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RESEARCH ARTICLE

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# Near theoretical upper bound new loosely synchronous complementary spreading sequences



**Abstract** A new class of loosely synchronous (LS) spreading sequences with zero correlation window (ZCW) was presented. It was constructed by making Kronecker product of orthogonal matrix and ZCW complementary sequences. This new LS sequence increases the number of perfect complementary pairs and extends ZCW within the same group. Moreover, both auto-correlation and cross-correlation of ZCW in the same group remain identical. The minimum ZCW among different groups is the same as that of basic LS sequences. The method for constructing these new LS sequences is presented and ZCW properties are also verified. The number of these new LS sequences is only smaller than theoretical upper bound by one.

**Keywords** loosely synchronous spreading sequence, Kronecker product, ZCW

## 1 Introduction

As a result of traditional spreading sequences with imperfect aperiodic auto-correlation and cross- correlation properties, multiple-access interference (MAI) and inter-symbol interference (ISI) are caused in random time-varying channels [1]. Zero correlation window (ZCW) complementary sequences are basically characterized by properties that their auto-correlation or cross-correlation vector sum is zero at a certain window shift, which can remarkably mitigate MAI and ISI. In particular, loosely synchronous (LS) spreading sequences with ZCW properties have been frequently studied for application in spread-spectrum systems and in code division multiple-access (CDMA) communications [2–4]. It is a pity that ZCW sequence length, sequence quantity and ZCW width react upon each other in a conflict. Therefore, it was proposed in Ref. [5] that group sequences are used to increase the quantity of spreading sequence. Subsequently, a kind of ZCW group sequences with ZCW existing in both internal group and cross group was presented in Ref. [6]. In this paper, a new class of strong anti-interference LS spreading sequences is constructed. New LS spreading sequences with good auto-correlation function (ACF) and cross-correlation function (CCF) can enormously reduce MAI and ISI in multi-user communication system.

This paper is organized as follows. The basic definitions and theorems of LS sequences are described in Section 2. In Section 3, the specific method for constructing new LS sequences will be showed. Section 4 investigates some properties of new LS sequences. Examples and simulation results will be drawn in Setcion 5. Finally, some main conclusions are summarized in the last section.

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## 2 Basic definitions and theorems of LS sequences

**Definition 1** Given two complex sequences $c=[c_1,c_2,\ldots,c_N]$ and $s=[s_1,s_2,\ldots,s_N]$ ($c_i$ and $s_i$ are complex numbers, and the module of $c_i$ and $s_i$ is equal to one. $i=1,2,\ldots,N$), the complementary pair of sequence $(c,s)$ is called LS sequence. The normalized aperiodic ACF of $(c,s)$ is defined as follows

$$R_{(c,s)}(\tau)=\begin{cases}\dfrac{1}{2N}\displaystyle\sum_{n=\tau+1}^{N}\left[c_nc_{n-\tau}^{*}+s_ns_{n-\tau}^{*}\right], & 0\leqslant\tau<N\\ \dfrac{1}{2N}\displaystyle\sum_{n=1}^{N+\tau}\left[c_nc_{n-\tau}^{*}+s_ns_{n-\tau}^{*}\right], & -N<\tau<0\end{cases}\tag{1}$$

where * denotes conjugate operand in this paper.

If $0<\Delta<N$ and $R_{(c,s)}(\tau)=\begin{cases}0; & 0<|\tau|\leqslant\Delta\\ 1; & \tau=0\end{cases}$, let $\tau_w=\max\{\Delta\}$. Here $\max\{\square\}$ is the maximum value function. $(c,s)$ is called window auto-complementary sequence with double side ZCW of $(c,s)$ equal to $2\tau_w+1$. Especially, when $\tau_w = N-1$, $(c,s)$ is called auto-complementary sequence.

**Definition 2** Given two distinct LS sequences $(c_1,s_1)$ and $(c_2,s_2)$ (the length of $c_i$ or $s_i$ is $N$, and $i = 1, 2$), it is defined that the normalized aperiodic CCF of $c_1$ and $c_2$ is $\boldsymbol{R}_{c_1,c_2}(\tau)$. Similarly, $R_{s_1,s_2}(\tau)$ denotes the normalized aperiodic CCF of $s_1$ and $s_2$. Then

$$R_{c_1,c_2}(\tau)=\begin{cases}\dfrac{1}{N}\displaystyle\sum_{n=\tau+1}^{N}c_{1,n}c_{2,n-\tau}^{*}, & 0\leqslant\tau<N\\ \dfrac{1}{N}\displaystyle\sum_{n=1}^{N+\tau}c_{1,n}c_{2,n-\tau}^{*}, & -N<\tau<0\end{cases}\tag{2}$$

$$R_{s_1,s_2}(\tau)=\begin{cases}\dfrac{1}{N}\displaystyle\sum_{n=\tau+1}^{N}s_{1,n}s_{2,n-\tau}^{*}, & 0\leqslant\tau<N\\ \dfrac{1}{N}\displaystyle\sum_{n=1}^{N+\tau}s_{1,n}s_{2,n-\tau}^{*}, & -N<\tau<0\end{cases}\tag{3}$$

If $0\leqslant\Delta<N$ and $(R_{c_1,c_2}(\tau)+R_{s_1,s_2}(\tau))=0(0\leqslant|\tau|\leqslant\Delta)$, let $\tau_w=\max\{\Delta\}$. $(c_1,s_1)$ and $(c_2,s_2)$ are called window cross-complementary sequences, with the double side ZCW of $(c_1,s_1)$ and $(c_2,s_2)$ equal to $2\tau_w+1$. Especially, when $\tau_w = N-1$, $(c_1,s_1)$ and $(c_2,s_2)$ are called cross-complementary sequences.

**Definition 3** If two sequences $c_1 = (c_{1,1}, c_{1,2},\dots, c_{1,N})$ and $c_2 = (c_{2,1}, c_{2,2},\dots, c_{2,N})$ are concatenated to generate a new sequence $c_1c_2 = (c_{1,1}, c_{1,2},\dots, c_{1,N}, c_{2,1}, c_{2,2},\dots, c_{2,N})$, then $c_1,c_2$ is called concatenated sequence.

**Theorem 1** If two auto-complementary sequences $(c_1,s_1)$ and $(c_2,s_2)$ (the length of $c_i$ or $s_i$ is $N$, and $i = 1, 2$) comprise cross-complementary sequences, the concatenated sequences $(c_1c_2,s_1s_2)$ or $(c_1(-c_2), s_1(-s_2))$ is also auto-complementary sequence. Simultaneously, $(c_1c_2,s_1s_2)$ and $(c_1(-c_2),s_1(-s_2))$ comprise cross-complementary sequences. Moreover, the double side ZCW between $(c_1c_2,s_1s_2)$ and $(c_2c_1,s_2s_1)$ is at least equal to $2N-1$.

**Corollary 1** If two auto-complementary sequences $(c_1,s_1)$ and $(c_2,s_2)$ (the length of $c_i$ or $s_i$ is $N$, and $i = 1, 2$) comprise cross-complementary sequences, the minimum double side ZCW of $(c_1(-c_2),s_1(-s_2))$ and $(c_2c_1,s_2s_1)$ is equal to $2N-1$. The similar correlation goes for $(c_2(-c_1),s_2(-s_1))$ and $(c_1c_2,s_1s_2)$.

## 3 Methods for constructing new LS spreading sequences

For convenience, only select 1 or −1 as the entry of basic LS sequences.

**Step 1** Choose two basic LS sequences $(c_1,s_1)$ and $(c_2,s_2)$ as initial seeds.

Here, $c_1 = [+\ +]$, $s_1 = [+\ -]$, $c_2 = [-\ +]$, $s_2 = [-\ -]$. + denotes 1, and – denotes −1.

**Step 2** Using binary tree to generate two complementary concatenated sequences pair and each pair of concatenated sequences is cross-complementary sequences.

$$\left\{\begin{matrix}c_1, s_1\\ c_2, s_2\end{matrix}\right\}\Rightarrow\left\langle\begin{matrix}\left\{\begin{matrix}c_1c_2, s_1s_2\\ c_1(-c_2), s_1(-s_2)\end{matrix}\right\}\Rightarrow\cdots\\ \left\{\begin{matrix}c_2c_1, s_2s_1\\ c_2(-c_1), s_2(-s_1)\end{matrix}\right\}\Rightarrow\cdots\end{matrix}\right.$$

Arbitrary cross-complementary sequences generated from the previous step can generate the new cross- complementary sequences according to the same binary tree structure.

**Step 3** Take all the $2^j$ LS sequences from the $j$th order of the binary tree, with their original order not changed, and then these $2^j$ LS sequences comprise a matrix $\boldsymbol{G}$.

$$\boldsymbol{G}=[\boldsymbol{g}_1^{\mathrm{T}},\dots,\boldsymbol{g}_i^{\mathrm{T}},\dots,\boldsymbol{g}_{\mathrm{M}}^{\mathrm{T}}]^{\mathrm{T}}$$

where $M = 2^j$, $\boldsymbol{g}_i = (c_{i1},c_{i2},\dots,c_{iN},s_{i1},s_{i2},\dots,s_{iN})$. $i = 1,2,\dots,M$. Here $[\cdot]^{\mathrm{T}}$ denotes the transpose of $[\cdot]$. $N$ is the length of $c$ sequence or $s$ sequence of the $j$th order.

**Step 4** Choose Hadamard matrix $\boldsymbol{H}$ of size $K\times K$ as orthogonal matrix.

Here, $\boldsymbol{H} = (h_{mn})$ $(m, n = 1, 2,\dots, K)$

Do Kronecker product to generate a new matrix $\boldsymbol{G}'$.

$$\boldsymbol{G}'=\begin{pmatrix}\boldsymbol{H}\otimes(c_{11},c_{12},\dots,c_{1N}),\boldsymbol{H}\otimes(s_{11},s_{12},\dots,s_{1N})\\ \vdots\\ \boldsymbol{H}\otimes(c_{i1},c_{i2},\dots,c_{iN}),\boldsymbol{H}\otimes(s_{i1},s_{i2},\dots,s_{iN})\\ \vdots\\ \boldsymbol{H}\otimes(c_{M1},\mathrm{c}_{M2},\dots,c_{MN}),\boldsymbol{H}\otimes(\mathrm{s}_{M1},\mathrm{s}_{M2},\dots,\mathrm{s}_{MN})\end{pmatrix}$$

where ⊗ denotes Kronecker product.

$\boldsymbol{g}_i'$ is the $i$th group of new LS spreading sequence.

$$\boldsymbol{g}_i' = (\boldsymbol{H}\otimes(c_{i1}, c_{i2},\dots, c_{iN}), \boldsymbol{H}\otimes(s_{i1}, s_{i2},\dots, s_{iN}))$$

Thus, we can construct $M$ groups of LS sequences, and each group contains $K$ LS sequences.

## 4 ACF and CCF properties of new LS spreading sequences

In this section, the correlation properties of new LS ZCW spreading sequences are investigated.

**Theorem 2** Double side ZCW of CCF within the same group is at least equal to $2N-1$ for all $\boldsymbol{g}_i'$ $(i = 1, 2,\dots, M)$. $N$ is the length of $c$ sequence or $s$ sequence.

**Proof** Without loss of generality, CCF of $\boldsymbol{g}_1'$ is proved. $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ are supposed to be the $i$th row and the $j$th row respectively in $\boldsymbol{g}_1'$. Moreover, $i\neq j$.

$$\boldsymbol{x}_i=(h_{i1}c_{11},h_{i1}c_{12},\dots,h_{i1}c_{1N},\dots,h_{iK}c_{11},h_{iK}c_{12},\dots,h_{iK}c_{1N},\\ h_{i1}s_{11},h_{i1}s_{12},\dots,h_{i1}s_{1N},\dots,h_{iK}s_{11},h_{iK}s_{12},\dots,h_{iK}s_{1N})$$

$$\begin{aligned}\boldsymbol{x}_j = (&h_{j1}c_{11}, h_{j1}c_{12}, ..., h_{j1}c_{1N}, ..., h_{jK}c_{11}, h_{jK}c_{12}, ..., h_{jK}c_{1N},\\ &h_{j1}s_{11}, h_{j1}s_{12}, ..., h_{j1}s_{1N}, ..., h_{jK}s_{11}, h_{jK}s_{12}, ..., h_{jK}s_{1N})\end{aligned}$$

When $\tau = 0$, $R_{\boldsymbol{x}_i,\boldsymbol{x}_j}(0) = \dfrac{1}{2NK}\sum_{m=1}^{K} h_{im}h^{*}_{jm}\left[R_{\boldsymbol{c}_1,\boldsymbol{c}_1}(0) + R_{\boldsymbol{s}_1,\boldsymbol{s}_1}(0)\right]$.

Because $\boldsymbol{H}$ is an orthogonal matrix, $\sum_{m=1}^{K} h_{im}h^{*}_{jm} = 0$ ($i \neq j$). That is $R_{\boldsymbol{x}_i,\boldsymbol{x}_j}(0) = 0$.

If $\tau \neq 0$, let $\tau = \tau_1 N + \tau_2$, where $0 \leqslant \tau_1 < K$, $0 < \tau_2 < N$.

When $0 < \tau < KN$ (if $-KN < \tau < 0$, the similar proof can be obtained), the CCF of $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ can be expressed as follows:

$$\begin{aligned}R_{\boldsymbol{x}_i,\boldsymbol{x}_j}(\tau) = &\frac{1}{2NK}\Bigg[\sum_{m=\tau_1+1}^{K} h_{im}h^{*}_{j(m-\tau_1)}\sum_{n=\tau_2+1}^{N} c_{1,n}c^{*}_{1,n-\tau_2}\\ &+ \sum_{m=\tau_1+1}^{K} h_{im}h^{*}_{j(m-\tau_1)}\sum_{n=\tau_2+1}^{N} s_{1,n}s^{*}_{1,n-\tau_2} + \sum_{l=\tau_1+2}^{K} h_{il}h^{*}_{j(l-\tau_1-1)}\\ &\bullet\sum_{k=1}^{\tau_2} c_{1,k}c^{*}_{1,N-\tau_2+k} + \sum_{l=\tau_1+2}^{K} h_{il}h^{*}_{j(l-\tau_1-1)}\sum_{k=1}^{\tau_2} s_{1,k}s^{*}_{1,N-\tau_2+k}\Bigg]\\ = &\frac{1}{2NK}\Bigg[\sum_{m=\tau_1+1}^{K} h_{im}h^{*}_{j(m-\tau_1)}R_{\boldsymbol{c}_1,\boldsymbol{c}_1}(\tau_2) + \sum_{m=\tau_1+1}^{K} h_{im}h^{*}_{j(m-\tau_1)}\\ &\bullet R_{\boldsymbol{s}_1,\boldsymbol{s}_1}(\tau_2) + \sum_{n=\tau_1+2}^{K} h_{in}h^{*}_{j(n-\tau_1-1)}R_{\boldsymbol{c}_1,\boldsymbol{c}_1}(\tau_2 - N)\\ &+ \sum_{n=\tau_1+2}^{K} h_{in}h^{*}_{j(n-\tau_1-1)}R_{\boldsymbol{s}_1,\boldsymbol{s}_1}(\tau_2 - N)\Bigg]\end{aligned}$$

It is known that $c_1$ and $s_1$ are cross-complementary sequences.

Hence, when $0 < \tau_2 < N$, according to Eq. (1), the following formulas can be obtained

$$R_{\boldsymbol{c}_1,\boldsymbol{c}_1}(\tau_2) + \mathrm{R}_{\boldsymbol{s}_1,\boldsymbol{s}_1}(\tau_2) = 0,\ R_{\boldsymbol{c}_2,\boldsymbol{c}_2}(\tau_2 - N) + R_{\boldsymbol{s}_2,\boldsymbol{s}_2}(\tau_2 - N) = 0.$$

That is $R_{\boldsymbol{x}_i,\boldsymbol{x}_j}(\tau) = 0$ (when $0 \leqslant \tau_1 < K$, $0 < \tau_2 < N$).

To sum up, $R_{\boldsymbol{x}_i,\boldsymbol{x}_j}(\tau) = 0$ (when $\tau = 0$ or $\tau \neq \pm lN$, $l = 1, 2, \ldots, K-1$).

Double side ZCW of CCF within the same group is at least equal to $2N-1$ for all $\boldsymbol{g}_i'$ ($i = 1, 2, \ldots, M$).

**Theorem 3** Double side ZCW of ACF for all sequences in $\boldsymbol{g}_i'$ ($i = 1, 2, \ldots, M$) is also at least equal to $2N-1$. $N$ is the length of $c$ sequence or $s$ sequence.

**Proof** In Theorem 2, let $i = j$. According to Eq. (1), if $\tau = 0$, $R_{x_i,x_i}(0) = 1$. If $\tau \neq 0$ and $\tau \neq \pm lN$, $R_{x_i,x_i}(\tau) = 0$. $l = 1, 2, \ldots, K-1$

Therefore, double side ZCW of ACF for all sequences in $\boldsymbol{g}_i'$ ($i = 1, 2, \ldots, M$) is at least equal to $2N-1$.

**Theorem 4** The minimum double side ZCW of CCF between $\boldsymbol{g}_i'$ and $\boldsymbol{g}_j'$ ($i \neq j$, $i, j = 1, 2, \ldots, M$) is at least equal to $2\tau_v + 1$. $2\tau_v + 1$ is the minimum double side ZCW between $\boldsymbol{g}_i$ and $\boldsymbol{g}_j$.

**Proof** According to Theorem 1 and Corollary 1, it is known that $\boldsymbol{g}_1$ and $\boldsymbol{g}_M$ have the minimum double side ZCW which is assumed to be equal to $2\tau_w + 1$. Therefore, it has only been proved that the minimum double side ZCW between $\boldsymbol{g}_1'$ and $\boldsymbol{g}_M'$ is equal to $2\tau_w + 1$.

Clearly, $\tau_w < N$.

$\boldsymbol{x}_i$ is supposed to be the $i$th row of $\boldsymbol{g}_1'$, $\boldsymbol{y}_j$ is the $j$th row of $\boldsymbol{g}_M'$, where

$$\begin{aligned}\boldsymbol{x}_i = (&h_{i1}c_{11}, h_{i1}c_{12}, ..., h_{i1}c_{1N}, ..., h_{iK}c_{11}, h_{iK}c_{12}, ..., h_{iK}c_{1N},\\ &h_{i1}s_{11}, h_{i1}s_{12}, ..., h_{i1}s_{1N}, ..., h_{iK}s_{11}, h_{iK}s_{12}, ..., h_{iK}s_{1N})\\ \boldsymbol{y}_j = (&h_{j1}c_{M1}, h_{j1}c_{M2}, ..., h_{j1}c_{MN}, ..., h_{jK}c_{M1}, h_{jK}c_{M2}, ..., h_{jK}c_{MN},\\ &h_{j1}s_{M1}, h_{j1}s_{M2}, ..., h_{j1}s_{MN}, ..., h_{jK}s_{M1}, h_{jK}s_{M2}, ..., h_{jK}s_{MN})\end{aligned}$$

The double side ZCW between $(c_1, s_1)$ and $(c_M, s_M)$ is equal to $2\tau_w + 1$. When $0 \leqslant \tau \leqslant \tau_w$ (if $-\tau_w \leqslant \tau < 0$, the proof is similar, the CCF of $\boldsymbol{x}_i$ and $\boldsymbol{y}_j$ can be expressed as follows:

$$\begin{aligned}R_{\boldsymbol{x}_i,\boldsymbol{y}_j}(\tau) = &\frac{1}{2NK}\Bigg[\sum_{m=1}^{K} h_{im}h^{*}_{jm}\sum_{n=\tau+1}^{N} c_{1,n}c^{*}_{M,n-\tau}\\ &+ \sum_{m=1}^{K} h_{im}h^{*}_{jm}\sum_{n=\tau+1}^{N} s_{1,n}s^{*}_{M,n-\tau} + \sum_{l=2}^{K} h_{il}h^{*}_{j(l-1)}\\ &\bullet\sum_{k=1}^{\tau} c_{1,k}c^{*}_{M,N-\tau+k} + \sum_{l=2}^{K} h_{il}h^{*}_{j(l-1)}\sum_{k=1}^{\tau} s_{1,k}s^{*}_{M,N-\tau+k}\Bigg]\\ = &\frac{1}{2NK}\Bigg[\sum_{m=1}^{K} h_{im}h^{*}_{jm}R_{\boldsymbol{c}_1,\boldsymbol{c}_M}(\tau) + \sum_{m=1}^{K} h_{im}h^{*}_{jm}R_{\boldsymbol{s}_1,\boldsymbol{s}_M}(\tau)\\ &+ \sum_{n=2}^{K} h_{in}h^{*}_{j(n-1)}R_{\boldsymbol{c}_1,\boldsymbol{c}_M}(\tau - N)\\ &+ \sum_{n=2}^{K} h_{in}h^{*}_{j(n-1)}R_{\boldsymbol{s}_1,\boldsymbol{s}_M}(\tau - N)\Bigg]\end{aligned}$$

According to Eqs. (2) and (3), when $0 \leqslant \tau \leqslant \tau_w$,

$$R_{\boldsymbol{c}_1,\boldsymbol{c}_M}(\tau) + R_{\boldsymbol{s}_1,\boldsymbol{s}_M}(\tau) = 0,\ R_{\boldsymbol{c}_1,\boldsymbol{c}_M}(\tau - N) + R_{\boldsymbol{s}_1,\boldsymbol{s}_M}(\tau - N) = 0.$$

To sum up, $R_{\boldsymbol{x}_i,\boldsymbol{y}_j}(\tau) = 0$ (when $-\tau_w \leqslant \tau \leqslant \tau_w$). The minimum double side ZCW of CCF between $\boldsymbol{g}_i'$ and $\boldsymbol{g}_j'$ ($i \neq j$, $i, j = 1, 2, \ldots, M$) is at least equal to $2\tau_v + 1$.

**Corollary 2** According to the proof above, $\boldsymbol{H}$ can be any orthogonal matrix. For convenience, this paper selects Hardamard matrix as orthogonal matrix.

**Corollary 3** The number of cross-complementary sequences of each sequence within $\boldsymbol{g}_i'$ ($i = 1, 2, \ldots, M$) is at least equal to $K$. $K$ is the dimension of row of matrix $\boldsymbol{H}$.

## 5 Examples and simulation results

The length of new LS sequence is $2KN$, and the quantity of new LS sequence is $KM$. For any given sequence length, if $N$ increases, $K$ will decreases, and vice versa. Therefore, a trade-off between sequence quantity and ZCW width need to be considered according to practical radio channels. Taking 128 chips length of new LS sequence as an example, i.e. $128 = 2KN$, the following two schemes are considered.

**Scheme 1** Let $N = 16$ and $K = 4$. To select upper half eight sequences from the 4th branch of the binary tree as matrix $\boldsymbol{G}$. The minimum double side ZCW of these eight sequences is $2\tau_w + 1$, i.e. 7. The method in Section 2 is applied to construct eight group sequences, and each group has four sequences.

Let $e_{ij}$ denote the $j$th sequence in $i$th group, Where $i = 1, 2, \ldots, 8$ and $j = 1, 2, 3, 4$. Without loss of generality, $e_{11}$ is selected to verify ACF and CCF of new LS sequences. In Fig. 1, the ACF of $e_{11}$ and CCF of $e_{11}$ with $e_{12}$ are given. The minimum double side ZCW of ACF and CCF within the same group is equal to $2N-1$, i.e. 31.

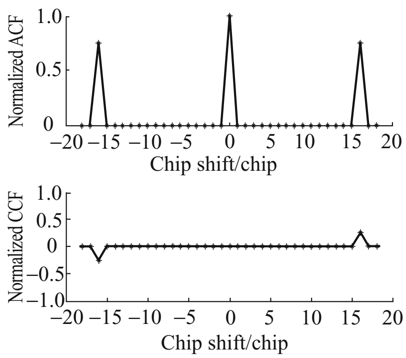


**Fig. 1** ACF of $e_{11}$ and CCF of $e_{11}$ with $e_{12}$

In Fig. 2, both the CCF of $e_{11}$ with $e_{21}$ and the CCF of $e_{11}$ with $e_{41}$ are given. It is apparent that arbitrary sequence in $\boldsymbol{g}_1'$ and arbitrary sequence in $\boldsymbol{g}_2'$ comprise cross-complementary sequences. The minimum double side CCF ZCW of $e_{11}$ and $e_{41}$ is equal to 15.

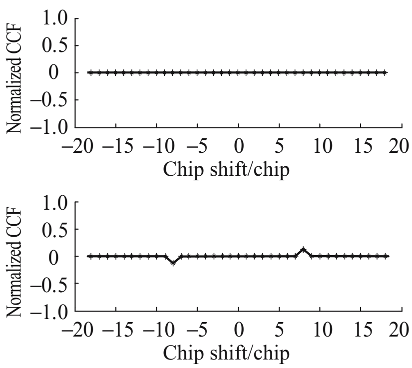


**Fig. 2** CCF of $e_{11}$ with $e_{21}$ and $e_{11}$ with $e_{41}$

Figure 3 shows the CCF of $e_{11}$ with $e_{51}$ and the CCF of $e_{11}$ with $e_{71}$. The minimum double side ZCW of $e_{11}$ and $e_{51}$ (or $e_{71}$) is equal to 7.

The other sequences of $\boldsymbol{g}_i'$ ($i = 1, 2, \ldots, M$) have the same properties as the first sequence of $\boldsymbol{g}_1'$.

**Scheme 2** Let $N = 8$ and $K = 8$. To take upper half four sequences from the third branch of the binary tree as matrix $\boldsymbol{G}$. The minimum double side ZCW of these four sequences is $2\tau_w + 1$, i.e. 7. The similar method is applied to construct four group sequences and each group has eight sequences.

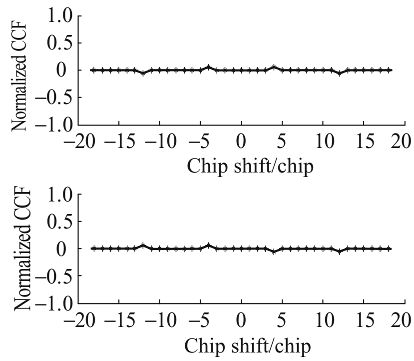


**Fig. 3** CCF of $e_{11}$ with $e_{51}$ and $e_{11}$ with $e_{71}$

Let $e_{ij}'$ denote the $j$th sequence in $i$th group using Scheme 2, where $i = 1, 2, 3, 4$ and $j = 1, 2, \ldots, 8$. Without loss of generality, $e_{11}'$ is selected to verify ACF and CCF of new LS sequences. Figure 4 shows the ACF of $e_{11}'$ and CCF of $e_{11}'$ with $e_{12}'$. The minimum double side ZCW of ACF and CCF within the same group is equal to 15.

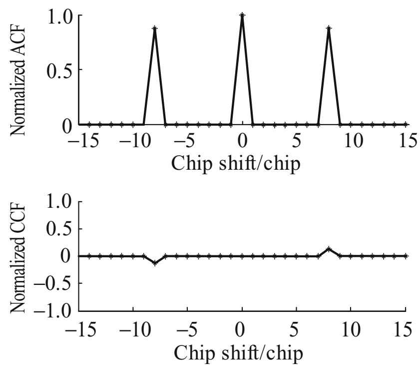


**Fig. 4** ACF of $e_{11}'$ and the CCF of $e_{11}'$ with $e_{12}'$

In Fig. 5, both the CCF of $e_{11}'$ with $e_{21}'$ and the CCF of $e_{11}'$ with $e_{41}'$ are given. It is apparent that arbitrary sequence in $\boldsymbol{g}_1'$ and arbitrary sequence in $\boldsymbol{g}_2'$ comprise cross-complementary sequences. The minimum double side ZCW of $e_{11}'$ and $e_{41}'$ is equal to 7.

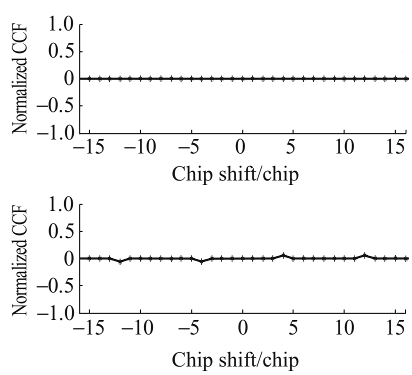


**Fig. 5** CCF of $e_{11}'$ with $e_{21}'$ and $e_{11}'$ with $e_{41}'$

The other sequences of $\boldsymbol{g}_i'$ ($i$=1, 2,…, $M$) have the same properties as the first sequence of $\boldsymbol{g}_i'$.

Comparing the two schemes, for a fixed sequence length and quantity, Scheme 1 constructs eight groups of sequences with four sequences in each group. Each sequence has four cross-complementary sequences and the double side ZCW of ACF and CCF in the same group is 31. Scheme 2 constructs four groups of sequences with eight sequences in each group. Each sequence has eight cross-complementary sequences and the double side ZCW of ACF and CCF in the same group is 15. It is obvious that if the number of cross-complementary sequences increases, the double side ZCW of ACF and CCF within the same group will decreases, and vice versa.

In schemes 1 and 2, the minimum double side ZCW among groups is equal to 7. The theoretical upper bound [7] is that when sequence length is L, there are $\lfloor L/(\tau_w+1)+1 \rfloor$ sequences with double side ZCW $2\tau_w+1$. Here $\lfloor \cdot \rfloor$ denotes to select the minimum integer.

The number of new LS spreading sequences is only smaller than theoretical upper bound by one.

## 5 Conclusion

This paper constructs a new class of LS ZCW spreading sequences by using Kronecker product of orthogonal matrix and basic LS spreading sequences. Compared with the traditional LS spreading sequences, the new ones increase the number of cross-complementary sequences for each sequence and extend ZCW width in each group. The double side ZCW of ACF and CCF in the same group remains the same. The minimum double side ZCW between groups is equal to that of basic LS spreading sequences. New LS spreading sequences with good ACF and CCF can enormously reduce MAI and ISI in multi-user communication system.

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