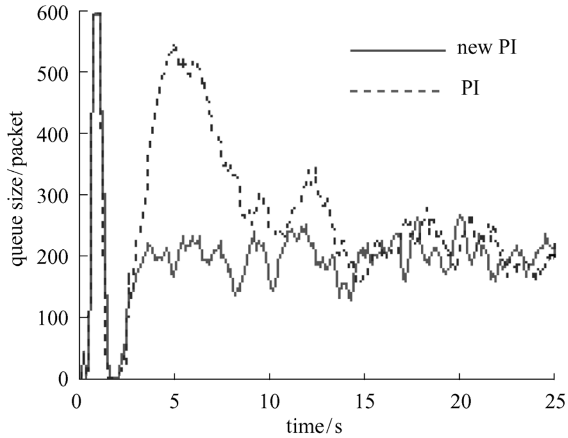

Fig. 7 Simulation results of Experiment 3

could become better at the cost of degrading the performance. To illustrate this, we carry out this simulation experiment. The network dumbbell topology and parameter settings are the same to that of Experiment 1. The sample frequency is 160 Hz. Let the expected queue length equal to 200 packets. We introduce 120 TCP flows and the simulation time is 100 s. When $t = 50$ s, 40 TCP flows are lost. The parameters of the PI controller corresponding to the adjusting parameter λ are listed in Table 1, and the dynamic responses are shown in Fig. 8. The relationship of the mark/drop probability with time is illustrated in Fig. 9.

5 Conclusions

In this paper, a robust PI control scheme for AQM routers supporting TCP flows has been proposed on the basis of H_∞ optimization and IMC theory. The main contributions of the paper are summarized as follows:

Table 1 Parameters of PI controller with adjusted λ

λ/R_0	$K_C/\times10^{-4}$	T_I
0.1	3.0045	1.3595
2.0	0.57216	1.0787
10	0.0070457	0.055793

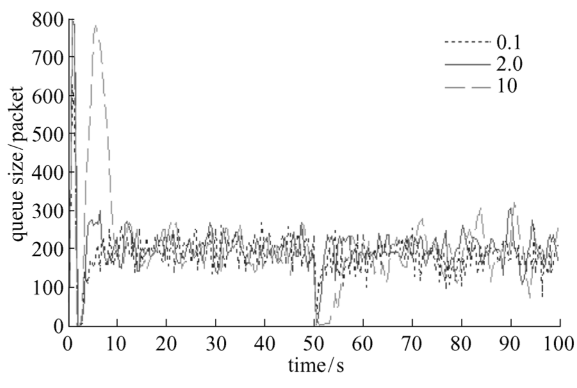

Fig. 8 Queue evolution of Experiment 4

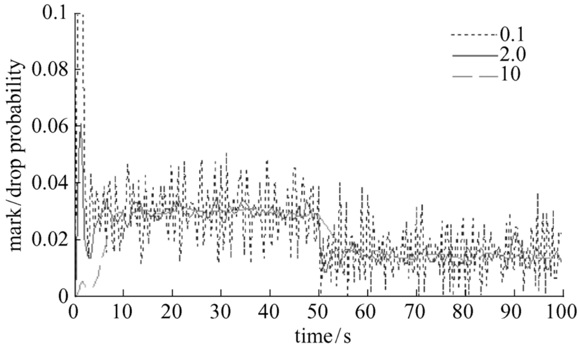

Fig. 9 Mark/drop probability of Experiment 4

1) The internal stability and asymptotical properties of the AQM control system are investigated so that the proposed PI control scheme is able to guarantee the stability of the instantaneous queue and to adapt to the variations in TCP window size.

2) The proposed PI control scheme is simple to implement and the trade-off between the nominal performance and robust stability of AQM closed-loop control system can be accomplished by tuning only one adjustable parameter λ.

3) The simulation experiments using ns-2 simulator show that the proposed AQM scheme has better performance than RED and Hollot's PI control scheme. It can result in higher link utilization, much lower packet loss rate and smaller queue fluctuations.

Acknowledgements This work was supported by the Specialized Research Fund for the Doctoral Program of Higher Education (No. 20070248010), NCET (No. 04-0383) and Australia-China Special Fund for Scientific & Technological Cooperation (No. 071107037).

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