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RESEARCH ARTICLE

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Enhancement of wave and acceleration of electron in plasma in the external field

Abstract This paper investigates the enhancement of Langmuir and ion-acoustic wave and the acceleration of the electron in collisionless plasma, in the presence of an external transverse field. Based on hydrodynamic equations, an equation formulizing the parametric instability was derived. Furthermore, the formula for ponderomotive force and the expression that describes the electron acceleration were obtained. The results show that Langmuir and ion-acoustic wave are enhanced and the charged particles can be accelerated by the coupling of wave-wave. In addition, it can be concluded that ponderomotive force, due to the coupling of the external field (pump) to the Langmuir wave (ion-acoustic wave), is the driving force to excite the parametric instability and comprises the high- and low-frequency components.

Keywords parametric instability, enhancement of wave, acceleration of electron, ponderomotive force

1 Introduction

In the 1960's, the parametric instability in plasma was theoretically advanced. Based on the hydrodynamic equations, Silin made a systematic study of the parametric instability for the first time and obtained the growth rate for parametric instability in cold plasma [1]. An experiment carried out by R. A. Stern and N. Tzoar showed that a relatively weak external field could generate a coupling of electron plasma waves and ion-acoustic waves [2]. D. F. Dubois and M. V. Goldman estimated the threshold of the excitation by considering Landau damping [3–5]. Lee and Su investigated the parametric coupling in thermal plasma employing the hydrodynamic description [6]. Using the Vlasov equations, Jackson took Landau damping into account and obtained a more useful solution of the problem [7]. Nishikawa considered collisions as the damping mechanism and obtained some interesting results for the threshold and the growth rate [8–10]. The above results can well describe some phenomenon of ionospheric heating, such as especially the enhancement of Langmuir and ion-acoustic [11–16]. In addition, the ponderomotive force that played an important role in parametric instability was obtained by neglecting the non-linear term of motion equation and obtaining its modification further [11]. In this paper, we present the expression of ponderomotive force based on the oscillation equations of Langmuir and ion-acoustic wave. Also, the charged particle can be accelerated by exciting the parametric instability.

2 Parametric coupling of wave in plasma

Consider a equilibrium and non-dissipation plasma which can be described by the motion equation

$$m^\alpha n^\alpha\left(\frac{\partial}{\partial t}+\boldsymbol{v}^\alpha\cdot\frac{\partial}{\partial \boldsymbol{r}}\right)\boldsymbol{v}^\alpha=e^\alpha n^\alpha\boldsymbol{E}-e^\alpha n^\alpha\nabla\Phi-T^\alpha\nabla n^\alpha, \quad (1)$$

the equation of continuity

$$\frac{\partial n^\alpha}{\partial t}+\nabla\cdot(n^\alpha\boldsymbol{v}^\alpha)=0 \quad (2)$$

and the Poisson equation

$$\nabla^2\Phi=-\frac{1}{\varepsilon_0}\sum(e^\alpha n^\alpha). \quad (3)$$

The superscript α denotes either electron (no α) or ion ($\alpha=i$). $\boldsymbol{E}$ denotes the external field, m^α the mass of particle α, n^α the number density, $\boldsymbol{v}^\alpha$ the velocity, e^α the charge, T^α

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the temperature, Φ the electrostatic potential, and ε_0 the permeability of free space. In the following analyses, we derive the ($\pm \boldsymbol{k}_0$, $\pm \omega_0$), ($\pm \boldsymbol{k}$, $\pm \omega$), and ($\pm \Delta\boldsymbol{k}$, $\pm \Delta\omega$) Fourier components from Eqs. (1)–(3).

We assume that the plasma is subject to a transverse electric field

$$\boldsymbol{E}_0 = \frac{1}{2}(\boldsymbol{E}_{k_0} + \boldsymbol{E}_{-k_0}), \tag{4}$$

$\boldsymbol{E}_{k_0} = \boldsymbol{E}_0 \exp[\mathrm{j}(\boldsymbol{k}_0 \cdot \boldsymbol{r} - \omega_0 t)]$, $\boldsymbol{E}_{-k_0} = \boldsymbol{E}_0 \exp[-\mathrm{j}(\boldsymbol{k}_0 \cdot \boldsymbol{r} - \omega_0 t)]$ and $(\boldsymbol{k}_0 \cdot \boldsymbol{E}_0) = 0$. In addition, $(\boldsymbol{k}_0, \omega_0)$ satisfies the matching relation, namely,

$$\boldsymbol{k}_0 = \boldsymbol{k} + \Delta\boldsymbol{k}, \tag{5}$$

$$\omega_0 = \omega + \Delta\omega, \tag{6}$$

where ($\boldsymbol{k}$, ω) and ($\Delta\boldsymbol{k}$, $\Delta\omega$) represent the electron plasma wave and ion-acoustic wave respectively.

For convenience, assume that the wavelength of pump is shorter than Debye length below the reflection height of pump, where Langmuir wave is usually excited. So the intensity of pump does not change in the domain where the parametric instability is excited, that is, $\boldsymbol{k}_0 \approx 0$.

Electrons and ions can be accelerated by pump. By neglecting the Lagrange term of equation of motion, we can obtain

$$\boldsymbol{v}^{\alpha}_{lk_0} = l\mathrm{j}\frac{{}_1 e^{\alpha}\boldsymbol{E}_{lk_0}}{2\omega_0 m^{\alpha}}, \tag{7}$$

where $\boldsymbol{v}^{\alpha}_{lk_0}$ denotes the velocity of the charged particle α in the direction perpendicular to $l\boldsymbol{k}_0$ and $l = \pm 1$. Then we have

$$\boldsymbol{k}\cdot\boldsymbol{v}^{\alpha}_{lk_0} = l\mathrm{j}\frac{{}_1 e^{\alpha}\sum_{lk_0}}{m^{\alpha}}, \tag{8}$$

where $\sum_{lk_0} = \dfrac{(\boldsymbol{k}\cdot\boldsymbol{E}_{lk_0})}{2\omega_0}$.

By linearizing the electron equation of continuity, its ($\pm \boldsymbol{k}$, $\pm \omega$), and ($\pm \Delta\boldsymbol{k}$, $\pm \Delta\omega$) Fourier components

$$(\boldsymbol{k}\cdot\boldsymbol{v}_{lk})n_0 = \omega n_{lk} + \frac{e\sum_{lk_0}}{m}n_{-l\Delta k}\mathrm{e}^{l\mathrm{j}\frac{\pi}{2}}, \tag{9}$$

$$(\Delta\boldsymbol{k}\cdot\boldsymbol{v}_{l\Delta k})n_0 = \Delta\omega n_{l\Delta k} + \frac{e\sum_{lk_0}}{m}n_{-lk}\mathrm{e}^{l\mathrm{j}\frac{\pi}{2}} \tag{10}$$

are obtained.

Similarly one can have the ($\pm \boldsymbol{k}$, $\pm \omega$), and ($\pm \Delta\boldsymbol{k}$, $\pm \Delta\omega$) Fourier components of ion equation of continuity

$$\left(\boldsymbol{k}\cdot\boldsymbol{v}^{i}_{lk}\right)n_0 = \omega n^{i}_{lk} + \frac{e\sum_{lk_0}}{M}n^{i}_{-l\Delta k}\mathrm{e}^{l\mathrm{j}\frac{\pi}{2}}, \tag{11}$$

$$\left(\Delta\boldsymbol{k}\cdot\boldsymbol{v}^{i}_{l\Delta k}\right)n_0 = \Delta\omega n^{i}_{l\Delta k} + \frac{e\sum_{lk_0}}{M}n^{i}_{-lk}\mathrm{e}^{l\mathrm{j}\frac{\pi}{2}}, \tag{12}$$

where m denote the mass of electron. M and e are the mass and the charge of ion respectively.

Following Eqs. (9)–(12), it is obvious that the $\boldsymbol{k}$ and $\Delta\boldsymbol{k}$ Fourier components of the velocity of electron and ion not only relate to their intrinsic oscillation (the term on the right side of equations), but also to the coupling of the pump and intrinsic oscillation (the term on the right side of equations).

The ($\boldsymbol{k}$,ω) Fourier component of the electron equation of the motion satisfies

$$mn_0\left(\frac{\partial}{\partial t}\boldsymbol{v}_{k} + \boldsymbol{v}_{k_0}\cdot\frac{\partial}{\partial \boldsymbol{r}}\boldsymbol{v}_{-\Delta k} + \boldsymbol{v}_{\Delta k}\cdot\frac{\partial}{\partial \boldsymbol{r}}\boldsymbol{v}_{k_0}\right) + mn_{-\Delta k}\frac{\partial}{\partial t}\boldsymbol{v}_{k_0}$$
$$= -\frac{e\boldsymbol{E}_{k_0}}{2}n_{-\Delta k} + en_0\frac{\partial}{\partial \boldsymbol{r}}\Phi_{k} - T_e\frac{\partial}{\partial \boldsymbol{r}}n_{k}, \tag{13}$$

where $\Phi_{k} = e\left(n^{i}_{k} - n_{k}\right)/\varepsilon_0 \boldsymbol{k}^2$ satisfying the Poisson equation.

Taking the scalar product of $\boldsymbol{k}$ with Eq. (13), one can obtain

$$n_0[\omega(\boldsymbol{k}\cdot\boldsymbol{v}_{k}) + (\boldsymbol{v}_{k_0}\cdot\Delta\boldsymbol{k})(\boldsymbol{k}\cdot\boldsymbol{v}_{-\Delta k})] + \omega_0(\boldsymbol{k}\cdot\boldsymbol{v}_{k_0})n_{-\Delta k}$$
$$= -\mathrm{j}\frac{e(\boldsymbol{k}\cdot\boldsymbol{E}_{k_0})}{2m}n_{-\Delta k} - \omega^2_{pe}\left(n^{i}_{n} - n_{k}\right) + \boldsymbol{k}^2\boldsymbol{v}^2_{T_e}n_{k}, \tag{14}$$

where $\boldsymbol{v}_{T_e} = T_e/m$ is the thermal velocity of the electron.

Apply Eqs. (8), (9) and (12) into Eq. (14). Then Eq. (14) is rewritten as

$$\omega^2 n_{k} + \left(-\omega^2_{pe} - \boldsymbol{k}^2\boldsymbol{v}^2_{1T_{1e}} + \frac{e^2\sum_{k_0}\sum_{-k_0}}{m^2}\right)n_{k}$$
$$= \frac{e(\omega - \Delta\omega)}{m}\left(\sum_{k_0} n_{-\Delta k}\right)\mathrm{e}^{-\mathrm{j}\frac{\pi}{2}} - \omega^2_{pe}n^{i}_{k}, \tag{15}$$

where $\omega^2_{pe} = n_0 e^2/\varepsilon_0 m$ is the plasma frequency of the electron.

Likewise, we can gain the ($\Delta\boldsymbol{k}$,$\Delta\omega$) Fourier component of the electron motion equation

$$\Delta\omega^2 n_{\Delta k} + \left(-\omega^2_{pe} - \Delta\boldsymbol{k}^2\boldsymbol{v}^2_{T_e} + \frac{e^2\sum_{k_0}\sum_{-k_0}}{m^2}\right)n_{\Delta k}$$
$$= \frac{e(\omega - \Delta\omega)}{m}\left(\sum_{k_0} n_{-k}\right)\mathrm{e}^{-\mathrm{j}\frac{\pi}{2}} - \omega^2_{pe}n^{i}_{\Delta k} \tag{16}$$

and the ($\boldsymbol{k}$,ω) and ($\Delta\boldsymbol{k}$,$\Delta\omega$) Fourier components of the ion motion equation

$$\omega^2 n^{i}_{k} + \left(-\omega^2_{pi} - \boldsymbol{k}^2\boldsymbol{v}^2_{1T_i} + \frac{e^2\sum_{k_0}\sum_{-k_0}}{M^2}\right)n^{i}_{k}$$
$$= \frac{e(\omega - \Delta\omega)}{M}\left(\sum_{k_0} n^{i}_{-\Delta k}\right)\mathrm{e}^{\mathrm{j}\frac{\pi}{2}} - \omega^2_{pi}n_{k}, \tag{17}$$

$$\Delta\omega^2 n^i_{\Delta\boldsymbol{k}} + \left(-\omega_{pi}^2 - \Delta\boldsymbol{k}^2\boldsymbol{v}_{1T_i}^2 + \frac{e^2\sum_{\boldsymbol{k}_0}\sum_{-\boldsymbol{k}_0}}{M^2}\right)n^i_{\Delta\boldsymbol{k}} = \frac{e(\omega-\Delta\omega)}{M}\left(\sum_{\boldsymbol{k}_0} n^i_{-\boldsymbol{k}}\right)e^{j\frac{\pi}{2}} - \omega_{pi}^2 n_{\Delta\boldsymbol{k}}, \quad (18)$$

here $\omega_{pi}^2 = n_0 e^2/\varepsilon_0 M$ denotes the plasma frequency of ion and $\boldsymbol{v}_{T_i} = T_i/M$ the thermal velocity of ion.

3 Discussions

3.1 Nonlinear coupling and enhancement of intrinsic wave

Rewrite Eq. (15) as

$$\frac{\partial^2 n_{\boldsymbol{k}}}{\partial t^2} + \left(\omega_{pe}^2 + \boldsymbol{k}^2\boldsymbol{v}_{1T_{1e}}^2 - \frac{e^2\sum_{\boldsymbol{k}_0}\sum_{-\boldsymbol{k}_0}}{m^2}\right)n_{\boldsymbol{k}} = \frac{e(\omega-\Delta\omega)}{m}\left(\sum_{\boldsymbol{k}_0} n_{-\Delta\boldsymbol{k}}\right)e^{j\frac{\pi}{2}} + \omega_{pe}^2 n^i_{\boldsymbol{k}}. \quad (19)$$

Obviously, Eq. (19) is an oscillation equation as regards the wave n_k with the frequency

$$\sqrt{\omega_{pe}^2 + \boldsymbol{k}^2\boldsymbol{v}_{T_e}^2 - \frac{e^2\sum_{\boldsymbol{k}_0}\sum_{-\boldsymbol{k}_0}}{m^2}}$$

and a driving force with the same frequency. On the right-hand of Eq. (19) is the driving factor, which features the same frequency and wave number to those of n_k, resulting in a resonance with n_k, thus enhancing n_k. The driving force is composed of the coupling of pump and ion-acoustic wave and the restricting effect of ion on electron.

Like Eq. (15), we rewrite Eq. (16) as

$$\frac{\partial^2 n_{\Delta\boldsymbol{k}}}{\partial t^2} + \left(\omega_{pe}^2 + \Delta\boldsymbol{k}^2\boldsymbol{v}_{T_e}^2 - \frac{e^2\sum_{\boldsymbol{k}_0}\sum_{-\boldsymbol{k}_0}}{m^2}\right)n_{\Delta\boldsymbol{k}} = \frac{e(\omega-\Delta\omega)}{m}\left(\sum_{\boldsymbol{k}_0} n_{-k}\right)e^{j\frac{\pi}{2}} + \omega_{pe}^2 n^i_{\Delta\boldsymbol{k}} \quad (20)$$

and obtain similar results to Eq. (19).

According to Eqs. (19) and (20), one can conclude that n_k and $n_{\Delta k}$ drive each other by coupling them respectively to pump and form a positive feedback circuit, thus resulting in the non-linear growth of n_k and $n_{\Delta k}$.

3.2 Ponderomotive force and its expression

Conduct a simply algebraic operation on Eqs. (15)–(18). They can be rewritten as

$$-m\frac{\omega^2}{\boldsymbol{k}^2}\nabla n_{\boldsymbol{k}} = m\frac{\omega_{pe}^2}{\boldsymbol{k}^2}\nabla\left(n^i_{\boldsymbol{k}} - n_{\boldsymbol{k}}\right) + \frac{e^2\sum_{\boldsymbol{k}_0}\sum_{-\boldsymbol{k}_0}}{m\boldsymbol{k}^2}\nabla n_{\boldsymbol{k}} - T_e\nabla n_{\boldsymbol{k}} + \frac{e(\omega-\Delta\omega)}{\boldsymbol{k}^2}\left(\sum_{\boldsymbol{k}_0}\nabla n_{-\Delta\boldsymbol{k}}\right)e^{j\frac{\pi}{2}}, \quad (21)$$

$$-m\frac{\omega^2}{\Delta\boldsymbol{k}^2}\nabla n_{\Delta\boldsymbol{k}} = m\frac{\omega_{pe}^2}{\Delta\boldsymbol{k}^2}\nabla\left(n^i_{\Delta\boldsymbol{k}} - n_{\Delta\boldsymbol{k}}\right) + \frac{e^2\sum_{\boldsymbol{k}_0}\sum_{-\boldsymbol{k}_0}}{m\Delta\boldsymbol{k}^2}\nabla n_{\Delta\boldsymbol{k}} - T_e\nabla n_{\Delta\boldsymbol{k}} + \frac{e(\omega-\Delta\omega)}{\Delta\boldsymbol{k}^2}\left(\sum_{\boldsymbol{k}_0}\nabla n_{-\boldsymbol{k}}\right)e^{j\frac{\pi}{2}}, \quad (22)$$

$$-M\frac{\omega^2}{\boldsymbol{k}^2}\nabla n^i_{\boldsymbol{k}} = -M\frac{\omega_{pi}^2}{\boldsymbol{k}^2}\nabla\left(n^i_{\boldsymbol{k}} - n_{\boldsymbol{k}}\right) + \frac{e^2\sum_{\boldsymbol{k}_0}\sum_{-\boldsymbol{k}_0}}{M\boldsymbol{k}^2}\nabla n^i_{\boldsymbol{k}} - T_i\nabla n^i_{\boldsymbol{k}} + \frac{e(\omega-\Delta\omega)}{\boldsymbol{k}^2}\left(\sum_{\boldsymbol{k}_0}\nabla n^i_{-\Delta\boldsymbol{k}}\right)e^{-j\frac{\pi}{2}}, \quad (23)$$

$$-M\frac{\Delta\omega^2}{\Delta\boldsymbol{k}^2}\nabla n^i_{\Delta\boldsymbol{k}} = -M\frac{\omega_{pi}^2}{\Delta\boldsymbol{k}^2}\nabla\left(n^i_{\Delta\boldsymbol{k}} - n_{\Delta k}\right) + \frac{e^2\sum_{\boldsymbol{k}_0}\sum_{-\boldsymbol{k}_0}}{M\Delta\boldsymbol{k}^2}\nabla n^i_{\Delta\boldsymbol{k}} - T_i\nabla n^i_{\Delta\boldsymbol{k}} + \frac{e(\omega-\Delta\omega)}{\Delta\boldsymbol{k}^2}\left(\sum_{\boldsymbol{k}_0}\nabla n^i_{-\boldsymbol{k}}\right)e^{-j\frac{\pi}{2}}. \quad (24)$$

The term on the left side of Eq. (21) is the causing force; the first term on the right side is the Coulomb force by electrostatic field; the second by the thermal pressure; the third by the Coulomb force through the external field and the fifth ponderomotive force driving the parametric instability. One can obtain similar results by examining Eqs. (22)–(24).

Due to the coupling of the intrinsic waves in plasma and pump, the ponderomotive force bears the same frequency and direction as Coulomb force and thermal pressure but phase with a discrepant of π/2, which is in direct proportion to the intensity of pump; for different directions and particles, the ponderomotive force is different, in addition, the ponderomotive is related to the pump, Langmuir wave and ion-acoustic wave. For example, the quantity of the ponderomotive force for $\boldsymbol{k}$ is

$$\boldsymbol{F}_{p\boldsymbol{k}} = \frac{e(\omega-\Delta\omega)}{\boldsymbol{k}^2}\left(\sum_{\boldsymbol{k}_0}\nabla n_{-\Delta\boldsymbol{k}}\right)e^{j\frac{\pi}{2}} \quad (25)$$

with which the wave-number $\boldsymbol{k}$ of Langmuir wave, the intensity of ion-acoustic wave $n_{\Delta k}$ and the pump are related.

Equations (22)–(24) are the oscillation equations of $n_{\Delta k}$, $n^i_{\boldsymbol{k}}$ and $n^i_{\Delta\boldsymbol{k}}$ respectively, from which one can arrive at a similar result.

3.3 Acceleration of charged particle due to the external field

Equations (9)–(12) show that a disturbance of the ($\pm \boldsymbol{k}$, $\pm\omega$) and ($\pm\Delta\boldsymbol{k}$, $\pm\Delta\omega$) components of the velocity of the electron and ion are induced by modulating Langmuir wave and ion-acoustic wave on pump respectively, that is, due to ponderomotive force.

The ratios of the ($\boldsymbol{k}$, ω) and ($\Delta\boldsymbol{k}$, $\Delta\omega$) components of the velocity of the charged particle in the presence of external field to that in the absence of external field are presented as follows respectively

$$\frac{\boldsymbol{k}\cdot\boldsymbol{v}_{\boldsymbol{k}}^{\alpha}}{\boldsymbol{k}\cdot\boldsymbol{v}_{0\boldsymbol{k}}^{\alpha}}=1+\frac{e\sum_{\boldsymbol{k}_0}n_{-\Delta\boldsymbol{k}}^{\alpha}}{\omega m^{\alpha}n_{\boldsymbol{k}}^{\alpha}}\mathrm{e}^{\gamma\mathrm{j}\frac{\pi}{2}}, \tag{26}$$

$$\delta_{\boldsymbol{k}}^{\alpha}=\sqrt{1+\left(\frac{e\sum_{\boldsymbol{k}_0}n_{-\Delta\boldsymbol{k}}^{\alpha}}{\omega m^{\alpha}n_{\boldsymbol{k}}^{\alpha}}\right)^2}, \tag{27}$$

and

$$\frac{\Delta\boldsymbol{k}\cdot\boldsymbol{v}_{\Delta\boldsymbol{k}}^{\alpha}}{\Delta\boldsymbol{k}\cdot\boldsymbol{v}_{0\Delta\boldsymbol{k}}^{\alpha}}=1+\frac{e\sum_{\boldsymbol{k}_0}n_{-\boldsymbol{k}}^{\alpha}}{\Delta\omega m^{\alpha}n_{\Delta\boldsymbol{k}}^{\alpha}}\mathrm{e}^{-\gamma\mathrm{j}\frac{\pi}{2}}, \tag{28}$$

$$\delta_{\Delta\boldsymbol{k}}^{\alpha}=\sqrt{1+\left(\frac{e\sum_{\boldsymbol{k}_0}n_{-\boldsymbol{k}}^{\alpha}}{\Delta\omega m^{\alpha}n_{\Delta\boldsymbol{k}}^{\alpha}}\right)^2}, \tag{29}$$

where $\boldsymbol{v}_{0\boldsymbol{k}}^{\alpha}$ and $\boldsymbol{v}_{0\Delta\boldsymbol{k}}^{\alpha}$ are the velocity of particle α for $\boldsymbol{k}$ and $\Delta\boldsymbol{k}$ respectively, $\delta_{\boldsymbol{k}}^{\alpha}$ and $\delta_{\Delta\boldsymbol{k}}^{\alpha}$ the growth of the velocity. Also, it is $\gamma=1$ for electron and $\gamma=-1$ for ion.

Obviously, $\delta_{\boldsymbol{k}}^{\alpha}>1$ and $\delta_{\Delta\boldsymbol{k}}^{\alpha}>1$. The above discussions show that the charged particle in plasma is accelerated by exciting the parametric instability.

4 Conclusions

By analyzing the effect of external field on the charged particle and the intrinsic wave in thermal plasma, one can draw conclusion as follows:

1) Langmuir wave and ion-acoustic wave can be enhanced by exciting the parametric instability.

2) The ponderomotive force driving the parametric instability results from the coupling of pump to Langmuir wave and ion-acoustic wave.

3) The ponderomotive force is composed of high- and low-frequency components.

4) The charged particle is accelerated and its energy is transferred from pump to plasma due to a parametric coupling of an external field to Langmuir wave and ion-acoustic wave.

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