Front. Electr. Electron. Eng. China 2007, 2(4): 432–434
DOI 10.1007/s11460-007-0080-1

RESEARCH ARTICLE

YIN Zhongke, WANG Jianying, Pierre Vandergheynst

A fast algorithm for image reconstruction based on sparse decomposition

Abstract It is very slow at present to reconstruct an image from its sparse decomposition results. To overcome this one of the main drawbacks in image sparse decomposition, the property of the energy distribution of atoms is studied in this paper. Based on the property that energy of most atoms is highly concentrated, an algorithm is proposed to fast reconstruct an image from atoms' parameters by limiting atom reconstruction calculating within the atom energy concentrating area. Moreover, methods for fast calculating atom energy and normalization are also put forward. The fast algorithm presented in this paper improves the speed of the image reconstructing by approximately 32 times without degrading the reconstructed image quality.

Keywords image processing, sparse decomposition, matching pursuit, image reconstruction

1 Introduction

In order to realize more flexible, concise and self-adaptive representation for signals, one new method for signal decomposition on an over-complete dictionary on the basis of the wavelet analysis was proposed in Ref. [1]. The method can achieve a very concise sparse representation for signals and the process of obtaining signal sparse representation is called signal sparse decomposition. Because of its excellent properties, the representation method is soon extended from one-dimensional signals to two-dimensional images [2]. Since then, many algorithms have been developed to sparsely decompose images, among which matching pursuit (MP) is the most commonly used. Almost at the same time, sparse decomposition of images has been applied in many aspects of image processing successfully, such as image compression, noise removing, image recognition and so on [2–4]. Sparse decomposition of images has drawn more and more researchers' attention and the high tide of research on it was coming because of its successful application in many aspects of image processing. In 2003, IEEE ICIP especially discussed about sparse decomposition of images for the first time [5].

Translated from *Journal of University of Electronic Science and Technology of China*, 2006, 35(4): 447–449 [译自: 电子科技大学学报]

YIN Zhongke (✉), WANG Jianying, Pierre Vandergheynst
School of Information Science and Technology, Southwest Jiaotong University, Chengdu 610031, China
E-mail: zkyin@home.swjtu.edu.cn

2 Image sparse decomposition and reconstruction

Suppose that the researched image is f and its size $M_1 \times M_2$, where M_1 and M_2 are the length and width of the image respectively. If the image is decomposed on a set of complete and orthogonal bases, the basis number of the set is $M_1 \times M_2$. Because of the orthogonal property of the bases, the distribution of them in the space consisting of images is sparse. Therefore, the energy of the decomposed image is dispersedly distributed on different bases after decomposition. When the image is represented with one set of bases, the dispersive distribution of the energy leads to non-concise representation of the image using the combination of the bases. In other words, the representation of the image is not sparse. Non-sparse representation does not benefit for later image processing, such as recognition, compression and so on. In order to obtain sparse representation of images, bases must be dense enough in the space consisting of images. As a result, the orthogonal property of the bases will not be guaranteed. Then the bases in fact are not bases, but should be renamed as atoms. The set of atoms is over-complete and called over-complete dictionary of atoms. The result of the image decomposition on an over-complete dictionary of atoms must be sparse.

Let $D = \{g_\gamma\}_{\gamma \in \Gamma}$ be an over-complete dictionary used in sparse decomposition of images, where g_γ is the atom defined by the parameter group γ. Different methods may be used to constitute atoms, leading to different parameters and numbers in the parameter group γ. The size of atom g_γ is the same as that of the image, and the atom should be normalized, that is $\|g_\gamma\| = 1$. Γ is the set of the parameter group γ. Because the atom dictionary is over-complete, the number of the parameter groups should be much larger than the size of the decomposed image. If p is the number of atoms in the over-complete dictionary $D = \{g_\gamma\}_{\gamma \in \Gamma}$, $p \gg M_1 \times M_2$. By sparsely decomposing one image, a linear representation of the image can be obtained [2]

$$f = \sum_{k=0}^{M_1 \times M_2} \langle R^k f,\, g_{\gamma_k} \rangle g_{\gamma_k} \tag{1}$$

where $\langle R^k f, g_{\gamma_k} \rangle$ is the projection of the image f or the image residual $R^k f$ on the selected best atom g_{γ_k}. Due to convergent characters of $\|R^k f\|$ [1,2], the important parts of image can be represented by minor atoms (compared with the size of the image). It is

$$f = \sum_{k=0}^{n-1} \langle R_k f,\, g_{\gamma_k} \rangle g_{\gamma_k} \tag{2}$$

Equation (2) and the condition $n \ll M_1 \times M_2$ show the meaning of sparse representation completely.

The process of image sparse decomposition is obtaining parameters of n atoms which represent major components of the image and the weights of the corresponding atom g_γ in the image f or the image residual $R^k f$. The process of image reconstruction is the process of approximate reconstruction of the original image from the above parameters.

At present, most researches are concentrated on the process of image sparse decomposition, while little on image reconstruction. The reason is that the computational burden of image sparse decomposition is very large. The problem of computational burden in image reconstruction is also outstanding in many applications. For example, when browsing images on internet, the time spent in reconstructing the images is much longer than that in transmitting them if the images are compressed, coded and transmitted on the basis of the result of image sparse decomposition. Generally speaking, it takes several minutes to reconstruct the image with Eq. (2) at present, but this speed is not endurable in image browsing. To improve the speed in image reconstruction with Eq. (2), in this paper, a new algorithm was proposed on the basis of the energy property and formation methods of atoms which constitute the image.

3 Fast algorithms for image reconstruction

3.1 Energy property of the atom

For the atom used in image sparse decomposition, it is true that the energy of one atom is highly concentrated in the center area of the atom just as the energy distribution in real physical atoms as shown in Figs. 1 and 2. One atom used in image sparse decomposition and its energy distribution are represented in Figs. 1 and 2 respectively. In Fig. 2, the energy is concentrated in the center of the atom, as the white area shows. On the other hand, in the rest area in the atom, as the black part shows, there is almost no energy.

Fig. 1 Atom

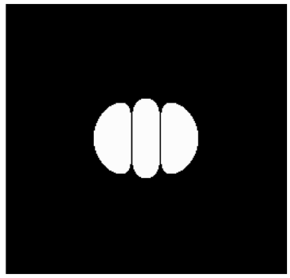

Fig. 2 Energy distribution of atom

3.2 Fast Algorithm 1 for image reconstruction

Compared with image reconstruction based on coefficients of discrete cosine transform (DCT) or wavelet transform (WT), the computational load of reconstructing one image with Eq. (2) is much larger on the basis of the parameters of image sparse decomposition. Each time one atom g_{γ_k} is obtained according to parameters, one time a function which is determined by the selection of atoms will be calculated [6]. The essential reason of huge calculation burden is that the calculation scale of the function is the same as the size of the image.

It is found from this research that it is not necessary to calculate on this scale. Because the energy of the atom is mainly concentrated in the center, it just needs to calculate its center when calculating the atom. When the center part of the atom is big, it is called big atom. Otherwise, it is called small atom when the center part is small. It is found that, in order to represent the image, the number of big atoms used is very finite, while the small atoms are mostly used, for only small atoms represent the details of the image and big atoms represent the background of the image. By calculating the center of the atom instead of the whole atom the calculating speed will be greatly increased, and as a result, the speed of image reconstruction according to Eq. (2) will be increased. If the image is reconstructed using this fast algorithm and the selected center is suitable, the quality of reconstructed image will not be degraded.

3.3 Fast Algorithm 2 for image reconstruction

When the image is decomposed sparsely, in order to represent the image with fewer parameters, the energy parameters of the atoms are not considered as the parameters in the results of sparse decomposition. This is because they are related with the scale parameters of the atoms, from which the energy of atoms can be worked out. When the image is being reconstructed with Eq. (2), the atom g_{γ_k} itself must be normalized. Quite a lot of calculation in the process of image reconstruction is cost in calculating the energy of atoms and the normalization process. For one atom, once its scale parameters are determined, its energy is determined too. Therefore, it is not necessary to calculate the energy of the atom in the process of image reconstruction. It only needs to construct a table according to the relationship between the scale and energy. The energy of the atoms can be obtained through searching the table with scale parameter of every parameter group. In this way the normalization process for atom g_{γ_k} is simplified and the atom formation speed is accelerated. Consequently the speed of image reconstruction is improved accordingly.

In addition, the above two fast algorithms can be combined serially together to further improve the image reconstruction speed.

4 Experimental results

The standard Lena image with the size of $256 \times 256 \times 8$ is used in the experiment, and the atom conformation method of the over-complete dictionary mentioned in Ref. [6] is adopted. First, the MP algorithm is applied to decompose the image into linear combination of 500 atoms. Second, Eq. (2) is used to reconstruct the original image approximately. Table 1 shows the speed comparison between the fast algorithms proposed in this paper and the algorithm for image reconstruction with Eq. (2) directly. Under the condition that the constructed images have the same quality, the speed of fast algorithms proposed in this paper is 32 times faster than that of the original algorithm. Similar results and conclusions can be obtained if other kinds of atoms are used in experiments, such as atoms in Ref. [2].

Table 1 Speed comparison between the fast algorithms and the algorithm with Eq. (2)

Algorithm	Ratio of the reconstruction time with Eq. (2) directly to that with fast algorithms
Fast Algorithm 1 for image reconstruction	20
Fast Algorithm 2 for image reconstruction	1.6
(Algorithm 1) + (Algorithm 2)	32

Acknowledgements This work is supported by the National Natural Science Foundation of China (Grant No. 60602043), China Scholarship Council Found (No. 2001-3048) and Applied Foundational Research of Sichuan Province (No. 03JY029-048-2).

References

1. Mallat S, Zhang Z. Matching pursuit with time-frequency dictionaries. IEEE Transactions on Signal Processing, 1993, 41(12): 3 397−3 415
2. Bergeau F, Mallat S. Matching pursuit of images. In: Proceedings of IEEE-SP, Piladelphia, PA. USA: IEEE Press, 1994, 330−333
3. Neff R, Zakhor A. Very low bit-rate video coding based on matching pursuit. IEEE Transactions on Circuits and Systems for Video Technology, 1997, 7(1): 158−171
4. Phillips P. Matching pursuit filter design. In: Proceedings of the 12th IAPR International Conference on SP, Jerusalem, Israel, 1994, 57−61
5. SESSION MA-S2: Redundant representations for visual communications. In: Proceedings of IEEE ICIP, Barcelona. IEEE Press, 2003, 33−64
6. Vandergheynst P, Frossard P. Efficient image representation by anistropic refinement in matching pursuit. In: Proceedings of IEEE on ICASSP, Salt Lake City. IEEE Press, 2001, 1,757−1,760