Front. Electr. Electron. Eng. China 2008, 3(1): 79–85
DOI 10.1007/s11460-008-0004-8

RESEARCH ARTICLE

**Anquan WEI, Lianfeng SHEN**

# Performance analysis of the periodic sequence DSSS system against CW interference



**Abstract** Based on the brief account of the performance analysis result of the direct sequence spread spectrum (DSSS) system against a single tone continuous wave (CW) interference obtained from the traditional standard Gaussian approximation (SGA) hypothesis, the mathematical expression of the interference component of the symbol decision variable in the periodic sequence DSSS system under CW interference was deduced and the actual performance of the periodic sequence DSSS system against CW interference was researched through theoretical analysis and numerical simulations. The results indicate that the interference component of the symbol decision variable in the periodic sequence DSSS system under CW interference operates at a constant level or fluctuate monochromatically, which does not approach the standard Gaussian distribution, and the actual performance of the periodic sequence DSSS system against CW interference is completely different from the analytic result resorted to the standard Gaussian approximation (SGA). The bit error performance is correlative not only with the interference-signal ratio (ISR), the frequency offset and the phase of the CW interference sensitively, but also with the individual spread spectrum code sequence.

**Keywords** direct sequence spread spectrum, performance analysis, periodic sequence, CW interference, standard Gaussian approximation

Translated from *Journal on Communications*, 2007, 28(5): 1–6 [译自: 通信学报]

Anquan WEI (✉), Lianfeng SHEN
National Mobile Communications Research Laboratory, Southeast University, Nanjing 210096, China

Anquan WEI (✉)
Institute of Communications Engineering, PLA University of Science and Technology, Nanjing 210007, China
E-mail: weianquan@126.com

## 1 Introduction

Performance analysis of direct sequence spread spectrum (DSSS) communication under several familiar interference types, such as wide-band, narrow-band, single-tone and multiple-access interference, is usually based on the hypothesis of the standard Gaussian approximation (SGA) or the improved Gaussian approximation (IGA). That is, no matter what kind of interference it is, a symbol decision variable produced by the correlative de-spreader in a DSSS system receiver is regarded approximately as a Gaussian distribution. Under this hypothesis, as long as the mean of the signal component and the variance of the interference component in the symbol decision variable are estimated, the bit-error-rate (BER) can be evaluated with the expression of the error function (erfc-function or Q-function). This kind of analysis is widely applied and presented in abundant literatures for its theoretical convenience in application and conciseness in its conclusion [1–12].

For a long-code DSSS system, the period of the spread spectrum sequence is far longer than the interval of a message symbol. Each message symbol uses a different pseudo-noise (PN) sequence to spread spectrum, so that the spread spectrum sequence can be regarded as an idea random sequence with independent and identical distribution. This analytic method is accurate enough under this circumstance. But as known to us, for the ease of realizing the code synchronization and control of the cross-correlation characteristic of the codes in CDMA communication network, some DSSS systems utilize short-code to spread spectrum, where each message symbol uses the same length-limited PN sequence, such as an m-sequence or a Gold-sequence, to spread spectrum, and the spread spectrum sequence is a fixed periodic sequence with the periods equal to the interval of a message symbol. This kind of spread spectrum manner is also called periodic sequence spread spectrum. The characteristic of a periodic sequence is not the same as that of an ideal random sequence with independent and identical distribution, which in consequence results in that the SGA hypothesis in the traditional performance analysis method does not always support a periodic sequence DSSS system.

The single-tone continuous wave (CW) interference is a simple but important interference type. But in the process of theoretic analysis and simulation on the performance of a periodic sequence DSSS system, it has been found that a symbol decision variable produced by the correlative de-spreader in a periodic sequence DSSS system receiver under CW interference does not approach the standard Gaussian distribution. It has been proven that the interference component produced by the CW interference passing through the correlative de-spreader in a periodic sequence DSSS system receiver varies sinusoidally and a method to compute the frequency and magnitude is presented in Ref. [13]. This paper focuses on the practical performance analysis of a periodic sequence DSSS system under CW interference. First, the result obtained from the traditional SGA hypothesis is briefly presented, then the mathematical expression of the interference component of the symbol decision variable in a periodic sequence DSSS system under CW interference is deduced with theoretical analysis. Following that the performance of the system is simulated, and finally the results of the theoretical analysis and the numerical simulations are discussed and the conclusions are presented.

## 2 System model

For convenience of the analysis, the DSSS system with a rectangular pulse-shaping BPSK modulation is taken into account, and the chip-matched filter is equivalent to an integration-dump. The analysis model of the system is shown in Fig. 1. A message symbol interval is supposed to be $T_b$, the corresponding message symbol rate is $R_b = 1/T_b$, the chip interval of a spread spectrum code is $T_c$, and the spread multiple is $N$, and naturally $T_b = NT_c$.

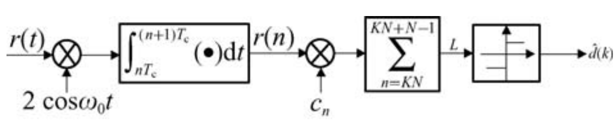


**Fig. 1** DSSS system model

Through ignoring the influence of thermal noise and assuming the case with only CW interference, the received signal can be expressed as

$$r(t) = s(t) + j(t), \tag{1}$$

where $s(t)$ is DSSS signal, and $j(t)$ is CW interference. They can be individually expressed as

$$s(t) = \sqrt{2S}d(t)\cos\omega_0 t, \tag{2}$$

$$j(t) = \sqrt{2J}\cos(\omega_j t + \theta_j). \tag{3}$$

Here, $S$ and $J$ denote the power of the input signal and interference respectively, $d(t) = \pm 1$ is the wave of the transmitted message sequence, $\omega_0$ is the signal carrier frequency, and $\omega_j$ and $\theta_j$ is the frequency and phase of CW interference respectively.

The output of the chip-matched filter is

$$r(n) = \pm\sqrt{2S}T_c + \sqrt{2J}\int_{nT_c}^{(n+1)T_c}\cos(\Delta\omega t - \theta_j)\mathrm{d}t, \tag{4}$$

where $\Delta\omega = 2\pi\Delta f = \omega_0 - \omega_j$ is the frequency offset of the interference. Thus the $k$th symbol decision variable can be expressed as

$$L(k) = \pm\sqrt{2S}T_b + L_1(k). \tag{5}$$

$L(k)$, as a variable for a fixed $k$, includes two parts, of which $\pm\sqrt{2S}T_b$ is the signal component, and $L_1(k)$ is the interference component:

$$L_1(k) = \sqrt{2J}\sum_{n=Nk}^{Nk+N-1} c_n \int_{nT_c}^{(n+1)T_c}\cos(\Delta\omega t - \theta_j)\mathrm{d}t. \tag{6}$$

## 3 Performance analysis of DSSS system against CW interference based on the SGA hypothesis

Suppose that the spread spectrum sequence is completely an ideal random sequence with independent and identical distribution, $c_i = \pm 1$. When $\Delta\omega \neq 0$, $L_1(k)$, as a variable for a fixed $k$, can be regarded as the sum of many independent random variables. According to the central limit theorem, the probability distribution of variable $L_1(k)$ approaches the Gaussian distribution gradually along with the increase of $N$, so the distribution of the variable $L(k)$ observes the Gaussian distribution with the mean of $\pm\sqrt{2S}T_b$. As long as the variance of the interference component $L_1(k)$ is estimated, its probability distribution can be determined completely, and consequently the symbol decision error rate can be evaluated. By this way, the BER expression of DSSS system under CW interference can be deduced as follows [6].

When $\omega_j = \omega_0$,

$$P_b = Q\left(\sqrt{\frac{SN}{J\cos^2\theta_j}}\right). \tag{7}$$

When $\omega_j \neq \omega_0$,

$$P_b = Q\left(\sqrt{\frac{2SN}{J\sin c^2(\Delta\omega T_c/2)\left[1+\frac{\cos(N\Delta\omega T_c-2\theta_j)\sin(N\Delta\omega T_c)}{N\sin(\Delta\omega T_c)}\right]}}\right). \quad (8)$$

When the phase of CW interference is uncertain, if the interference phase distributes uniformly between [0, 2π], the average BER can be evaluated by the following expression:

$$P_b = \frac{1}{2\pi}\int_0^{2\pi} P_b(\theta_j)\,d\theta_j. \quad (9)$$

The analytic curves and simulated results of the BER changing with interference-signal ratio ISR = $J/S$ dB in four cases of $R_b = 1$ bit/s, $N = 31$, $\theta_j = 0$, $\Delta f = 0$ Hz, 5 Hz, 10 Hz and 15 Hz are presented in Fig. 2.

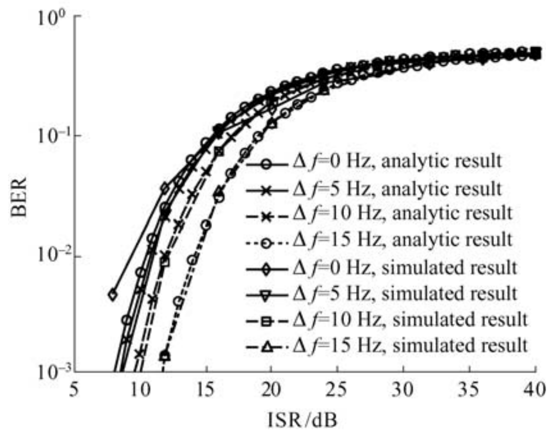


**Fig. 2** Analytic and simulated results of the performance of DSSS system against CW interference

Based on the comparison between the analytic results and the numerical simulations in Fig. 2, one can see that, for the performance of the DSSS system with ideal random sequence against CW interference, except $\Delta f = 0$ Hz, the simulated results are entirely consistent with the results of the theoretical analysis grounded on SGA hypothesis at $\Delta f = 5$ Hz, 10 Hz and 15 Hz. This demonstrates that the analysis method based on the SGA hypothesis is sufficiently accurate under the condition that the spread spectrum sequence is ideally random. Moreover, it is shown that according to the analytic results based on the SGA hypothesis, within the range of the frequency offset determined in Fig. 2, the greater the $\Delta f$, the better the BER performance.

## 4 Practical performance analysis of periodic sequence DSSS system against CW interference

In the analysis of Sect. 3, the spread spectrum sequence is supposed to be an ideal random sequence with independent and identical distribution. However, in the short-code DSSS system, every massage symbol uses the same and fixed sequence to spread spectrum, so the spread spectrum sequence is length-limited and deterministic, with characteristics differing from a non-periodic sequence that is absolutely random. This leads to the fact that the SGA hypothesis used extensively in the anti-interference performance analysis of the DSSS system is not always applicable to these kinds of short-code DSSS systems.

When the input interference obeys Gaussian distribution, such as an additive white Gaussian noise, the interference component in a symbol decision variable is still Gaussian though the spread spectrum sequence is periodic and deterministic in the DSSS system. Besides, the analytical result based on the SGA hypothesis is still exact in this case. But when the input interference is CW, the frequency and phase of the input CW interference are deterministic, and the spread spectrum code by itself of each symbol is deterministic and identical, one can see, by the symbol decision variable in Eq. (5) and its interference component expression of Eq. (6), that the interference component of the decision variable of symbol is also deterministic. In the following section, the characteristics of the interference component sequence $L_1(k)$ and its effect on the BER performance are analyzed.

From Eq. (6), when $\omega_j = \omega_0$, we obtain

$$\begin{aligned} L_1(k) &= \sqrt{2J}\sum_{n=Nk}^{Nk+N-1} c_n \int_{nT_c}^{(n+1)T_c} \cos(\Delta\omega t-\theta_j)\,dt \\ &= \sqrt{2J}\,T_c\cos\theta_j \sum_{n=Nk}^{Nk+N-1} c_n. \end{aligned} \quad (10)$$

Because the spread spectrum codes used by every message symbol are identical, Eq. (10) can be expressed as

$$L_1(k) = \sqrt{2J}\,T_c\cos\theta_j \sum_{i=0}^{N-1} c_i. \quad (11)$$

From the above expression, it can be seen that, for the CW interference with the same frequency and phase as the spread spectrum carrier, the interference component $L_1(k)$ in a symbol decision variable is a fixed value that has nothing to do with $k$. Taking no account of the influence of thermal noise, if $|L_1(k)| < \sqrt{2S}T_b$, the interference has no impact on the symbol decision. Conversely, if $|L_1(k)| > \sqrt{2S}T_b$, the system will decide the output as a fixed symbol and the BER of the system is 0.5. The relation of BER and ISR is presented as a threshold effect, and the ISR threshold can be expressed as

$$M_j = 20\lg \frac{N}{\left|\cos\theta_j \sum_{i=0}^{N-1} c_i\right|} \mathrm{dB}. \quad (12)$$

If $\omega_j \neq \omega_0$, we can obtain

$$\begin{aligned} L_1(k) &= \sqrt{2J} \sum_{n=Nk}^{Nk+N-1} c_n \int_{nT_c}^{(n+1)T_c} \cos(\Delta\omega t - \theta_j)\mathrm{d}t \\ &= \sqrt{2J}T_c \sin c\frac{\Delta\omega T_c}{2} \sum_{n=Nk}^{Nk+N-1} c_n \\ &\quad \times \cos\left[\Delta\omega\left(n+\frac{1}{2}\right)T_c - \theta_j\right] \end{aligned} \quad (13)$$

where $\sin c(x) = \sin x/x$.

Since the spread spectrum codes used by each symbol are identical, Eq. (13) can be expressed as

$$L_1(k) = \sqrt{2J}T_c \sin c\frac{\Delta\omega T_c}{2} \sum_{i=0}^{N-1} c_i \cos\left[\Delta\omega\left(Nk+i+\frac{1}{2}\right)T_c - \theta_j\right]. \quad (14)$$

Equation (14) can be expanded as

$$\begin{aligned} L_1(k) &= \sqrt{2J}T_c \sin c\frac{\Delta\omega T_c}{2} \\ &\quad \times \sum_{i=0}^{N-1} c_i \cos\left[\Delta\omega\left(i+\frac{1}{2}\right)T_c - \theta_j\right] \cos(\Delta\omega NkT_c) \\ &\quad - \sqrt{2J}T_c \sin c\frac{\Delta\omega T_c}{2} \\ &\quad \times \sum_{i=0}^{N-1} c_i \sin\left[\Delta\omega\left(i+\frac{1}{2}\right)T_c - \theta_j\right] \sin(\Delta\omega NkT_c). \end{aligned} \quad (15)$$

Let

$$A = \sqrt{2J}T_c \sin c\frac{\Delta\omega T_c}{2} \sum_{i=0}^{N-1} c_i \cos\left[\Delta\omega\left(i+\frac{1}{2}\right)T_c - \theta_j\right], \quad (16)$$

$$B = \sqrt{2J}T_c \sin c\frac{\Delta\omega T_c}{2} \sum_{i=0}^{N-1} c_i \sin\left[\Delta\omega\left(i+\frac{1}{2}\right)T_c - \theta_j\right]. \quad (17)$$

Both $A$ and $B$ have nothing to do with $k$, so $L_1(k)$ can be expressed as

$$\begin{aligned} L_1(k) &= A\cos(\Delta\omega T_b k) - B\sin(\Delta\omega T_b k) \\ &= C\cos(\Delta\omega T_b k - \phi), \end{aligned} \quad (18)$$

where

$$C = \sqrt{A^2 + B^2}, \ \phi = \mathrm{arctg}\frac{B}{A}. \quad (19)$$

Equation (18) can be divided into two cases. If $\Delta\omega T_b = 2\pi n$, namely $\Delta f = nR_b$ ($n$ is an integer number), we can get $L_1(k) = -C\cos\phi$, which is a fixed value that has nothing to do with $k$. In this case, if $C < \sqrt{2S}T_b$, the system will have no error code, while if $C > \sqrt{2S}T_b$, the system will decide the output as a fixed symbol and the BER of the system is 0.5. Besides, the sequence $L_1(k)$ is a CW waveform with the amplitude $C$ and the angle frequency $\Delta\omega T_b$. If $C < \sqrt{2S}T_b$, the system will have no error code, on the contrary, if $C > \sqrt{2S}T_b$, it corresponds to continual and periodic error code in the system.

As shown in Eq. (18), the interference component in the decision variable of symbol exhibits a constant value or single frequency fluctuation. Apparently, the rule of constant value or single frequency fluctuation does not obey the standard Gaussian distribution, and the analysis of BER should not be conducted on the SGA hypothesis, and thus the simple expression for the performance of the periodic sequence DSSS system sequence against CW interference cannot be obtained.

## 5 Performance simulations of the periodic sequence DSSS system against CW interference

Because the distribution of interference component is not Gaussian, the analytical result does not accord with the practical performance of the periodic sequence DSSS system against the CW interference if the SGA is still adopted to analyze the performance. It is difficult to give a rigorous BER expression under specific ISR, and the performance of the periodic sequence DSSS system against CW interference is analyzed through numerical simulations.

The simulation model is shown in Fig. 1. In the simulation, let $R_b = 1$ bit/s, $N = 31$, and the spread spectrum sequence be a periodic $m$-sequence generated by the generator polynomial $1 + x^2 + x^5$.

The rule of the change of $L_1(k)$ under different interference frequency offsets is depicted in Fig. 3, where the transmitted massage sequence is all null, making the mean of the decision variable to be a constant value, and the ISR is set to be 30 dB. The interference frequency offsets $\Delta f$ are set to be 0 Hz, 0.01 Hz, 0.1 Hz, and 1 Hz, which represent the two cases of $\Delta f = nR_b$ and $\Delta f \neq nR_b$ respectively. It can be seen from Fig. 3 that the interference component of the decision variable is a constant value when the frequency offset of the interference to the signal is 0 Hz and 1 Hz respectively. Otherwise, the rule of the change of the interference component of the decision variable is a single frequency

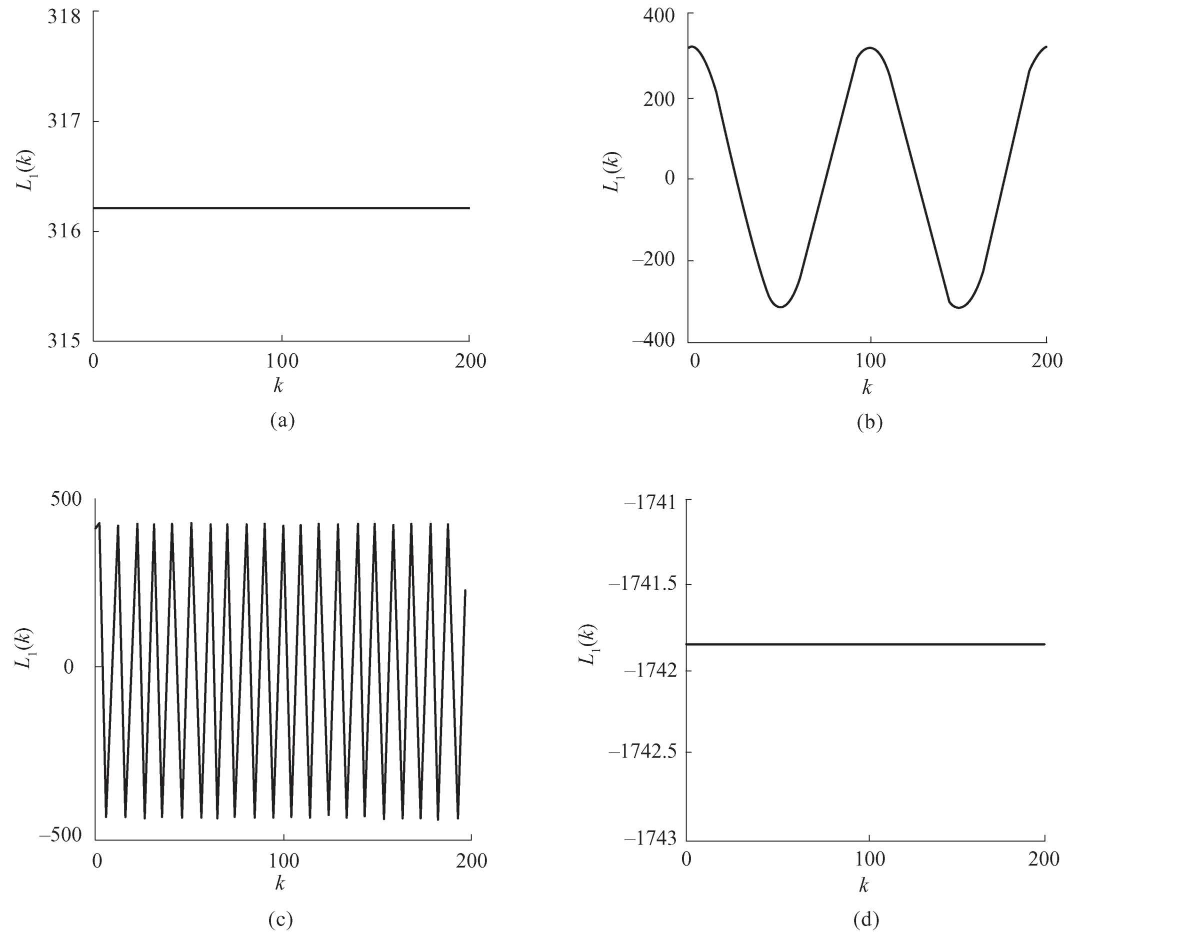


**Fig. 3** Rule of change of the interference component produced by CW interference passing through the correlative de-spreader under different interference frequency offsets. (a) $\Delta f = 0$ Hz; (b) $\Delta f = 0.01$ Hz; (c) $\Delta f = 0.1$ Hz; (d) $\Delta f = 1$ Hz

waveform with the frequency $\Delta f$ when the frequency offset of interference is 0.01 Hz and 0.1 Hz respectively.

The performance of the periodic sequence DSSS system against CW interference is demonstrated in Fig. 4, where only CW interference having the same frequency and phase with the carrier is considered and the thermal noise is ignored. In Fig. 4, the performance of BER of DSSS system has obvious threshold effect. There is no error code when the ISR is lower than 30 dB. Conversely, the BER jumps to 0.5. This result can be interpreted from the performance analysis of the periodic sequence DSSS system against CW interference in Sect. 4. As for $m$-sequence, we have $\left|\sum_{i=0}^{N-1} c_i\right| = 1$. Meanwhile, $N = 31$, $\theta_j = 0$, so the threshold of ISR of the system that can be acquired from Eq. (12) equates to $20\lg \dfrac{N}{\left|\cos\theta_j \sum_{i=0}^{N-1} c_i\right|} = 20\lg 31 = 29.8$ dB.

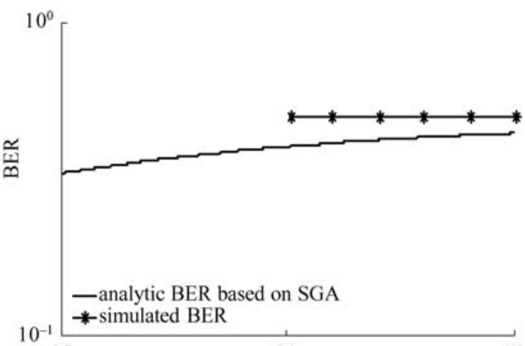


**Fig. 4** Performance of the periodic sequence DSSS system with existing CW interference only

The performance of the periodic sequence DSSS system against the CW interference under different interference frequency offsets is shown in Fig. 5 when CW interference and Gaussian white noise exist simultaneously. The ISR is fixed to a 20 dB value, and different curves represent different frequency offsets of interference to signal. Figure 5 indicates that BER performance of these curves is completely different when the frequency offset difference is set to 0.5 Hz only. This result illuminates that the CW interference deteriorates the system performance, and the extent of deterioration is closely related with the frequency offset. That is, the performance of the system varies to the frequency offset.

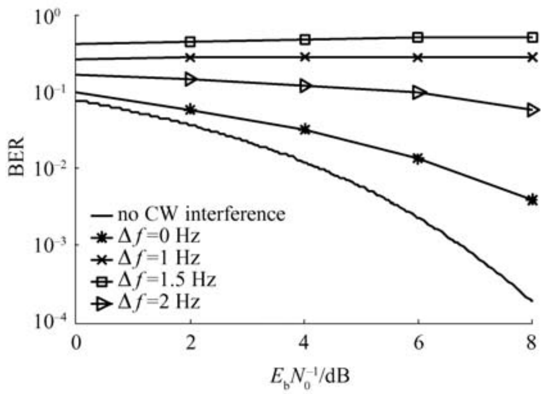


**Fig. 5** Performance of the periodic sequence DSSS system under different interference frequency offsets in the presence of CW interference and AWGN

In order to possibly clarify the relationship between the BER performance of the system and the frequency offset of the interference, the BER performance of the periodic sequence DSSS system under different interference frequency offsets is presented in Fig. 6, where CW interference and Gaussian white noise exist simultaneously. In this figure, the $E_b/N_0$ is fixed to 6.79 dB, which corresponds to the BER of $10^{-3}$ of BPSK-DSSS system with only AWGN, and the ISR of CW interference to DSSS signal is fixed to 20 dB. The two curves in the figure correspond to two kinds of spread spectrum sequence, *m*-sequences with generation polynomials $1+x^2+x^5$ and $1+x^2+x^3+x^4+x^5$, respectively. The BER of the system changes sharply with a trivial change of the interference frequency offset, which demonstrates that the performance of the periodic sequence DSSS system against CW interference is highly sensitive to the interference frequency offset if no error correction coding/decoding and interference suppression measures are taken. This phenomenon results from the fact that a correlative de-spreader corresponds to a filter with certain frequency response, which has different interference suppression effect with different frequency. The BER varies to the change of the CW interference frequency offset. Apart from that, it is shown that the BER has something to do with a specific spread spectrum sequence in the periodic sequence DSSS system. The reason is that the frequency characteristic of a correlative de-spreader is different when spread spectrum sequence differs.

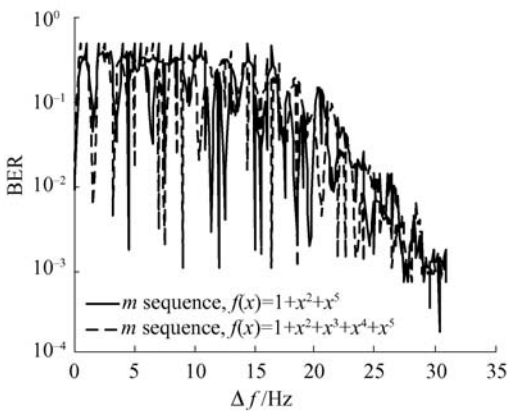


**Fig. 6** Effect of different interference frequency offsets to the performance of the periodic sequence DSSS system with different m-sequences

## 6 Conclusions

CW interference is simple. However, the analysis of the performance of the periodic sequence DSSS system against CW interference is far from what we usually imagined. Because the symbol decision variable is not close to the Gaussian distribution under CW interference, the analytical result of the practical performance of the periodic sequence DSSS system against CW interference is completely diverse from the result derived based on the SGA hypothesis, nor can the simple common expression be easily obtained. The performance of the periodic sequence DSSS system against CW interference is analyzed theoretically in this paper and some specific simulation results are presented. The theoretical analysis and the simulation result show some unique features of the performance of the periodic sequence DSSS system against CW interference, which completely differs from the analytical result based on the SGA hypothesis. The BER performance of the periodic sequence DSSS system under CW interference is not only closely related with ISR and the frequency offset and phase of the interference, but also concerned with a specific spread spectrum sequence. In order to improve the performance of the periodic sequence DSSS system against CW interference, it is very important to adopt adaptive interference suppression filters and channel coding/decoding.

**Acknowledgements** This work was supported by the National Natural Science Foundation of China (Grant No. 60472053), the Natural Science Foundation of Jiangsu Province (No. BK2003055) and the Specialized Research Fund for the Doctoral Program of Higher Education (No. 20030286017).

## References

1. Torrieri D J. Principles of Military Communication Systems. Artech House, 1981
2. Simon M K, Omura J K, Scholtz R A, et al. Spread Spectrum Communications Handbook. New York: McGraw-Hill, 2002
3. Proakis J G. Digital Communications. 4th ed. New York: McGraw-Hill, 2001
4. Rappaport T S. Wireless Communications: Principle and Practice. NJ: Prentice-Hall, 1996
5. Zhang B N, Wei A Q, Guo D S. Anti-Jamming Techniques in Communications. Beijing: China Machine Press, 2006 (in Chinese)

6. Liu C X. Anti-Jamming Performance Research on Military Spread Spectrum Anti-Jamming Microwave Relay Station. Beijing University of Aeronautics and Astronautics, 2002 (in Chinese)
7. Torrieri D J. Performance of direct-sequence system with long pseudonoise sequence. IEEE Journal on Selected Areas in Communications, 1992, 10(4): 770–781
8. Yoon Y C. Quadri-phase DS-CDMA with pulse shaping and the accuracy of the Gaussian approximation for matched filter receiver performance analysis. IEEE Transactions on Wireless Communications, 2002, 1(4): 761–768
9. Cheng J, Beaulieu N C. Accurate DS-CDMA bit-error probability calculation in Rayleigh fading. IEEE Transactions on Wireless Communications, 2002, 1(1): 3–15
10. Joon H C, Jeong Y K, Lehnert J S. Average bit-error-rate performance of band-limited DS/SSMA communications. IEEE Transactions on Communications, 2002, 50(7): 1150–1159
11. Gu X, Olafsson S. On the accuracy of the Gaussian approximation for performance analysis of spread-spectrum channels. PIMRC 2003, China: Beijing, 2003, 2037–2041
12. Yeon K J, Joon H C, Lehnert J S. Performance bounds on chip-matched-filter receivers for band-limited DS/SSMA communications. IEEE Transactions on Communications, 2005, 53(1): 131–141
13. Li Z Q, Zhang B J, Li G X. Analysis of anti-jamming performances of periodic PN sequences under CW interference. Journal of University of Electronic Science and Technology of China, 2004, 33(2): 121–124