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RESEARCH ARTICLE

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A new TDOA algorithm based on Taylor series expansion in cellular networks

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Abstract Time difference of arrival (TDOA) is the positioning technique with the most potential in cellular mobile telecommunication systems. The Taylor series expansion method has been widely used in solving nonlinear equations for its high accuracy and good robustness. However, the performance of the Taylor's method depends highly on the initial estimation. Therefore, one new algorithm, hybrid optimizing algorithm (HOA) was proposed, which combines the Taylor series expansion method with the steepest decent method. The steepest decent method features fast convergence at the initial iteration and small computation complexity. HOA takes great advantage of both methods. Simulation results show that HOA achieves better performance on positioning accuracy and efficiency.

Keywords positioning, hybrid optimizing algorithm (HOA), steepest decent method, cellular radio networks

1 Introduction

Because of the emerging demand for location-aware services, mobile position location in wireless cellular networks has been a hot research topic in recent years for its wide applications [1,2]. According to the measurement of a specific set of signal characteristics, positioning methods can be classified as follows: angle of arrival (AOA), time of arrival (TOA), time difference of arrival (TDOA), frequency difference of arrival (FDOA), carrier phase of arrival (POA), phase difference of arrival (PDOA) and hybrid techniques composed of two or more of the above methods [3]. TDOA has been found the most promising technique in wireless cellular networks, and has been applied in IS-95 CDMA and GSM [4]. When employing TDOA measurements, a set of nonlinear location equations will be set up thereafter. Usually, these equations can be solved after being linearized. Fang [5] gave an exact solution when the number of equations equates that of unknown coordinates. This solution, however, cannot make use of extra measurements when there are extra sensors, to improve position accuracy. The more general situation based on least squares algorithm with extra measurements was considered by Friedlander [6]. Although a closed-form solution has been developed, the estimators are not optimum. Chan gave a closed-form, non-iterative solution utilizing the least squares algorithm two times, which performs well when the TDOA estimation errors are small. However, as the estimation errors increase, the performance declines quickly [7]. The Taylor-series method [8] is commonly employed in terms of solving nonlinear equations due to its properties of high accuracy and good robustness. It is an iterative method under the prerequisite that the initial guess in the particular condition is close to the true solution to avoid local minima. However, the selection of such a starting point is not simple in practice.

Thus, a new algorithm called hybrid optimizing algorithm (HOA) is proposed, which starts with the steepest decent method with properties of fast convergence at the initial iterativeness and small computation complexity. When an initial guess is close to the true solution, the convergence of the steepest decent method becomes slow, so the Taylor-series method is applied after that and the final solutions can be worked out. HOA takes great advantage of both the Taylor series expansion method and the steepest decent method to optimize the whole iterative process. The analysis and simulation results are presented in the end.

2 The proposed HOA algorithm

When TDOA is measured based on the TDOA cellular networks, a set of equations can be described as follows:

$$R_{i,1} = c(t_i - t_1) = c\Delta\tau_i = R_i - R_1, \quad (1)$$

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where

$$R_i = \sqrt{(X_i - x)^2 + (Y_i - y)^2}, \quad i = 1, 2, \ldots, N. \qquad (2)$$

Here, (X_i, Y_i) is the base station (BS) coordinate, (x, y) is the mobile station (MS) location, R_i is the distance between the BS and MS, N is the number of BS, c is the light speed, and $\Delta\tau_i$ is the TDOA between the service BS and the ith BS_i. From a geometric perspective, each equation presents a hyperbolic curve. Equation (1) is a set of nonlinear equations. The solution to the nonlinear equations, which needs to be linearized first, is in effect equal to a non-constrained minimization problem. The Taylor series expansion method has been widely used in solving nonlinear equations for its high accuracy and good robustness.

2.1 Taylor series method

Equation (1) can be rewritten as a function:

$$f_i(x,y) = \sqrt{(x - X_{i+1})^2 + (y - Y_{i+1})^2} - \sqrt{(x - X_1)^2 + (y - Y_1)^2}, \quad i = 1, 2, \ldots, N-1. \qquad (3)$$

Let $\hat{t}_i$ be the corresponding time of arrival at BS_i. Then,

$$f_i(x,y) = \hat{d}_{i+1,1} + \varepsilon_{i+1,1}, \quad i = 1,2,\ldots,N-1, \qquad (4)$$

where

$$\hat{d}_{i+1,1} = c(\hat{t}_{i+1} - \hat{t}_1), \qquad (5)$$

and $\varepsilon_{i,1}$ is the corresponding range differences estimation error with covariance R.

If (x_0, y_0) is the initial guess of the MS coordinates, then

$$x = x_0 + \delta_x, \quad y = y_0 + \delta_y. \qquad (6)$$

Expanding Eq. (3) in Taylor series and retaining the first two terms produce:

$$f_{i,0} + a_{i,1}\delta_x + a_{i,2}\delta_y \approx \hat{d}_{i+1,1} + \varepsilon_{i+1,1}, \; i = 1, 2, \ldots, N-1, \qquad (7)$$

where

$$\begin{cases} f_{i,0} = f_i(x_0, y_0) \\ a_{i,1} = \left.\dfrac{\partial f_i}{\partial x}\right|_{x_0,y_0} = \dfrac{X_1 - x_0}{\hat{d}_1} - \dfrac{X_{i+1} - x_0}{\hat{d}_{i+1}} \\ \hat{d}_i = \sqrt{(x_0 - X_i)^2 + (y_0 - Y_i)^2} \\ a_{i,2} = \left.\dfrac{\partial f_i}{\partial y}\right|_{x_0,y_0} = \dfrac{Y_1 - y_0}{\hat{d}_1} - \dfrac{Y_{i+1} - y_0}{\hat{d}_{i+1}} \end{cases}. \qquad (8)$$

Equation (7) can be rewritten as

$$\boldsymbol{A}\delta = \boldsymbol{D} + \boldsymbol{e}, \qquad (9)$$

where

$$\boldsymbol{A} = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \\ \vdots & \vdots \\ a_{N-1,1} & a_{N-1,2} \end{bmatrix}, \; \delta = \begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix},$$

$$\boldsymbol{D} = \begin{bmatrix} \hat{d}_{2,1} - f_{1,0} \\ \hat{d}_{3,1} - f_{2,0} \\ \vdots \\ \hat{d}_{N,1} - f_{N-1,0} \end{bmatrix}, \; \boldsymbol{e} = \begin{bmatrix} \varepsilon_{2,1} \\ \varepsilon_{3,1} \\ \vdots \\ \varepsilon_{N,1} \end{bmatrix}.$$

The weighted least square estimator for Eq. (9) produces

$$\delta = \left[\boldsymbol{A}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{A}\right]^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{D}. \qquad (10)$$

2.2 Steepest decent based method for initial coordinates correction

From the above analysis, the convergence of the Taylor series expansion method and the convergence speed directly depend on the choice of the MS initial coordinates. This iterative method must start with an initial guess that is close to the true solution to avoid local minima. The selection of such a starting point is not simple in practice.

To solve this problem, steepest decent method with the properties of fast convergence at the initial iteration and small computation complexity is applied at the first several iterations to get corrected MS coordinates that satisfy the Taylor series expansion method. The algorithm is described as follows.

Equation (4) can be rewritten as

$$\varphi_i(x,y) = f_i(x,y) - \hat{d}_{i+1,1} + \varepsilon_{i+1,1}, \quad i = 1,2,\ldots,N-1. \qquad (11)$$

Construct a set of module functions from Eq. (11),

$$\Phi(x,y) = \sum_{i=1}^{N-1} [\varphi_i(x,y)]^2. \qquad (12)$$

The solution to Eq. (11) is translated to compute the point of minimum Φ. In geometry, $\Phi(x,y)$ is a three-dimensional curve, the minimum point equates to the tangent point between $\Phi(x,y)$ and Oxy. In the region D of $\Phi(x,y)$, any point is passed through by an equal high line. If starting with an initial guess (x_0,y_0) in the region D, declining $\Phi(x,y)$ in the direction of steepest descent until $\Phi(x,y)$ declines to its minimum, we can obtain the solution.

Usually, the normal direction of an equal high line is the direction of the gradient vector of $\Phi(x,y)$, which is denoted by

$$\boldsymbol{G}=\left(\frac{\partial\Phi}{\partial x},\frac{\partial\Phi}{\partial y}\right)^{\mathrm{T}}. \tag{13}$$

The opposite direction to the gradient vector is the steepest descent direction.

The specific steps are as follows.

Suppose given (x_0,y_0) is an approximate solution, compute the gradient vector at this point:

$$\boldsymbol{G}_0=(g_{10},g_{20})^{\mathrm{T}},$$

where

$$\begin{cases} g_{10}=\left.\dfrac{\partial\Phi}{\partial x}\right|_{(x_0,y_0)}=2\left[\displaystyle\sum_{i=1}^{N-1}\left(\frac{\partial\varphi_i}{\partial x}\right)\varphi_i\right]_{(x_0,y_0)} \\ g_{20}=\left.\dfrac{\partial\Phi}{\partial y}\right|_{(x_0,y_0)}=2\left[\displaystyle\sum_{i=1}^{N-1}\left(\frac{\partial\varphi_i}{\partial y}\right)\varphi_i\right]_{(x_0,y_0)} \end{cases} \tag{14}$$

Then, start from (x_0,y_0), cross an appropriate step-size in the direction of $-\boldsymbol{G}_0$. λ is the step-size parameter. Then get a new point (x_1,y_1) as

$$\begin{cases} x_1=x_0-\lambda g_{10} \\ y_1=y_0-\lambda g_{20} \end{cases}. \tag{15}$$

Choose an appropriate λ in order to let (x_1,y_1) be the relative minimum in $-\boldsymbol{G}_0$, i.e., $\Phi(x_1,y_1)\approx\min\{\Phi(x_0-\lambda g_{10},y_0-\lambda g_{20})\}$.

To fix on another approximation close to (x_0,y_0), expand $\varphi_i(x_0-\lambda g_{10},y_0-\lambda g_{20})$ at (x_0,y_0). Omit the high order terms in the expansion and keep only the zero and first order terms of λ, then get the approximation of Φ as follows:

$$\Phi(x_0-\lambda g_{10},y_0-\lambda g_{20})=\sum_{i=1}^{N-1}[\varphi_i(x_0-\lambda g_{10},y_0-\lambda g_{20})]^2$$
$$\approx\left\{\sum_{i=1}^{N-1}(\varphi_i)^2-2\lambda\left[\sum_{i=1}^{N-1}\varphi_i\left(g_{10}\frac{\partial\varphi_i}{\partial x}+g_{20}\frac{\partial\varphi_i}{\partial y}\right)\right]\right.$$
$$\left.+\lambda^2\left[\sum_{i=1}^{N-1}\left(g_{10}\frac{\partial\varphi_i}{\partial x}+g_{20}\frac{\partial\varphi_i}{\partial y}\right)^2\right]\right\}_{(x_0,y_0)}.$$

Let $\partial\Phi/\partial\lambda=0$, then

$$\lambda=\left[\frac{\displaystyle\sum_{i=1}^{N-1}\varphi_i\left(g_{10}\frac{\partial\varphi_i}{\partial x}+g_{20}\frac{\partial\varphi_i}{\partial y}\right)}{\displaystyle\sum_{i=1}^{N-1}\left(g_{10}\frac{\partial\varphi_i}{\partial x}+g_{20}\frac{\partial\varphi_i}{\partial y}\right)^2}\right]_{(x_0,y_0)}. \tag{16}$$

Subtract Eq. (16) from Eq. (15), we obtain a new (x_1, y_1), and consider it as a relative minimum point of Φ in the direction of $-G_0$. Then start at this new point (x_1,y_1), update the position estimate according to the above steps until Φ is sufficiently small.

3 Analysis procedure and simulation results of HOA

In general, the convergence of the steepest descent method is fast when the initial guess is far from the true solution, otherwise, the convergence rate will slow down. The Taylor series expansion method has been widely used in solving nonlinear equations for its high accuracy and good robustness. This method performs well under the condition that the initial estimation is close to the true solution. However, it converges slowly or even not convergent when the initial estimation is far from the true solution. Therefore, HOA is proposed in the combination of both Taylor series expansion method and steepest descent method, taking advantage of both methods to optimize the whole iterative process and improve positioning accuracy and efficiency.

In HOA, at the beginning of iteration, steepest descent method is applied to let the rough initial guess be close to the true solution. Then, a further precise adjustment is implemented by Taylor series expansion method to make sure that the final estimation is close enough to the true solution. HOA has the properties of good convergence and improved efficiency. The specific flow is presented as follows.

1) Give a free initial guess (x_0,y_0), compute $\frac{\partial\varphi_i}{\partial x}$, $\frac{\partial\varphi_i}{\partial y}$, $i=1, 2, \ldots, N-1$.

2) Compute the gradient vector g_{10}, g_{20} at the point (x_0, y_0) from Eq. (14).

3) Compute λ from Eq. (16).

4) Compute (x_1,y_1) from Eq. (15).

5) If $\Phi\approx0$, stop; otherwise, substitute (x_1,y_1) for (x_0, y_0), iterate step 2)-5).

6) Compute $\hat{d}_{i+1,1}$, $i=1, 2, \ldots, N-1$ from Eq. (5).

7) Compute $\hat{d}_1$, $\hat{d}_{i+1}$, $f_{i,0}$, $a_{i,1}$ and $a_{i,2}$, $i=1, 2, \ldots, N-1$ from Eq. (8).

8) Compute δ from Eq. (10).

9) Continually refine the position estimation by steps 7)–9) until δ satisfies the accuracy.

According to the above flow, the performance of the proposed HOA is evaluated via Matlab simulation software. In the simulation, we model a cellular system with one central BS and two other adjacent BS. More assistant BSs can be utilized for more accuracy. However, in cellular communication systems, one of the main design philosophies is to make the link loss between the target mobile and the home BS as small as possible, while the other link loss as large as possible to reduce the interference and to increase the signal-to-interference ratio for the desired communication link. This design philosophy is not favorable to position location (PL), and leads to major problems in the current PL technologies, i.e., hearability and accuracy. Considering the balance between communication link and position accuracy, two assistant BSs are chosen. We assume that the coordinates of the central BS is ($x_1 = 0$ m; $y_1 = 0$ m), and the two assistant BSs coordinates are ($x_2 = 2500$ m; $y_2 = 0$ m), ($x_3 = 0$ m; $y_3 = 2500$ m) respectively, MS coordinates are ($x = 300$; $y = 400$). A comparison of HOA and the Taylor series expansion method is presented.

Numerous simulation computation results are divided into the following 3 situations. In the first situation, HOA is more accurate and efficient under the precondition of the same initial guess and the accurate measured time. In the second situation, HOA is more convergent to any given initial guess than the Taylor series expansion method with the same initial guess and the accurate measured time. In the third situation, under the condition of inaccurate measured time and the same initial guess, HOA is proved to be more accurate and efficient. Simulation results are given in Tables 1, 2, 3 respectively.

As shown in Table 1, the steepest decent method performs much better at the convergence speed. Indeed, the location error of HOA is about 10^3 times smaller than that of the Taylor series expansion method. Meanwhile, the computation efficiency of the former is improved by 23.35%, which demonstrates that HOA features more accuracy and efficiency.

As shown in Table 2, when the initial guess is far from the true location, the Taylor series expansion method is not convergent while HOA is still convergent, which consequently declines the constraints of the initial guess.

As shown in Table 3, when the measurements are inaccurate, the HOA location error is smaller than that of the Taylor series expansion method by 10 times. Meanwhile, the computation efficiency is improved by 23.14%.

Table 1 Comparison of HOA and Taylor series expansion method when the initial guess is close to the true solution and the measured time is accurate

algorithm	iterative result/m	error/m	time/ms
HOA	$x = 299.9985$	$xx = -0.0015$	0.374530
	$y = 400.0006$	$yy = 0.0006$	
Taylor	$x = 301.1$	$xx = 1.1000$	0.488590
	$y = 400.4482$	$yy = 0.4482$	

Table 2 Comparison of HOA and Taylor series expansion method when the initial guess is far from the true solution and the measured time is accurate

algorithm	iterative result/m	error/m	time/ms
HOA	$x = 299.9985$	$xx = -0.0015$	1.025930
	$y = 400.0006$	$yy = 0.0006$	
Taylor	$x = +\infty$	not convergent	
	$y = +\infty$		

Table 3 Comparison of HOA and Taylor series expansion method when the initial guess is the same and the measured time is inaccurate

algorithm	iterative result/m	error/m	time/ms
HOA	$x = 301.1297$	$xx = 1.1297$	0.376400
	$y = 400.4492$	$yy = 0.4492$	
Taylor	$x = 317.8$	$xx = 17.8000$	0.489680
	$y = 396.0549$	$yy = -3.9451$	

4 Conclusions

The analysis and simulation results demonstrate that the proposed HOA that combines both Taylor series expansion method and steepest descent method takes great advantages of both methods, optimizes the whole iterative process, and improves positioning accuracy and efficiency. In the comparison with the Taylor series expansion method, it has the advantages of fast convergence and high accuracy.

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