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RESEARCH ARTICLE

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Electromagnetic-scattering by bi-sphere groups and coherent-beam scattering by homogeneous spheres

Abstract By using Mie's theory, the boundary conditions, and some advanced mathematical knowledge, the scattering problem of a plane-wave by bi-sphere groups and of cores-traversed coherent Gauss-beams by one sphere was addressed. In each, the coefficients of the scattering-field expressions were deduced. Finally, the result was predigested and transfigured so that the available form for programming was achieved. On deducing, the former adopted the undetermined coefficient method and the latter used the plane geometry method. Moreover, the complexity of the calculation was decreased here.

Keywords electromagnetic-scattering, undetermined coefficient method, bi-sphere scattering, beam scattering, scattering cross section

1 Introduction

Scattering of spherical particles is one of the top interests of space physicists, astronomers and meteorologists, and the most precise theoretical method for interpreting an overwhelming majority of phenomena related with colors in the atmosphere around the Earth. Presently, scattering by single particle [1] and scattering by double conductive spheres with comparatively simple boundary conditions have already been precisely resolved [2]. This paper focuses on rigorous general resolutions of plane wave scattering by bi-sphere groups of homogeneous media and coherent-beam scattering by any homogeneous sphere of core-traversed Gauss-beams through employing Mie's theory, while having acquired the fundamental approach for the precise analysis of particle scattering problems. The methods and analytical results of this paper can be adopted to precisely calculate the scattering of separate spherical particles.

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2 Mathematical disposal and calculations

This paper utilizes Mie's theory to dispose the scattering of bi-sphere groups, which involves two precise methods and one approximate method. The precise methods are: 1) Separating the scattering field of each particle into two parts, of which one is the direct expression of the scattering of a single particle, and the other is the reciprocity of the two spheres. The ultimate result of the scattering field is the superposition of the two parts; 2) Through employing Mie's theory, using the undetermined coefficient method to achieve the assumed series representation of each sphere in its respective self-centered coordinate system (including the reciprocity of the two spheres), and then solving the 8 independent equations derived by simplifying the boundary conditions of each sphere. The approximate method is: 3) neglecting the reciprocity of the two spheres, using the direct result of the scattering of each single particle, and then by using the translational addition theorem we can obtain the total scattering results expressed in any one of the two above-mentioned self-centered coordinate systems. The paper aimed at precise resolution, so the method 3) is not available. Methods 1) and 2) have no difference in nature, whereas, although method 1) has a more explicit physical meaning, the calculation of the reciprocity of the two spheres has to use the boundary conditions repeatedly. To avoid the unnecessary repetition, method 2) is adopted in this paper.

2.1 Series representation of scattering field

First, by using Mie's theory, we expanded the incoming field. Second, by taking the axis of the two spheres along the z-axis and setting the incoming plane wave as x-polarized (see Fig. 1), the form of the expanded representation of the incoming field is invariable in the self-centered coordinate system, and consequently the expanded representations in Ref. [1] can be employed directly.

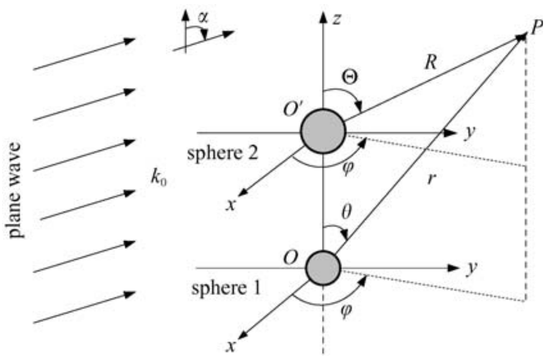

Fig. 1 Plane wave scattering of bi-sphere

Place sphere 1 at the origin of the laboratory coordinate system (x, y, z) (see Fig. 1) and construct an appropriate orthonormal coordinate system that is the spherical coordinates (r, θ, φ) based on sphere 1 (as the origin). φ is the same in both the self-centered coordinate systems of the two spheres because the two spheres are set along the z-axis. Suppose the self-centered coordinates of sphere 2 are (R, Θ, φ), we can achieve the expanded representations of vectors $\boldsymbol{E}_i(r,\theta,\varphi)$, $\boldsymbol{H}_i(r,\theta,\varphi)$, $\boldsymbol{E}_{s1}(r,\theta,\varphi)$, $\boldsymbol{H}_{s1}(r,\theta,\varphi)$, $\boldsymbol{E}_1(r,\theta,\varphi)$, $\boldsymbol{H}_1(r,\theta,\varphi)$ and $\boldsymbol{E}_i(R,\Theta,\varphi)$, $\boldsymbol{H}_i(R,\Theta,\varphi)$, $\boldsymbol{E}_{s2}(R,\Theta,\varphi)$, $\boldsymbol{H}_{s2}(R,\Theta,\varphi)$, $\boldsymbol{E}_2(R,\Theta,\varphi)$, $\boldsymbol{H}_2(R,\Theta,\varphi)$ of spherical vector wave functions, which are

$$\begin{cases} \boldsymbol{E}_i(r,\theta,\varphi)=\sum_{n=1}^{\infty}\boldsymbol{E}_n(\boldsymbol{M}_{o1n}^{(1)}-\mathrm{i}\boldsymbol{N}_{e1n}^{(1)}) \\ \boldsymbol{H}_i(r,\theta,\varphi)=\dfrac{-k}{\omega\mu}\sum_{n=1}^{\infty}\boldsymbol{E}_n(\boldsymbol{M}_{e1n}^{(1)}+\mathrm{i}\boldsymbol{N}_{o1n}^{(1)}) \end{cases}, \tag{1}$$

$$\begin{cases} \boldsymbol{E}_{s1}(r,\theta,\varphi)=\sum_{n=1}^{\infty}\boldsymbol{E}_n(\mathrm{i}a_n\boldsymbol{N}_{e1n}^{(3)}-b_n\boldsymbol{M}_{o1n}^{(3)}) \\ \boldsymbol{H}_{s1}(r,\theta,\varphi)=\dfrac{k}{\omega\mu}\sum_{n=1}^{\infty}\boldsymbol{E}_n(\mathrm{i}b_n\boldsymbol{N}_{o1n}^{(3)}+a_n\boldsymbol{M}_{e1n}^{(3)}) \end{cases}, \tag{2}$$

$$\begin{cases} \boldsymbol{E}_1(r,\theta,\varphi)=\sum_{n=1}^{\infty}\boldsymbol{E}_n(A_n\boldsymbol{M}_{o1n}^{(1)}-\mathrm{i}B_n\boldsymbol{N}_{e1n}^{(1)}) \\ \boldsymbol{H}_1(r,\theta,\varphi)=\dfrac{-k_1}{\omega\mu_1}\sum_{n=1}^{\infty}\boldsymbol{E}_n(B_n\boldsymbol{M}_{e1n}^{(1)}+\mathrm{i}A_n\boldsymbol{N}_{o1n}^{(1)}) \end{cases}, \tag{3}$$

$$\begin{cases} \boldsymbol{E}_i(R,\Theta,\varphi)=\mathrm{e}^{-\mathrm{i}kb\cos\alpha}\sum_{n=1}^{\infty}\boldsymbol{E}_n\Big(\boldsymbol{M}_{o1n}^{(1)}(R,\Theta,\varphi) \\ \qquad -\mathrm{i}\boldsymbol{N}_{e1n}^{(1)}(R,\Theta,\varphi)\Big) \\ \boldsymbol{H}_i(R,\Theta,\varphi)=\dfrac{-k\mathrm{e}^{-\mathrm{i}kb\cos\alpha}}{\omega\mu}\sum_{n=1}^{\infty}\boldsymbol{E}'_n\Big(\boldsymbol{M}_{e1n}^{(1)}(R,\Theta,\varphi) \\ \qquad +\mathrm{i}\boldsymbol{N}_{o1n}^{(1)}(R,\Theta,\varphi)\Big) \end{cases}. \tag{4}$$

Equations (1), (2) and (3) denote respectively the incoming, the scattering and the total field near sphere 1 in spherical coordinate system (r, θ, φ) that possesses variable parameters r, θ, φ, with spherical vector wave functions $\boldsymbol{M}_{1n}$ and $\boldsymbol{N}_{1n}$ [1–3]. In the spherical coordinate system (R, Θ, φ) of sphere 2, the scattering field and the inner field have identical forms as Eqs. (2) and (3), with only r, θ, φ changed to R, Θ, φ, and a_n, b_n, A_n, B_n to c_n, d_n, C_n, D_n. The detailed expressions of the spherical vector wave functions, such as $\boldsymbol{M}_{o1n}^{(1)}$, $\boldsymbol{N}_{e1n}^{(1)}$,are given as follows:

$$\boldsymbol{M}_{emn}=\frac{-m}{\sin\theta}\sin m\varphi P_n^m(\cos\theta)z_n(\rho)\hat{e}_\theta - \cos m\varphi\frac{\mathrm{d}P_n^m(\cos\theta)}{\mathrm{d}\theta}z_n(\rho)\hat{e}_\varphi,$$

$$\boldsymbol{M}_{omn}=\frac{m}{\sin\theta}\cos m\varphi P_n^m(\cos\theta)z_n(\rho)\hat{e}_\theta - \sin m\varphi\frac{\mathrm{d}P_n^m(\cos\theta)}{\mathrm{d}\theta}z_n(\rho)\hat{e}_\varphi,$$

$$\boldsymbol{N}_{emn}=\frac{z_n(\rho)}{\rho}\cos(m\varphi)n(n+1)P_n^m(\cos\theta)\hat{e}_r + \cos m\varphi\frac{\mathrm{d}P_n^m(\cos\theta)}{\mathrm{d}\theta}\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}[\rho z_n(\rho)]\hat{e}_\theta - m\sin m\varphi\frac{P_n^m(\cos\theta)}{\sin\theta}\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}[\rho z_n(\rho)]\hat{e}_\varphi,$$

$$\boldsymbol{N}_{omn}=\frac{z_n(\rho)}{\rho}\sin(m\varphi)n(n+1)P_n^m(\cos\theta)\hat{e}_r + \sin m\varphi\frac{\mathrm{d}P_n^m(\cos\theta)}{\mathrm{d}\theta}\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}[\rho z_n(\rho)]\hat{e}_\theta + m\cos m\varphi\frac{P_n^m(\cos\theta)}{\sin\theta}\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}[\rho z_n(\rho)]\hat{e}_\varphi.$$

Here, P_n^m is the associated Legendre polynomial, z_n is any one of the four kinds of Bessel functions, superscripts 1 and 3 (see Eqs.(5) and (6)) correspond to j_n and $h_n^{(1)}$ respectively, $\rho = ka$ is the size-parameter (dimension) of sphere 1, k is the wavelength of the incoming wave in the homogeneous sphere 1, a is the radius of sphere 1 [1].

2.2 Translational addition theorem

To derive the solution of the total electromagnetic field around the spheres, a uniform coordinate system is needed, and a comparatively complicated translation, that is, the translational addition theorem is also required [2,5].

To get the translating coefficients of the translational addition theorem, we introduced a direct but approximate method, which was realized by transforming the infinite summing to the interceptive finite summing. First, derive the translating coefficients in sub-functions, and then substitute them into the acquired expanded expressions of the scattering field. Therefore, when the terms of truncation is large enough, which can be estimated by the convergence, we can achieve reasonable scattering coefficients and scattering cross sections.

The translational addition theorem of the vector is shown as follows:

$$\begin{cases} \boldsymbol{M}_{1n}^{(3)}(R,\Theta,\varphi)=\sum\limits_{\upsilon=0}^{\infty}[A_{\upsilon 1}^{n1}\boldsymbol{M}_{1\upsilon}^{(1)}(r,\theta,\varphi) \\ \qquad +B_{\upsilon 1}^{n1}\boldsymbol{N}_{1\upsilon}^{(1)}(r,\theta,\varphi)] \\ \boldsymbol{N}_{1n}^{(3)}(R,\Theta,\varphi)=\sum\limits_{\upsilon=0}^{\infty}[A_{\upsilon 1}^{n1}\boldsymbol{N}_{1\upsilon}^{(1)}(r,\theta,\varphi) \\ \qquad +B_{\upsilon 1}^{n1}\boldsymbol{M}_{1\upsilon}^{(1)}(r,\theta,\varphi)] \end{cases}, \quad (r\leqslant b), \quad (5)$$

$$\begin{cases} \boldsymbol{M}_{1n}^{(3)}(r,\theta,\varphi)=\sum\limits_{\upsilon=0}^{\infty}[C_{\upsilon 1}^{n1}\boldsymbol{M}_{1\upsilon}^{(1)}(R,\Theta,\varphi) \\ \qquad +D_{\upsilon 1}^{n1}\boldsymbol{N}_{1\upsilon}^{(1)}(R,\Theta,\varphi)] \\ \boldsymbol{N}_{1n}^{(3)}(r,\theta,\varphi)=\sum\limits_{\upsilon=0}^{\infty}[C_{\upsilon 1}^{n1}\boldsymbol{N}_{1\upsilon}^{(1)}(R,\Theta,\varphi) \\ \qquad +D_{\upsilon 1}^{n1}\boldsymbol{M}_{1\upsilon}^{(1)}(R,\Theta,\varphi)] \end{cases}, \quad (R\leq b). \quad (6)$$

As to the translational addition theorem of the far field (r, $R\geqslant b$), we just need to replace $A_{\upsilon 1}^{n1}$, $B_{\upsilon 1}^{n1}$, $C_{\upsilon 1}^{n1}$, $D_{\upsilon 1}^{n1}$ by $\bar{A}_{\upsilon 1}^{n1}$, $\bar{B}_{\upsilon 1}^{n1}$, $\bar{C}_{\upsilon 1}^{n1}$, $\bar{D}_{\upsilon 1}^{n1}$, and replace the superscripts 1 of the spherical vector wave functions by 3. The detailed expressions of $A_{\upsilon 1}^{n1}$, $B_{\upsilon 1}^{n1}$, $C_{\upsilon 1}^{n1}$, $D_{\upsilon 1}^{n1}$ can be found in Refs. [2] and [5].

Now, we can see that in the near field, the scattering field of the distant sphere becomes part of the incoming field of the self-centered sphere by using the translational addition theorem, so that the scattering field of the self-centered sphere becomes the neighboring total scattering field. As to the far field, the total scattering field is still the superposed scattering fields of both spheres. Considering the difference of the translational addition theorem in the far field and the near field, we should use it separately in disposing problems.

2.3 Boundary conditions

First, we deal with the scattering of the near field ($r\leqslant b$ and $R\leqslant b$, b is the distance between the centers of the two spheres). As to obtaining the 8 independent equations needed for calculation, we should make the best use of the boundary conditions around the two spheres. The method in this paper is translating the scattering field of sphere 2 to part of the incoming field of sphere 1, and then deriving the total field around sphere 1. By using the continuous boundary conditions at the interface r_0 (r_0 is the radius of sphere 1) of the surrounding medium and sphere 1, we can derive 4 independent equations. The same way with sphere 2. By using the boundary conditions at the interface R_0 (R_0 is the radius of sphere 2) around sphere 2, we can also derive 4 independent equations.

The useful boundary conditions are: $(\boldsymbol{E}_i+\boldsymbol{E}_s-\boldsymbol{E}_1)\times\hat{\boldsymbol{e}}_r=(\boldsymbol{H}_i+\boldsymbol{H}_s-\boldsymbol{H}_1)\times\hat{\boldsymbol{e}}_r=\boldsymbol{0}$ ($r=r_0$ or $R=R_0$).

To the near field, by using the boundary conditions above and a few mathematical calculations, we can get the following 8 independent equations:

$$\begin{cases} a_n[x_1h_n^{(1)}(x_1)]'+c_n\dfrac{x_1}{x_2}\sum\limits_{\upsilon=0}^{\infty}A_{\upsilon 1}^{n1}[x_2j_\upsilon(x_2)]'+\dfrac{B_n}{m_1}[m_1x_1j_n(m_1x_1)]' \\ =[x_1j_n(x_1)]' \\ b_nh_n^{(1)}(x_1)+d_n\sum\limits_{\upsilon=0}^{\infty}A_{\upsilon 1}^{n1}j_\upsilon(x_2)+A_nj_n(m_1x_1)=j_n(x_1) \\ a_n\dfrac{x_2}{x_1}\sum\limits_{\upsilon=0}^{\infty}C_{\upsilon 1}^{n1}[x_1j_\upsilon(x_1)]'+c_n[x_2h_n^{(1)}(x_2)]'+\dfrac{D_n}{m_2}[m_2x_2j_n(m_2x_2)]' \\ =t[x_2j_n(x_2)]' \\ b_n\sum\limits_{\upsilon=0}^{\infty}C_{\upsilon 1}^{n1}j_\upsilon(x_1)+d_nh_n^{(1)}(x_2)+C_nj_n(m_2x_2)=tj_n(x_2) \end{cases}, \quad (7)$$

$$\begin{cases} a_nh_n^{(1)}(x_1)+c_n\sum\limits_{\upsilon=0}^{\infty}A_{\upsilon 1}^{n1}j_\upsilon(x_2)+\dfrac{\mu}{\mu_1}m_1B_nj_n(m_1x_1)=j_n(x_1) \\ b_n[x_1h_n^{(1)}(x_1)]'+d_n\dfrac{x_1}{x_2}\sum\limits_{\upsilon=0}^{\infty}A_{\upsilon 1}^{n1}[x_2j_\upsilon(x_2)]'+\dfrac{\mu}{\mu_1}A_n[m_1x_1j_n(m_1x_1)]' \\ =[x_1j_n(x_1)]' \\ a_n\sum\limits_{\upsilon=0}^{\infty}C_{\upsilon 1}^{n1}j_\upsilon(x_1)+c_nh_n^{(1)}(x_2)+\dfrac{\mu}{\mu_2}m_2D_nj_n(m_2x_2)=tj_n(x_2) \\ b_n\dfrac{x_2}{x_1}\sum\limits_{\upsilon=0}^{\infty}C_{\upsilon 1}^{n1}[x_1j_\upsilon(x_1)]'+d_n[x_2h_n^{(1)}(x_2)]'+\dfrac{\mu}{\mu_2}C_n[m_2x_2j_n(m_2x_2)]' \\ =t[x_2j_n(x_2)]' \end{cases}, \quad (8)$$

where μ, μ_1 and μ_2 are permeabilities of the surrounding medium (can be regarded as vacuum, approximately), spheres 1 and 2 respectively; $m_1=k_1/k$ and $m_2=k_2/k$ are comparative refractivities of spheres 1 and 2 (k, k_1 and k_2 are wavelengths of incoming wave in the surrounding medium, spheres 1 and 2); $x_1=ka_1$, $x_2=ka_2$ are size-parameters of spheres 1 and 2; t is the incident angular related translation [2] $\exp(-ikb\cos\alpha)$, with b as the distance between the centers of the two spheres and α as the angle formed of the incident direction and z-axis (see Fig. 1).

Assume

$$H_n^{(1)}=\mu m_1 D_n^{(1)}(x_1)/\mu_1-D_n^{(1)}(m_1x_1),\ H_n^{(2)}=\mu m_2 D_n^{(3)}(x_2)/\mu_2-D_n^{(1)}(m_2x_2),$$

$$H_n^{(3)}=\mu m_1 D_n^{(1)}(m_1x_1)/\mu_1-D_n^{(1)}(x_1),\ H_n^{(4)}=\mu m_2 D_n^{(1)}(m_2x_2)/\mu_2-D_n^{(3)}(x_2),$$

$$K_n^{(1)}=\mu m_2[\psi_n'(x_2)/\xi_n(x_2)]/\mu_2-[\psi_n(x_2)/\xi_n(x_2)]D_n^{(1)}(m_2x_2),$$

$$K_n^{(2)}=(\mu m_2/\mu_2)D_n^{(1)}(m_2x_2)[\psi_n(x_2)/\xi_n(x_2)]-\psi_n'(x_2)/\xi_n(x_2),$$

$$L_n^{(1)}=\mu m_1[\xi_n'(x_1)/\psi_n(x_1)]/\mu_1-D_n^{(1)}(m_1x_1)[\xi_n(x_1)/\psi_n(x_1)],$$

$$L_n^{(2)}=(\mu m_1/\mu_1)D_n^{(1)}(m_1x_1)[\xi_n(x_1)/\psi_n(x_1)]-\xi_n'(x_1)/\psi_n(x_1),$$

where $x_1=kr_0$ and $x_2=kR_0$. Then we can achieve the useful scattering coefficients conveniently:

$$a_n=\frac{H_n^{(2)}H_n^{(1)}-(x_1/x_2)K_n^{(1)}[\mu m_1A_{n2}/\mu_1-D_n^{(1)}(m_1x_1)A_{n1}]t}{H_n^{(2)}L_n^{(1)}-[\mu m_2C_{n2}/\mu_2-D_n^{(1)}(m_2x_2)C_{n1}][\mu m_1A_{n2}/\mu_1-D_n^{(1)}(m_1x_1)A_{n1}]},\tag{9}$$

$$c_n=\frac{(x_2/x_1)[\mu m_2C_{n2}/\mu_2-D_n^{(1)}(m_2x_2)C_{n1}]H_n^{(1)}-K_n^{(1)}L_n^{(1)}t}{[\mu m_2C_{n2}/\mu_2-D_n^{(1)}(m_2x_2)C_{n1}][\mu m_1A_{n2}/\mu_1-D_n^{(1)}(m_1x_1)A_{n1}]-H_n^{(3)}L_n^{(1)}},\tag{10}$$

$$b_n=\frac{H_n^{(4)}H_n^{(3)}-(x_1/x_2)K_n^{(2)}[(\mu m_1/\mu_1)D_n^{(1)}(m_1x_1)A_{n1}-A_{n2}]t}{H_n^{(4)}L_n^{(2)}-[(\mu m_2/\mu_2)D_n^{(1)}(m_2x_2)C_{n1}-C_{n2}][(\mu m_1/\mu_1)D_n^{(1)}(m_1x_1)A_{n1}-A_{n2}]},\tag{11}$$

$$d_n=\frac{(x_2/x_1)[(\mu m_2/\mu_2)D_n^{(1)}(m_2x_2)C_{n1}-C_{n2}][(\mu m_1/\mu_1)D_n^{(1)}(m_1x_1)-D_n^{(1)}(x_1)]-K_n^{(2)}L_n^{(2)}t}{[(\mu m_2/\mu_2)D_n^{(1)}(m_2x_2)C_{n1}-C_{n2}][(\mu m_1/\mu_1)D_n^{(1)}(m_1x_1)A_{n1}-A_{n2}]-H_n^{(4)}L_n^{(2)}},\tag{12}$$

where

$$A_{n1}=\sum_{\upsilon=0}^{\infty}A_{\upsilon1}^{n1}\frac{\psi_\upsilon(x_2)}{\psi_n(x_1)},\quad A_{n2}=\sum_{\upsilon=0}^{\infty}A_{\upsilon1}^{n1}\frac{\psi_\upsilon'(x_2)}{\psi_n(x_1)},$$
$$C_{n1}=\sum_{\upsilon=0}^{\infty}C_{\upsilon1}^{n1}\frac{\psi_\upsilon(x_1)}{\xi_n(x_2)},\quad C_{n2}=\sum_{\upsilon=0}^{\infty}C_{\upsilon1}^{n1}\frac{\psi_\upsilon'(x_1)}{\xi_n(x_2)}.$$

However, what people are much more concerned with is the scattering problem of the far field ($r\gg b$ or at least $r\geqslant b$). Fortunately, on the mathematical basis, the total field around the spheres can be achieved by using the uniformly same method and with quite similar boundary conditions. Just by simply rewriting the previous results, we can acquire the equations of scattering coefficients of the far field

$$\begin{cases} a_n[x_1h_n^{(1)}(x_1)]'+c_n\dfrac{x_1}{x_2}\displaystyle\sum_{\upsilon=0}^{\infty}\bar{A}_{\upsilon1}^{n1}[x_2h_\upsilon^{(1)}(x_2)]'+\dfrac{B_n}{m_1}[m_1x_1j_n(m_1x_1)]'=[x_1j_n(x_1)]' \\ b_nh_n^{(1)}(x_1)+d_n\displaystyle\sum_{\upsilon=0}^{\infty}\bar{A}_{\upsilon1}^{n1}h_\upsilon^{(1)}(x_2)+A_nj_n(m_1x_1)=j_n(x_1) \\ a_n\dfrac{x_2}{x_1}\displaystyle\sum_{\upsilon=0}^{\infty}\bar{C}_{\upsilon1}^{n1}[x_1h_\upsilon^{(1)}(x_1)]'+c_n[x_2h_n^{(1)}(x_2)]'+\dfrac{D_n}{m_2}[m_2x_2j_n(m_2x_2)]'=t[x_2j_n(x_2)]' \\ b_n\displaystyle\sum_{\upsilon=0}^{\infty}\bar{C}_{\upsilon1}^{n1}h_\upsilon^{(1)}(x_1)+d_nh_n^{(1)}(x_2)+C_nj_n(m_2x_2)=tj_n(x_2) \end{cases},\tag{13}$$

$$\begin{cases} a_n h_n^{(1)}(x_1) + c_n \sum_{\upsilon=0}^{\infty} \bar{A}_{\upsilon 1}^{n1} h_\upsilon^{(1)}(x_2) + \dfrac{\mu}{\mu_1} m_1 B_n j_n(m_1 x_1) = j_n(x_1) \\ b_n [x_1 h_n^{(1)}(x_1)]' + d_n \dfrac{x_1}{x_2} \sum_{\upsilon=0}^{\infty} \bar{A}_{\upsilon 1}^{n1} [x_2 h_\upsilon^{(1)}(x_2)]' + \dfrac{\mu}{\mu_1} A_n [m_1 x_1 j_n(m_1 x_1)]' = [x_1 j_n(x_1)]' \\ a_n \sum_{\upsilon=0}^{\infty} \bar{C}_{\upsilon 1}^{n1} h_\upsilon^{(1)}(x_1) + c_n h_n^{(1)}(x_2) + \dfrac{\mu}{\mu_2} m_2 D_n j_n(m_2 x_2) = t j_n(x_2) \\ b_n \dfrac{x_2}{x_1} \sum_{\upsilon=0}^{\infty} \bar{C}_{\upsilon 1}^{n1} [x_1 h_\upsilon^{(1)}(x_1)]' + d_n [x_2 h_n^{(1)}(x_2)]' + \dfrac{\mu}{\mu_2} C_n [m_2 x_2 j_n(m_2 x_2)]' = t [x_2 j_n(x_2)]' \end{cases} . \tag{14}$$

So that

$$a_n = \frac{H_n^{(2)} H_n^{(1)} - (x_1/x_2) K_n^{(1)} [(\mu m_1/\mu_1) A_{n4} - D_n^{(1)}(m_1 x_1) A_{n3}] t}{H_n^{(2)} L_n^{(1)} - [(\mu m_2/\mu_2) C_{n4} - D_n^{(1)}(m_2 x_2) C_{n3}][(\mu m_1/\mu_1) A_{n4} - D_n^{(1)}(m_1 x_1) A_{n3}]}, \tag{15}$$

$$b_n = \frac{H_n^{(4)} H_n^{(3)} - (x_1/x_2) K_n^{(2)} [(\mu m_1/\mu_1) D_n^{(1)}(m_1 x_1) A_{n3} - A_{n4}] t}{H_n^{(4)} L_n^{(2)} - [(\mu m_2/\mu_2) D_n^{(1)}(m_2 x_2) C_{n3} - C_{n4}][(\mu m_1/\mu_1) D_n^{(1)}(m_1 x_1) A_{n3} - A_{n4}]}, \tag{16}$$

$$c_n = \frac{(x_2/x_1)[(\mu m_2/\mu_2) C_{n4} - D_n^{(1)}(m_2 x_2) C_{n3}] H_n^{(1)} - K_n^{(1)} L_n^{(1)} t}{[(\mu m_2/\mu_2) C_{n4} - D_n^{(1)}(m_2 x_2) C_{n3}][(\mu m_1/\mu_1) A_{n4} - D_n^{(1)}(m_1 x_1) A_{n3}] - H_n^{(2)} L_n^{(1)}}, \tag{17}$$

$$d_n = \frac{(x_2/x_1)[(\mu m_2/\mu_2) D_n^{(1)}(m_2 x_2) C_{n3} - C_{n4}] H_n^{(3)} - K_n^{(2)} L_n^{(2)} t}{[(\mu m_2/\mu_2) D_n^{(1)}(m_2 x_2) C_{n3} - C_{n4}][(\mu m_1/\mu_1) D_n^{(1)}(m_1 x_1) A_{n3} - A_{n4}] - H_n^{(4)} L_n^{(2)}}, \tag{18}$$

where [2,5]

$$A_{n3} = \sum_{\upsilon=0}^{\infty} \bar{A}_{\upsilon 1}^{n1} \frac{\xi_\upsilon(x_2)}{\psi_n(x_1)}, \quad A_{n4} = \sum_{\upsilon=0}^{\infty} \bar{A}_{\upsilon 1}^{n1} \frac{\xi'_\upsilon(x_2)}{\psi_n(x_1)}, \ C_{n3} = \sum_{\upsilon=0}^{\infty} \bar{C}_{\upsilon 1}^{n1} \frac{\xi_\upsilon(x_1)}{\xi_n(x_2)}, \quad C_{n4} = \sum_{\upsilon=0}^{\infty} \bar{C}_{\upsilon 1}^{n1} \frac{\xi'_\upsilon(x_1)}{\xi_n(x_2)},$$

$$D_n^{(1)}(x) = \frac{\mathrm{d}}{\mathrm{d}x} \ln \psi_n(x) = \frac{\psi'_n(x)}{\psi_n(x)}, \ D_n^{(3)}(x) = \frac{\mathrm{d}}{\mathrm{d}x} \ln \xi_n(x) = \frac{\xi'_n(x)}{\xi_n(x)}.$$

$$A_{\upsilon m}^{nm} = (-1)^m i^{\upsilon-n} \sum_p [h_p^{(1)}(k_0 b) \alpha(m:p,n,\upsilon)], \ B_{\upsilon m}^{nm} = (-1)^m i^{\upsilon-n} \sum_p [h_p^{(1)}(k_0 b) \beta(m:p,n,\upsilon)],$$

$$\bar{A}_{\upsilon m}^{nm} = (-1)^m i^{\upsilon-n} \sum_p [j_p(k_0 b) \alpha(m:p,n,\upsilon)], \ \bar{B}_{\upsilon m}^{nm} = (-1)^m i^{\upsilon-n} \sum_p [j_p(k_0 b) \beta(m:p,n,\upsilon)],$$

$$C_{\upsilon m}^{nm} = (-1)^m i^{\upsilon-n} \sum_p [h_p^{(1)}(k_0 b) \alpha * (m:p,n,\upsilon)], \ D_{\upsilon m}^{nm} = (-1)^m i^{\upsilon-n} \sum_p [h_p^{(1)}(k_0 b) \beta * (m:p,n,\upsilon)],$$

$$\bar{C}_{\upsilon m}^{nm} = (-1)^m i^{\upsilon-n} \sum_p [j_p(k_0 b) \alpha * (m:p,n,\upsilon)], \ \bar{D}_{\upsilon m}^{nm} = (-1)^m i^{\upsilon-n} \sum_p [j_p(k_0 b) \beta * (m:p,n,\upsilon)].$$

With [2,5]

$$\alpha(m:p,n,\upsilon) = i^{-p} [(2\upsilon+1) a(m,-m:p,n,\upsilon) + i \frac{(\upsilon-m)}{\upsilon} (k_0 b) a(m,-m:p,n,\upsilon-1) - i \frac{(\upsilon+m+1)}{(\upsilon+1)} (k_0 b) a(m,-m:p,n,\upsilon+1)];$$

$$\beta(m:p,n,\upsilon) = i^{-p} \left[-i \frac{m(2\upsilon+1)}{\upsilon(\upsilon+1)} (k_0 b) a(m,-m:p,n,\upsilon) \right].$$

2.4 Applications and degenerateness of scattering coefficients

Amplitude relations of the incident and scattering field is [1]

$$\begin{pmatrix} \boldsymbol{E}_{//s} \\ \boldsymbol{E}_{\perp s} \end{pmatrix} = \frac{\mathrm{e}^{ik(r-z)}}{-ikr} \begin{pmatrix} S_2 & 0 \\ 0 & S_1 \end{pmatrix} \begin{pmatrix} \boldsymbol{E}_{//i} \\ \boldsymbol{E}_{\perp i} \end{pmatrix}, \tag{19}$$

where

$$\begin{cases} S_1 = \sum_n \frac{2n+1}{n(n+1)}(a_n\pi_n + b_n\tau_n) \\ S_2 = \sum_n \frac{2n+1}{n(n+1)}(a_n\tau_n + b_n\pi_n) \end{cases}. \tag{20}$$

Here, $\pi_n = P_n^1 / \sin\theta$ and $\tau_n = \mathrm{d}P_n^1/\mathrm{d}\theta$ are angle-dependent functions [1].

According to Eq. (19), we can derive the Stokes parameters of the incident and scattering field:

$$\begin{pmatrix} I_s \\ Q_s \\ U_s \\ V_s \end{pmatrix} = \frac{1}{k^2r^2} \begin{bmatrix} S_{11} & S_{12} & 0 & 0 \\ S_{12} & S_{11} & 0 & 0 \\ 0 & 0 & S_{33} & S_{34} \\ 0 & 0 & -S_{34} & S_{33} \end{bmatrix} \begin{pmatrix} I_i \\ Q_i \\ U_i \\ V_i \end{pmatrix}, \tag{21}$$

where

$$\begin{cases} S_{11} = \frac{1}{2}(|S_2|^2 + |S_1|^2),\ S_{12} = \frac{1}{2}(|S_2|^2 - |S_1|^2) \\ S_{33} = \frac{1}{2}(S_2^* S_1 + S_2 S_1^*),\ S_{34} = \frac{i}{2}(S_2^* S_1 - S_2 S_1^*) \end{cases}. \tag{22}$$

Among the matrix elements, only 3 are completely independent, satisfying the relation $S_{11}^2 = S_{12}^2 + S_{33}^2 + S_{34}^2$.

Based on the above scattering coefficients, we can easily derive the scattering cross sections, scattering amplitude and angle-dependent polarization of the scattering field. Accordingly, we can calculate these useful parameters by computer programming. For some hints of the detailed programming disposal, see Ref. [1].

According to the computational figures, we can see that the scattering of the two spheres is related with size-parameters and the distance of the two spheres, and related with the angle formed by the incident direction and the pointing direction of centers of the two spheres. For instance, we use the results above to analyze the relations of the scattering field along with the distance b, and a pair of numerical calculation results is shown in Fig. 2.

Figure 2 shows the angular distribution of scattering field parameter S_{11} (actually $\log(S_{11})$ for sharpening the tendency of changes) by two uniform spheres (with the same size and refractivity) of the same incident wave (with the same incident direction, wavelength, and polarization). The size parameter of the spheres is $x_1 = x_2 = 3.083$; the comparative refractivity is $m_1 = m_2 = 1.61 + 0.0004\mathrm{i}$; the incident wave is x-polarized and with the wavelength $\lambda = 0.6328$ μm (In fact, the wavelength can be arbitrary, only if the size parameters of the spheres are assured, the results are unchangeable with the wavelength), and the incident direction is along the z-axis.

For an exhaustive description of the scattering field, the parallel and vertical components to the scattering plane (i.e., to the x-polarized plane wave, the parallel and vertical components are Oxz and Oyz) should be derived. In Fig. 2, as an example, we just present the parallel component to the scattering plane (in fact, the vertical component has the similar relations). According to the figure, we can see obviously that: the larger the distance, the stronger the interference and the more dispersive of the energy of the scattering field.

In addition, it is similar to the scattering by a single sphere. The refractivity of the spheres cannot only change the polarization of the scattering field, but also can affect the angular distribution of the electric intensity of the

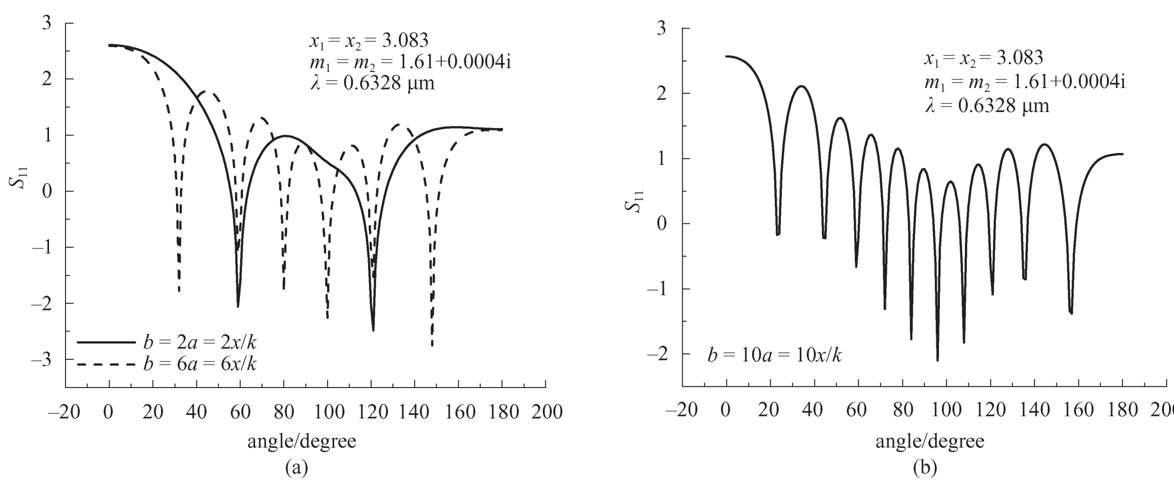

Fig. 2 Scattering fields of two uniform spheres

scattering field. Further theoretical analysis of the calculation results above is given as below.

On the physical meaning, if one of the two spheres is absent, its scattering field will also disappear, that is, the scattering by bi-sphere group will be degenerated to the scattering by a single sphere [1]. Based on this conclusion, we can analyze our scattering coefficients, and can use this method for further verifying the correctness of our derivation in a certain extent.

If $\mu_2 = \mu$, and $m_2 = 1$ or $x_2 = 0$, it denotes that sphere 2 is absent, then the expressions of the scattering coefficients should become the condition of the scattering by sphere 1 alone. Here we just demonstrate the expression of a_n in both the near and far fields. We also verified all the other scattering coefficients satisfying that conclusion, which can be easily perceived.

According to the calculation formulae of the translational addition theorem [2,5], we can see that, if $\mu_2 = \mu$, and $m_2 = 1$ or $x_2 = 0$, then $\mu m_1 A_{n2}/\mu_1 - D_n^{(1)}(m_1x_1)A_{n1}$, $\mu m_2 C_{n2}/\mu_2 - D_n^{(1)}(m_2x_2)C_{n1}$, $(\mu m_1/\mu_1)A_{n4} - D_n^{(1)}(m_1x_1)A_{n3}$, and $(\mu m_2/\mu_2)C_{n4} - D_n^{(1)}(m_2x_2)C_{n3}$ of

$$a_n = \frac{H_n^{(2)}H_n^{(1)} - (x_1/x_2)K_n^{(1)}[\mu m_1 A_{n2}/\mu_1 - D_n^{(1)}(m_1x_1)A_{n1}]t}{H_n^{(2)}L_n^{(1)} - [\mu m_2 C_{n2}/\mu_2 - D_n^{(1)}(m_2x_2)C_{n1}][\mu m_1 A_{n2}/\mu_1 - D_n^{(1)}(m_1x_1)A_{n1}]}, \qquad r \leqslant b \quad \text{and} \quad R \leqslant b,$$

$$a_n = \frac{H_n^{(2)}H_n^{(1)} - (x_1/x_2)K_n^{(1)}[(\mu m_1/\mu_1)A_{n4} - D_n^{(1)}(m_1x_1)A_{n3}]t}{H_n^{(2)}L_n^{(1)} - [(\mu m_2/\mu_2)C_{n4} - D_n^{(1)}(m_2x_2)C_{n3}][(\mu m_1/\mu_1)A_{n4} - D_n^{(1)}(m_1x_1)A_{n3}]}, \quad r > b \quad \text{and} \quad R > b$$

will become zero elements.

So we achieve $a_n = H_n^{(1)}/L_n^{(1)}$, i.e., the expression of scattering by a single sphere [1].

3 Coherent beams scattering by a homogeneous sphere

3.1 Analysis and calculation of beam scattering by a homogeneous sphere

Now, we consider a homochromous Gaussian beam TEM_{00} propagating along the positive direction of the z-axis, and the polarization direction of the beam is in the xz plane (see Fig. 3). The beam waist is ω_0 at the plane $z = 0$, and the curvature radius of the wave front is $R = \infty$. With suppression of the time harmonic dependence factor $\mathrm{e}^{-\mathrm{i}\omega t}$, the x-component of the electric field is

$$\boldsymbol{E}_{ix}(x,y,0) = \boldsymbol{E}_0 \exp\left[\frac{-(x^2+y^2)}{\omega_0^2}\right]. \tag{23}$$

And the y-component $\boldsymbol{E}_{iy}(x, y, 0) = 0$.

Similar to plane waves, by using spherical vector wave functions, the incident beam can also be expanded as follows:

$$\boldsymbol{E}_i = \sum_{n=1}^{\infty} \boldsymbol{E}_n g_n [\boldsymbol{M}_{o1n}^{(1)} - \mathrm{i}\boldsymbol{N}_{e1n}^{(1)}], \tag{24}$$

$$\boldsymbol{H}_i = \frac{k}{\omega\mu} \sum_{n=1}^{\infty} \boldsymbol{E}_n g_n [\boldsymbol{M}_{e1n}^{(1)} + \mathrm{i}\boldsymbol{N}_{o1n}^{(1)}], \tag{25}$$

where g_n is the beam factor. When $g_n = 1$, the expressions above will be degenerated to the form of the incident of a plane wave. Taking the scattering coefficients of the plane wave as a_n^P and b_n^P, then $a_n = g_n a_n^P$ and $b_n = g_n b_n^P$.

Thus we acquired the scattering amplitude functions, which is similar to that of a plane wave as follows:

$$\begin{aligned} S_1 &= \sum_{n=1}^{\infty} \tfrac{2n+1}{n(n+1)} g_n [a_n^P \pi_n(\cos\theta) + b_n^P \tau_n(\cos\theta)], \\ S_2 &= \sum_{n=1}^{\infty} \tfrac{2n+1}{n(n+1)} g_n [a_n^P \tau_n(\cos\theta) + b_n^P \pi_n(\cos\theta)]. \end{aligned} \tag{26}$$

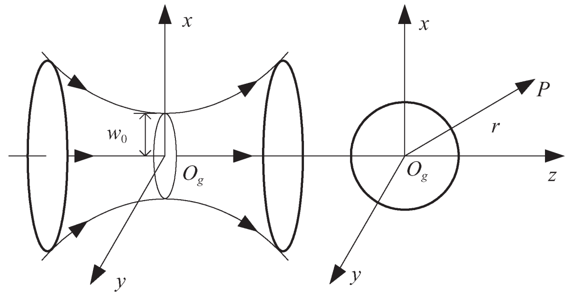

Fig. 3 Scattering of a single Gaussian beam

The corresponding attenuation cross section is

$$C_{\text{ext}} = \frac{\lambda^2}{2\pi}\sum_{n=1}^{\infty}(2n+1)|g_n|^2 \text{Re}\{a_n^P + b_n^P\},$$
$$C_{\text{sca}} = \frac{\lambda^2}{2\pi}\sum_{n=1}^{\infty}(2n+1)|g_n|^2 \text{Re}\{|a_n^P|^2 + |b_n^P|^2\}. \quad (27)$$

3.2 Analysis and calculation of coherent-beam scattering by a homogeneous sphere

Here, we still only analyze the scattering of traverse-center Gaussian beams so that the center of the sphere lies at the plane formed by the central axes of the two incident beams. Meanwhile, the scattering plane of the two beams is the same to the beams' incident plane (if the two beams are of the same axis, any plane including the central axis of the beams can be taken as the scattering plane).

Because of the linear properties of Maxwell equations and of the boundary conditions at the interface of two media, we can superpose the vector scattering fields of the two beams along with the planar angles directly, and we will easily acquire the coherent scattering result of the two beams.

What needs to be explained is that, considering the arbitrariness of the widths of beam-waists and of the distance between the beam-waists and the scattering sphere, for an exhaustive description of the scattering result, the omni-directional scattering of 360° angular directions at the scattering plane (see to Fig. 4) should be calculated. In this way, the problem becomes a plane geometrical one.

For the convenience of disposing this problem, the bisector of the angle formed by the incident directions of the two beams can be taken as the positive direction of the z-axis (see Fig. 4). By using plane geometry transformations, we can calculate the angular scattering fields of 180° at each side of the z-axis at the scattering plane. Because of the calculation results or theoretical analysis, if the two incident beams are uniform or along

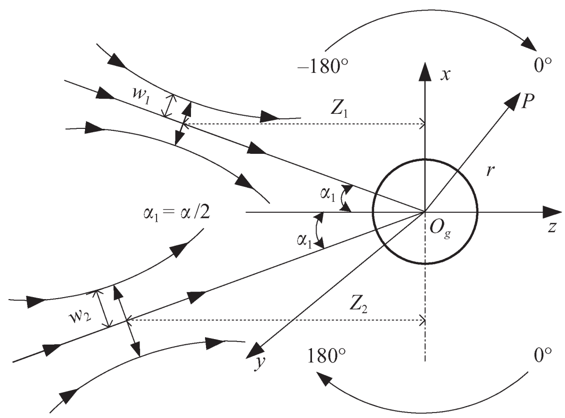

Fig. 4 Scattering of coherent beams

the same line, the scattering result is symmetrical to the z-axis.

4 Conclusions

There are mainly two methods employed in this paper: Mie's theory and the undetermined coefficient method. In deriving the scattering by bi-sphere groups, we utilized Mie's theory by performing the undetermined coefficient method on the total scattering field of the two spheres (the undetermined coefficient form of each sphere includes the reciprocity of the two spheres in each self-centered coordinate system). Then by using the translational additional theorem and the continuous boundary conditions, we acquired the needed useful scattering coefficients. In this way, the calculation is predigested and simplified, which is part of the innovations in this paper. For disposal of the coherent-beam scattering by a single homogeneous sphere, we introduced a method of simple geometrical transformation. By using the linear properties (capable of using the superposition principle) of the electromagnetic field (can be taken as a linear system in the three-dimensional Cartesian coordinate system), we just needed to superpose the separate and independent scattering field of each single beam in the scattering plane, and derive the useful total scattering field. In this way, we also simplified the complexity of calculation.

Although only the scattering of the x-polarized incident plane wave was discussed here, the fact is that for any linear polarized incident wave, and even arbitrarily polarized incident wave, the scattering result only relied on the symmetry of the scattering particles. For example, to the incident waves of the x-polarized and y-polarized of the same amplitude, the scattering fields satisfy the relation: $E_s(\varphi\ ;\ x\text{-polarized}) = E_s\ (\varphi + \pi/2;\ y\text{-polarized})$. Therefore, if we can acquire the scattering coefficients a_n and b_n of the x-polarized (or y-polarized) of the incident wave, we can also achieve all the useful scattering and absorption results of any kind of polarization of the same particles, such as scattering cross section and all the scattering matrix elements. These statements and analyses are also available for beam scattering.

Scattering of spherical particles is an important model for particle scatterings. Moreover, the analytical results of this paper are helpful and stimulative for researches about particle scatterings.

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