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RESEARCH ARTICLE

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Delay-dependent robust H_∞ controller design for a class of nonlinear uncertainty time-delay systems with input delay

Abstract Based on an appropriate Lyapunov function, this paper analyzes the design of a delay-dependent robust H_∞ state feedback control, with a focus on a class of nonlinear uncertainty linear time-delay systems with input delay using linear matrix inequalities. Under the condition that the nonlinear uncertain functions are gain bounded, a sufficient condition dependent on the delays of the state and input is presented for the existence of H_∞ controller. The proposed controller not only stabilized closed-loop uncertain systems but also guaranteed a prescribed H_∞ norm bound of closed-loop transfer matrix from the disturbance to controlled output. By solving a linear matrix inequation, we can obtain the robust H_∞ controller. An example is given to show the effectiveness of the proposed method.

Keywords non-linear uncertainty, time-delay system, linear matrix inequation, H_∞ control

1 Introduction

Time-delay is frequently encountered in a variety of dynamic systems, such as nuclear reactors, chemical engineering systems, biological systems, and population dynamics models. They are often a source of instability and degradation in control performance in many control systems. The analysis of the stability of dynamic control systems with delay and the synthesis of controllers for them are important both in theory and in practice. This has raised much interest from many researchers.

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In recent years, the problem of robust H_∞ control of time-delay systems has also received considerable attention. In Refs. [1,2], for example, memoryless H_∞ state feedback controllers are obtained. In Refs. [3–5], robust H_∞ state feedback controllers design is researched. In Ref. [5], based on the Riccati equation approach, the static state feedback robust controller of uncertain systems with delay in state and input is presented, while in which nonlinear uncertainty is not considered. In Ref. [6], robust stability problems of nonlinear uncertain time-delay systems are investigated. However, all these conclusions of literatures are independent of the size of the delay. Thus, in general, they are to some extent conservative. Therefore, efforts have been devoted to the research on the design of delay-dependent controllers, such as the research in Refs. [7–10], but they did not consider the case of delay in the control input. On the other hand, in Ref. [10], nonlinear uncertainty and state delay term were not considered.

This paper mainly contributes to stating linear matrix inequation (LMI) sufficient delay-dependent conditions for the robust H_∞ state feedback control design, which guarantees an H infinite-level of disturbance attenuation for nonlinear uncertain continuous delay systems with delay in the control input. The approach allows for including lumped time delays in the state vector and the control input.

Based on selected appropriate Lyapunov–Krasovskii functional, a novel delay-dependent sufficient condition for robust H_∞ controller design of nonlinear uncertain delay systems is derived in terms of LMIs. Virtually, this paper is an immediate extension of Refs. [4,6,10]. Finally, simulation results display the effectiveness of the method.

2 Problem statement

Consider the following nonlinear uncertain system with time-varying state and input delays:

$$\dot{\boldsymbol{x}}(t)=\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{f}_1(\boldsymbol{x}(t),t)+\boldsymbol{A}_d\boldsymbol{x}(t-\tau)+\boldsymbol{f}_{1d}(\boldsymbol{x}(t-\tau),t)+\boldsymbol{B}\boldsymbol{w}(t)+\boldsymbol{B}_1\boldsymbol{u}(t)+\boldsymbol{B}_2\boldsymbol{u}(t-d), \tag{1a}$$

$$\boldsymbol{z}(t)=\boldsymbol{C}\boldsymbol{x}(t), \tag{1b}$$

$$\boldsymbol{x}(t)=\phi(t), \quad t\in[-\tau,0], \tag{1c}$$

where $\boldsymbol{x}(t)\in R^n$ and $\boldsymbol{u}(t)\in R^m$ denote the state and control inputs respectively; $\boldsymbol{z}(t)\in R^r$ is the controllable output; $\boldsymbol{A}$, $\boldsymbol{A}_d$, $\boldsymbol{B}$, $\boldsymbol{B}_1$, $\boldsymbol{B}_2$ are known constant matrices of appropriate dimensions; $\boldsymbol{w}(t)\in R^p$ is the exogenous disturbance vector; Scalar τ denotes the known state delay; d denotes the known input delay; $\phi(t)$ is the continuous vector-valued initial condition; the uncertainties $\boldsymbol{f}_1$ and $\boldsymbol{f}_{1d}$ are unknown and represent the nonlinear parameter perturbations with respect to the current state $\boldsymbol{x}(t)$ and the delayed state $\boldsymbol{x}(t-\tau)$ respectively. In general, it is assumed that $\boldsymbol{f}_1(\boldsymbol{x}(t),t)$ and $\boldsymbol{f}_{1d}(\boldsymbol{x}(t-\tau),t)$ satisfy the following norm-bounded condition:

$$\|\boldsymbol{f}_1(\boldsymbol{x}(t),t)\|\leqslant\|\boldsymbol{W}_1\boldsymbol{x}(t)\|, \tag{2}$$

$$\|\boldsymbol{f}_{1d}(\boldsymbol{x}(t-\tau),t)\|\leqslant\|\boldsymbol{W}_{1d}\boldsymbol{x}(t-\tau)\|. \tag{3}$$

Assume that $\boldsymbol{w}(t)=\boldsymbol{0}$, $\boldsymbol{u}(t)=\boldsymbol{0}$, then free Eq. (1a) has the solution when satisfying Eqs. (2) and (3) and the initial condition Eq. (1c).

In fact, Eqs. (2) and (3) implicate that $\boldsymbol{f}_1(\boldsymbol{0},t)=\boldsymbol{0}$, $\boldsymbol{f}_{1d}(\boldsymbol{0},t)=\boldsymbol{0}$, i.e., $\boldsymbol{x}(t)=\boldsymbol{0}$ is a equilibrium point of free system.

This paper chiefly aims to design a delay-dependent state feedback controller:

$$\boldsymbol{u}(t)=\boldsymbol{K}\boldsymbol{x}(t). \tag{4}$$

For a given $\gamma>0$, so as to make the closed loop system (see Eq. (5)) of Eqs. (1) and (4)

$$\dot{\boldsymbol{x}}(t)=(\boldsymbol{A}+\boldsymbol{B}_1\boldsymbol{K})\boldsymbol{x}(t)+\boldsymbol{A}_d\boldsymbol{x}(t-\tau)+\boldsymbol{B}_2\boldsymbol{K}\boldsymbol{x}(t-d)+\boldsymbol{B}\boldsymbol{w}(t)+\boldsymbol{f}_1(x(t),t)+\boldsymbol{f}_{1d}(x(t-\tau),t), \tag{5}$$

robust asymptotically stable, and H_∞ performance index less than the given bound γ.

Since $\boldsymbol{x}(t-\tau)=\boldsymbol{x}(t)-\int_{t-\tau}^{t}\xi(\theta)\mathrm{d}\theta$, where $\boldsymbol{\xi}(\theta)=(\boldsymbol{A}+\boldsymbol{B}_1\boldsymbol{K})\boldsymbol{x}(\theta)+\boldsymbol{f}_1(\boldsymbol{x}(\theta),\theta)+\boldsymbol{A}_d\boldsymbol{x}(\theta-\tau)+\boldsymbol{B}_2\boldsymbol{K}\boldsymbol{x}(\theta-d)+\boldsymbol{f}_{1d}(\boldsymbol{x}(\theta-\tau),\theta)+\boldsymbol{B}\boldsymbol{w}(\theta)$. Eq. (5) can be translate into

$$\dot{\boldsymbol{x}}(t)=\bar{\boldsymbol{A}}\boldsymbol{x}(t)+\boldsymbol{B}_2\boldsymbol{K}\boldsymbol{x}(t-d)+\boldsymbol{B}\boldsymbol{w}(t)+\boldsymbol{f}_1(x(t),t)+\boldsymbol{f}_{1d}(x(t-\tau),t)-\boldsymbol{A}_d\int_{t-\tau}^{t}\xi(\theta)\mathrm{d}\theta, \tag{6}$$

where $\bar{\boldsymbol{A}}=(\boldsymbol{A}+\boldsymbol{B}_1\boldsymbol{K})+\boldsymbol{A}_d=\tilde{\boldsymbol{A}}+\boldsymbol{A}_d$.

3 Main results

Initially, we will consider the problem of robust stability problem for the time-delay system Eq. (1), and obtain the following main conclusions.

Theorem 1 Consider the nonlinear uncertain time-delay system Eq. (1), if there exist a matrix $\boldsymbol{K}$ positive definite matrices $\boldsymbol{P}>\boldsymbol{0}$, $\boldsymbol{Q}_1>\boldsymbol{0}$, $\boldsymbol{Q}_2>\boldsymbol{0}$, $\boldsymbol{R}>\boldsymbol{0}$, and non-zero positive constants λ_1, λ_{1d} that make the following LMI

$$\begin{bmatrix} \boldsymbol{M}_{11} & \boldsymbol{M}_{12} & \boldsymbol{M}_{13} & \boldsymbol{M}_{14} \\ \boldsymbol{M}_{12}^{\mathrm{T}} & \boldsymbol{M}_{22} & \tau\boldsymbol{A}_d^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{B}_2\boldsymbol{K} & \tau\boldsymbol{A}_d^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{B}_s \\ \boldsymbol{M}_{13}^{\mathrm{T}} & \tau\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}_2^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{A}_d & \boldsymbol{M}_{33} & \tau\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}_2^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{B}_s \\ \boldsymbol{M}_{14}^{\mathrm{T}} & \tau\boldsymbol{B}_s^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{A}_d & \tau\boldsymbol{B}_s^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{B}_2\boldsymbol{K} & -\boldsymbol{I}+\tau\boldsymbol{B}_s^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{B}_s \end{bmatrix}<\boldsymbol{0} \tag{7}$$

holds, where

$$\boldsymbol{M}_{11}=\boldsymbol{P}\bar{\boldsymbol{A}}+\bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}+\tau\tilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{R}^{-1}\tilde{\boldsymbol{A}}+\tau\boldsymbol{P}\boldsymbol{A}_d\boldsymbol{R}\boldsymbol{A}_d^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{Q}_1+\boldsymbol{Q}_2+\frac{1}{\lambda_1^2}\boldsymbol{W}_1^{\mathrm{T}}\boldsymbol{W}_1,$$

$$\boldsymbol{M}_{12}=\tau(\boldsymbol{A}+\boldsymbol{B}_1\boldsymbol{K})^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{A}_d,$$

$$\boldsymbol{M}_{13}=\tau(\boldsymbol{A}+\boldsymbol{B}_1\boldsymbol{K})^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{B}_2\boldsymbol{K}+\boldsymbol{P}\boldsymbol{B}_2\boldsymbol{K},$$

$$\boldsymbol{M}_{14}=\tau(\boldsymbol{A}+\boldsymbol{B}_1\boldsymbol{K})^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{B}_s+\boldsymbol{P}\boldsymbol{B}_s,$$

$$\boldsymbol{M}_{22}=-\boldsymbol{Q}_1+\frac{1}{\lambda_{1d}^2}\boldsymbol{W}_{1d}^{\mathrm{T}}\boldsymbol{W}_{1d}+\tau\boldsymbol{A}_d^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{A}_d,$$

$$\boldsymbol{M}_{33}=-\boldsymbol{Q}_2+\tau\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}_2^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{B}_2\boldsymbol{K},$$

$$\boldsymbol{B}_s=[\lambda_1\boldsymbol{I},\lambda_{1d}\boldsymbol{I}].$$

It is safe to say that time-delay system Eq. (1) are robustly stable.

Proof For system Eq. (6), define a Lyapunov–Krasovskii function as

$$V(\boldsymbol{x}(t),t)=\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{x}(t)+\int_{t-\tau}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}_1\boldsymbol{x}(s)\mathrm{d}s+\int_{t-d}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}_2\boldsymbol{x}(s)\mathrm{d}s+\int_{-\tau}^{0}\mathrm{d}s\int_{t+s}^{t}\xi^{\mathrm{T}}(\theta)\boldsymbol{R}^{-1}\xi(\theta)\mathrm{d}\theta$$
$$+\int_0^t\left[\left\|\frac{1}{\lambda_1}\boldsymbol{W}_1\boldsymbol{x}(s)\right\|^2-\left\|\frac{1}{\lambda_1}\boldsymbol{f}_1(\boldsymbol{x}(s),s)\right\|^2\right]\mathrm{d}s+\int_0^t\left[\left\|\frac{1}{\lambda_{1d}}\boldsymbol{W}_{1d}\boldsymbol{x}(s-\tau)\right\|^2-\left\|\frac{1}{\lambda_{1d}}\boldsymbol{f}_{1d}(\boldsymbol{x}(s-\tau),s)\right\|^2\right]\mathrm{d}s.$$

Then, the time derivative of $V(\boldsymbol{x}(t),t)$ along the trajectory Eq. (6) satisfies

$$\begin{aligned}\dot V(\boldsymbol{x}(t),t)=&\,\dot{\boldsymbol{x}}(t)\boldsymbol{P}\boldsymbol{x}(t)+\boldsymbol{x}(t)\boldsymbol{P}\dot{\boldsymbol{x}}(t)+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_1\boldsymbol{x}(t)-\boldsymbol{x}^{\mathrm{T}}(t-\tau)\boldsymbol{Q}_1\boldsymbol{x}(t-\tau)+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_2x(t)-\boldsymbol{x}^{\mathrm{T}}(t-d)\boldsymbol{Q}_2\boldsymbol{x}(t-d)\\&+\frac{1}{\lambda_1^2}\|\boldsymbol{W}_1\boldsymbol{x}(t)\|^2-\frac{1}{\lambda_1^2}\|\boldsymbol{f}_1(\boldsymbol{x}(t),t)\|^2+\frac{1}{\lambda_{1d}^2}\|\boldsymbol{W}_{1d}\boldsymbol{x}(t-\tau)\|^2-\frac{1}{\lambda_{1d}^2}\|\boldsymbol{f}_{1d}(\boldsymbol{x}(t-\tau),t)\|^2+\tau\xi^{\mathrm{T}}(t)\boldsymbol{R}^{-1}\xi(t)\\&-\int_{t-\tau}^{t}\xi^{\mathrm{T}}(\theta)\boldsymbol{R}^{-1}\xi(\theta)\mathrm{d}\theta\\=&\,\boldsymbol{x}^{\mathrm{T}}(t)(\boldsymbol{P}\bar{\boldsymbol{A}}+\bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P})\boldsymbol{x}(t)+2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}(\boldsymbol{f}_1(\boldsymbol{x}(t),t)+\boldsymbol{f}_{1d}(\boldsymbol{x}(t-\tau),t))+2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{B}\boldsymbol{w}(t)+2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{B}_2K\boldsymbol{x}(t-d)\\&-2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{A}_d\int_{t-\tau}^{t}\xi(\theta)\mathrm{d}\theta+\boldsymbol{x}^{T}(t)\boldsymbol{Q}_1\boldsymbol{x}(t)-\boldsymbol{x}^{\mathrm{T}}(t-\tau)\boldsymbol{Q}_1\boldsymbol{x}(t-\tau)+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_2\boldsymbol{x}(t)-\boldsymbol{x}^{\mathrm{T}}(t-d)\boldsymbol{Q}_2\boldsymbol{x}(t-d)\\&+\frac{1}{\lambda_1^2}\|\boldsymbol{W}_1\boldsymbol{x}(t)\|^2-\frac{1}{\lambda_1^2}\|\boldsymbol{f}_1(\boldsymbol{x}(t),t)\|^2+\frac{1}{\lambda_{1d}^2}\|\boldsymbol{W}_{1d}\boldsymbol{x}(t-\tau)\|^2-\frac{1}{\lambda_{1d}^2}\|\boldsymbol{f}_{1d}(\boldsymbol{x}(t-\tau),t)\|^2+\tau\xi^{\mathrm{T}}(t)\boldsymbol{R}^{-1}\xi(t)\\&-\int_{t-\tau}^{t}\xi^{\mathrm{T}}(\theta)\boldsymbol{R}^{-1}\xi(\theta)\mathrm{d}\theta,\end{aligned}$$

and

$$-2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{A}_d\int_{t-\tau}^{t}\xi(\theta)\mathrm{d}\theta\leqslant\tau\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{A}_d\boldsymbol{R}\boldsymbol{A}_d^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}(t)+\int_{t-\tau}^{t}\xi^{\mathrm{T}}(\theta)\boldsymbol{R}^{-1}\xi(\theta)\mathrm{d}\theta,$$

result in

$$\dot V\leqslant\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{M}\tilde{\boldsymbol{x}}(t),$$

where

$$\tilde{\boldsymbol{x}}(t)=\begin{bmatrix}\boldsymbol{x}^{\mathrm{T}}(t) & \boldsymbol{x}^{\mathrm{T}}(t-\tau) & \boldsymbol{x}^{\mathrm{T}}(t-d) & \bar{\boldsymbol{f}}^{\mathrm{T}} & \boldsymbol{w}^{\mathrm{T}}(t)\end{bmatrix}^{\mathrm{T}},$$
$$\bar{\boldsymbol{f}}^{\mathrm{T}}=\left[\frac{1}{\lambda_1}(\boldsymbol{f}_1(\boldsymbol{x}(t),t))^{\mathrm{T}},\ \frac{1}{\lambda_{1d}}(\boldsymbol{f}_{1d}(\boldsymbol{x}(t-\tau),t))^{\mathrm{T}}\right],$$

$$\boldsymbol{M}=\begin{bmatrix}\boldsymbol{M}_{11} & \boldsymbol{M}_{12} & \boldsymbol{M}_{13} & \boldsymbol{M}_{14} & \boldsymbol{M}_{15}\\ \boldsymbol{M}_{12}^{\mathrm{T}} & \boldsymbol{M}_{22} & \tau\boldsymbol{A}_d^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{B}_2\boldsymbol{K} & \tau\boldsymbol{A}_d^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{B}_s & \tau\boldsymbol{A}_d^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{B}\\ \boldsymbol{M}_{13}^{\mathrm{T}} & \tau\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}_2^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{A}_d & \boldsymbol{M}_{33} & \tau\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}_2^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{B}_s & \tau\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}_2^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{B}\\ \boldsymbol{M}_{14}^{\mathrm{T}} & \tau\boldsymbol{B}_s^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{A}_d & \tau\boldsymbol{B}_s^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{B}_2\boldsymbol{K} & -\boldsymbol{I}+\tau\boldsymbol{B}_s^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{B}_s & \tau\boldsymbol{B}_s^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{B}\\ \boldsymbol{M}_{15}^{\mathrm{T}} & \tau\boldsymbol{B}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{A}_d & \tau\boldsymbol{B}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{B}_2\boldsymbol{K} & \tau\boldsymbol{B}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{B}_s & \tau\boldsymbol{B}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{B}\end{bmatrix}, \tag{8}$$

$\boldsymbol{M}_{11}$, $\boldsymbol{M}_{12}$, $\boldsymbol{M}_{13}$, $\boldsymbol{M}_{14}$, $\boldsymbol{M}_{22}$ and $\boldsymbol{M}_{33}$ are defined in Theorem 1.

$$\boldsymbol{M}_{15}=\tau(\boldsymbol{A}+\boldsymbol{B}_1\boldsymbol{K})^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{B}+\boldsymbol{P}\boldsymbol{B}.$$

Based on the above reasoning process, we know if $\boldsymbol{w}(t)=\boldsymbol{0}$ and matrix inequation (7) holds, then $\dot V<0$, and system Eq. (6) is asymptotically stable. Therefore, time-delay system Eq. (1) is robustly stabilized. The proof is completed. For H_∞ performance of system Eq. (6), let initial value $\phi(t)=0$, thus for any $T>0$, yield

$$J_T=\int_0^T\left(\boldsymbol{z}^{\mathrm{T}}\boldsymbol{z}-\gamma^2\boldsymbol{w}^{\mathrm{T}}\boldsymbol{w}\right)\mathrm{d}t\leqslant\int_0^T\left(\boldsymbol{z}^{\mathrm{T}}\boldsymbol{z}-\gamma^2\boldsymbol{w}^{\mathrm{T}}\boldsymbol{w}+\dot V(\boldsymbol{x}_t)\right)\mathrm{d}t\leqslant\int_0^T\tilde{\boldsymbol{x}}^{\mathrm{T}}(t)\tilde{\boldsymbol{M}}\,\tilde{\boldsymbol{x}}(t)\,\mathrm{d}t,$$

where

$$\tilde{\boldsymbol{M}}=\boldsymbol{M}+\mathrm{diag}\left(\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C},\ \boldsymbol{0},\ \boldsymbol{0},\ -\gamma^2\boldsymbol{I}\right).$$

Since when $\tilde{\boldsymbol{M}}<\boldsymbol{0}$, $J_T<0$ can be obtained and meanwhile matrix inequation (7) is guaranteed, we have following conclusions.

Theorem 2 Consider the nonlinear uncertain time-delay system Eq. (1), if there exist a matrix $\boldsymbol{K}$, positive definite matrices $\boldsymbol{P}>\boldsymbol{0}$, $\boldsymbol{Q}_1>\boldsymbol{0}$, $\boldsymbol{Q}_2>\boldsymbol{0}$, $\boldsymbol{R}>\boldsymbol{0}$, nonzero positive constants λ_1, λ_{1d} and a positive constant $\gamma>0$ that make linear matrix inequation $\tilde{\boldsymbol{M}}<\boldsymbol{0}$ hold, then it can be concluded that time-delay system Eq. (1) is robustly stable, and H_∞ performance index is less than the given bound γ.

According to Schur's lemma, for the solution of matrix inequation $\tilde{\boldsymbol{M}}<\boldsymbol{0}$, it is only necessary to solve inequality (9):

$$\begin{bmatrix} (1,1) & \boldsymbol{0} & \boldsymbol{PB}_2\boldsymbol{K} & \boldsymbol{PB}_s & \boldsymbol{PB} & \tau(\boldsymbol{A}+\boldsymbol{B}_1\boldsymbol{K})^{\mathrm{T}} & \tau\boldsymbol{PA}_d \\ * & -\boldsymbol{Q}_1+\dfrac{1}{\lambda_{1d}^2}\boldsymbol{W}_{1d}^{\mathrm{T}}\boldsymbol{W}_{1d} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \tau\boldsymbol{A}_d^{\mathrm{T}} & \boldsymbol{0} \\ * & * & -\boldsymbol{Q}_2 & \boldsymbol{0} & \boldsymbol{0} & \tau\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}_2^{\mathrm{T}} & \boldsymbol{0} \\ * & * & * & -\boldsymbol{I} & \boldsymbol{0} & \tau\boldsymbol{B}_s^{\mathrm{T}} & \boldsymbol{0} \\ * & * & * & * & -\gamma^2\boldsymbol{I} & \tau\boldsymbol{B}^{\mathrm{T}} & \boldsymbol{0} \\ * & * & * & * & * & -\tau\boldsymbol{R} & \boldsymbol{0} \\ * & * & * & * & * & * & -\tau\boldsymbol{P} \end{bmatrix}<\boldsymbol{0}, \tag{9}$$

where

$$\boldsymbol{R}\leqslant\boldsymbol{P}^{-1},$$

$$(1,1)=\boldsymbol{P}\bar{\boldsymbol{A}}+\bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{Q}_1+\boldsymbol{Q}_2+\frac{1}{\lambda_1^2}\boldsymbol{W}_1^{\mathrm{T}}\boldsymbol{W}_1+\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C},$$

and $\boldsymbol{X}=\boldsymbol{P}^{-1}$, $\boldsymbol{Y}=\boldsymbol{KX}$, $\tilde{\boldsymbol{Q}}_1=\boldsymbol{XQ}_1\boldsymbol{X}$, $\tilde{\boldsymbol{Q}}_2=\boldsymbol{XQ}_2\boldsymbol{X}$. Hence, by pre-multiplying and post-multiplying inequality (9) with $\mathrm{diag}\{\boldsymbol{X},\boldsymbol{X},\boldsymbol{X},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{X}\}$, and using Schur's lemma, it turns out that inequality (9) is equivalent to LMI inequality (11). At the same time, $\boldsymbol{X}$ and $\boldsymbol{R}$ satisfy inequality (10). The proof is completed.

$$\boldsymbol{R}\leqslant\boldsymbol{X}, \tag{10}$$

$$\begin{bmatrix} \psi_{11} & \boldsymbol{0} & \boldsymbol{B}_2\boldsymbol{Y} & \boldsymbol{B}_s & \boldsymbol{B} & \tau(\boldsymbol{AX}+\boldsymbol{B}_1\boldsymbol{Y})^{\mathrm{T}} & \tau\boldsymbol{A}_d\boldsymbol{X} & \dfrac{1}{\lambda_1}\boldsymbol{W}_1^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{XC}^{\mathrm{T}} \\ * & -\tilde{\boldsymbol{Q}}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \tau(\boldsymbol{A}_d\boldsymbol{X})^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} & \dfrac{1}{\lambda_{1d}}\boldsymbol{W}_{1d}^{\mathrm{T}} & \boldsymbol{0} \\ * & * & -\tilde{\boldsymbol{Q}}_2 & \boldsymbol{0} & \boldsymbol{0} & \tau(\boldsymbol{B}_2\boldsymbol{Y})^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & -\boldsymbol{I} & 0 & \tau\boldsymbol{B}_s^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & * & -\gamma^2\boldsymbol{I} & \tau\boldsymbol{B}^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & * & * & -\tau\boldsymbol{R} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & * & * & * & -\tau\boldsymbol{X} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & * & * & * & * & -\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & * & * & * & * & * & -\boldsymbol{I} & \boldsymbol{0} \\ * & * & * & * & * & * & * & * & * & -\boldsymbol{I} \end{bmatrix}<\boldsymbol{0}, \tag{11}$$

where $\psi_{11}=(\boldsymbol{A}+\boldsymbol{A}_d)\boldsymbol{X}+(\boldsymbol{A}+\boldsymbol{A}_d)^{\mathrm{T}}\boldsymbol{X}+\boldsymbol{B}_1\boldsymbol{Y}+\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{B}_1^{\mathrm{T}}+\tilde{\boldsymbol{Q}}_1+\tilde{\boldsymbol{Q}}_2$.

Consequently, we establish the following corollary:

Corollary 1 For the nonlinear uncertain time-delay system(1), if there exist a matrix $\boldsymbol{Y}$, positive definite matrices $\boldsymbol{X}>\boldsymbol{0}$, $\tilde{\boldsymbol{Q}}_1>\boldsymbol{0}$, $\tilde{\boldsymbol{Q}}_2>\boldsymbol{0}$, $\boldsymbol{R}>\boldsymbol{0}$ and a positive constant $\gamma>0$ that make the following LMIs (10) and (11) hold, then it can be concluded that time-delay system Eq. (1) are robustly stabilized, and that H_∞ performance index is less than the given bound γ. Here, the controller gain matrix is $\boldsymbol{K}=\boldsymbol{YX}^{-1}$.

4 Numerical examples

Consider the following uncertain time-delay system

$$\begin{cases} \dot{\boldsymbol{x}}(t)=\boldsymbol{Ax}(t)+\boldsymbol{A}_d(t-\tau)+\boldsymbol{f}_1(\boldsymbol{x}(t),t) \\ \qquad +\boldsymbol{f}_{1d}(\boldsymbol{x}(t-\tau),t)+\boldsymbol{Bw}+\boldsymbol{B}_1\boldsymbol{u}(t)+\boldsymbol{B}_2\boldsymbol{u}(t-d)\,, \\ \boldsymbol{z}(t)=\boldsymbol{Cx}(t) \end{cases}$$

where

$$A=\begin{bmatrix}-4 & 1\\ 0 & -5\end{bmatrix},\ A_d=\begin{bmatrix}0.1 & 0\\ 0 & 0.5\end{bmatrix},$$

$$B=\begin{bmatrix}0.1\\ 0.1\end{bmatrix},\ B_1=\begin{bmatrix}0\\ 1\end{bmatrix},\ B_2=\begin{bmatrix}1\\ 0\end{bmatrix},$$

$$C=[0.1,0.1],\ \tau=0.3,\ d=2,$$

$$f_1=\begin{bmatrix}\sin x_1(t)\\ \sin x_2(t)\end{bmatrix},f_{1d}=\begin{bmatrix}\sin x_1(t-0.3)\\ \sin x_2(t-0.3)\end{bmatrix}.$$

Let $\lambda_1=\lambda_{1d}=\gamma=1$. By solving LMIs (10) and (11) of Corollary 1, we have

$$X=\begin{bmatrix}142.0608 & -19.0179\\ -19.0179 & 35.3256\end{bmatrix},$$
$$Y=[-8.6440\quad 51.1463],$$
$$\tilde{Q}_1=\begin{bmatrix}71.0159 & -20.4931\\ -20.4931 & 26.5082\end{bmatrix},$$
$$\tilde{Q}_2=\begin{bmatrix}54.8808 & -12.6588\\ -12.6588 & 14.4306\end{bmatrix},$$

$$K=[0.1433,1.5250].$$

Finally, we can get the gain matrices of the memoryless delay-dependent state feedback H_∞ controller:

$$u(t)=YX^{-1}x(t)=[0.1433\quad 1.5250]x(t),$$

and the simulation results for the system with the initial condition

$$x_1(0)=-0.5,\ x_2(0)=0.9,\ w(t)=\sin t/(1+t^2).$$

With the chosen parameter settings, the results of a simulation are shown in Figs. 1 and 2. It can be observed from Figs. 1 and 2 that the closed-loop time-delay system resulting from the design in this paper is indeed robustly stable and has a rather good H_∞ performance bound.

5 Conclusions

In this paper, a delay-dependent robust H_∞ state feedback controller has been proposed for a class of uncertain time-delay systems with input delay. Based on the Lyapunov stability theory and Lyapunov-Krasovskii function, it is shown that the resulting closed-loop system is robustly stable. Finally, a numerical example and simulation

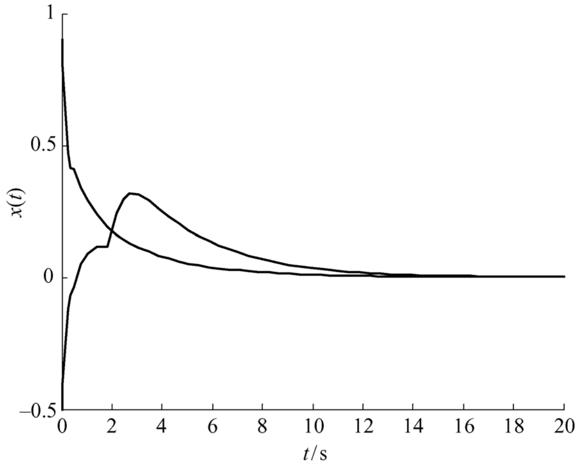

Fig. 1 Responses of closed-loop state variable $x(t)$ with $w(t)=\sin t/(1+t^2)$

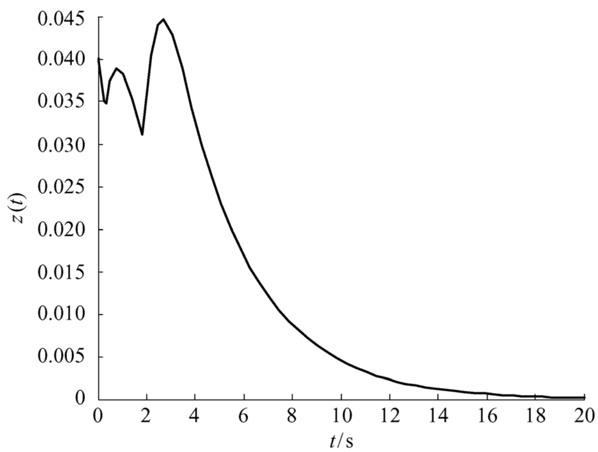

Fig. 2 Responses of controlled output $z(t)$

results demonstrate that the results obtained in this paper are effective and feasible.

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