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RESEARCH ARTICLE

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Analysis of the diffracted current basis functions used in the hybrid MoM-PO method

Abstract The combined moment method (MoM)-physical optics (PO) approach proposed by Bilow fails in some cases. Based on the theory of diffraction and the fundamental theory of electromagnetism, Bilow's diffracted current basis function was modified both within and outside the transition regions. The improved MoM-PO technique is validated by comparison with exact solutions for a right-angled perfectly conducting wedge at normal incidence.

Keywords theory of diffraction, wedge, diffracted current basis function (DCBF)

1 Introduction

In recent years, the electromagnetic scattering from targets coated by anisotropic materials has been an area of great interest and considerable value in engineering and military applications [1–9]. There is a great need for continuing and increasing the efforts launched on the research of scattering characteristics (frequency response, spatial distribution, and polarization) of these coating targets. On the other hand, the scattering from a wedge is an important problem to study because it is a very basic configuration and can be used to represent a wide class of scatterers. Studying the wedge can provide useful insights into the role that edges exert on the scattering form complex structures.

The moment method (MoM) [10] basically employs the equations of unknown fields in integral form to determine the field distribution in a given medium. As a low-frequency technique, the method has been applied to such problems for which the structure, owing to the memory storage, is small in terms of wavelength. At the same time, the physical optics (PO) and the geometrical theory of diffraction (GTD) [11] are applicable to bodies that are arbitrarily large in the electrical sense and may subsequently be referred to as high frequency techniques. The applications of diffraction theories, however, are limited to geometries for which the diffraction coefficient is known. Therefore, a combination of MoM and PO or GTD may lead to a power tool of handling a whole host of new problems.

Brunside utilized the hybrid MoM-GTD technique to solve scattering problems of incident plane waves by perfectly conducting right-angled wedge [12]. Bilow developed a hybrid MoM-PO solution for the currents induced by incident plane waves on infinite two-dimensional wedges with faces characterized by arbitrary tensor impedances [13]. The difference of their methods is essentially a variation in DCBF. One is based on GTD and another is based on uniform geometrical theory of diffraction (UTD). The results obtained by using Bilow's diffracted current basis function (DCBF) are in good agreement with the exact solutions in general cases. However, Bilow's basis function results in grave errors in some cases. In terms of the theory of diffraction and the fundamental theory of electromagnetism, we modify their basis functions both within and outside the transition regions.

2 Comparison between Burnside's DCBF and Bilow's

The geometry for the scattering problem is illustrated in Fig. 1. The wedge has its edge along the z-axis of a cylindrical reference frame and a monochromatic plane wave with an arbitrary polarization impinges on the edge from a direction determined by the two angles ϕ' and β'. β' refers to the edge of the wedge, and the case $\beta' = \pi/2$ corresponds to normal incidence. The exterior wedge angle is $n\pi$. The face $\phi = 0$ of the wedge relies on the x–z plane. An $e^{-j\omega t}$ time dependence is assumed and suppressed.

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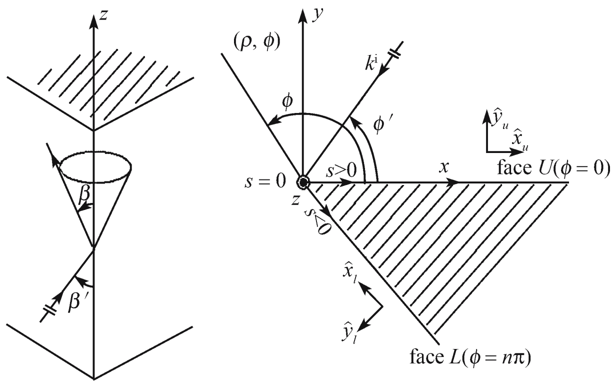

Fig. 1 Geometry for the scattering problem

The surface of the wedge is split into a MoM and a PO region (see Fig. 2). The moment method current (J^{MM}) around the edge can be defined by simple basis functions such as

$$\boldsymbol{J}^{\mathrm{MM}}=\sum_{m=1}^{N}\boldsymbol{\alpha}_m P(s-s_m) \tag{1}$$

Here, the u_n are pulse functions of unit amplitude with weight α_n

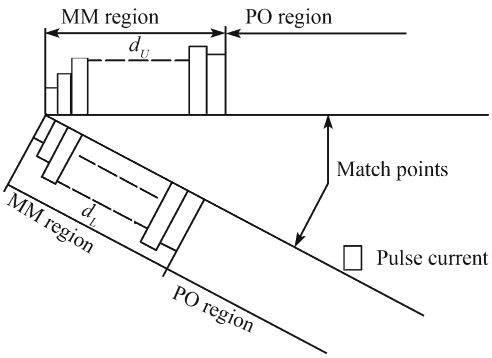

Fig. 2 Arrangement of pulse currents

In the PO region, the total current along the wedge walls is given by

$$\boldsymbol{J}^{\mathrm{o}}=\boldsymbol{J}^{\mathrm{PO}}+\boldsymbol{J}^{\mathrm{d}} \tag{2}$$

The diffracted current is associated with the diffracted field. Burnside's diffracted current basis function is

$$\frac{\exp(\mathrm{i}k_{xy}\rho)}{\sqrt{\rho}} \tag{3}$$

Bilow's diffracted current basis function can be written in the UTD form

$$f_{\mathrm{d}}(\rho,\gamma)=\mathrm{e}^{-\mathrm{i}k_{xy}\rho\cos\gamma}F_{+}\left[\sqrt{2k_{xy}\rho}\left|\cos\left(\frac{\gamma}{2}\right)\right|\right] \tag{4}$$

where k is the free-space wave number, and F_+ is the Fresnel integral

$$F_{+}(x)=\int_x^{\infty}\exp\ (\mathrm{i}t^2)\mathrm{d}t,\ k_{xy}=\sqrt{k^2-k_z^2},$$

$$\cos\gamma=\begin{cases}-\boldsymbol{k}^{\mathrm{i}}\bullet\boldsymbol{x}_{\boldsymbol{u}}/k_{xy}, & \text{when } \rho \text{ lies face } U\\ \boldsymbol{k}^{\mathrm{i}}\bullet\boldsymbol{x}_{\boldsymbol{l}}/k_{xy}, & \text{when } \rho \text{ lies face } L\end{cases}$$

$\boldsymbol{x_u}$, $\boldsymbol{x_l}$ are the face tangent unit vector (see Fig. 1). $f_{\mathrm{d}}(\rho,\gamma)$ can be rewritten as

$$f_{\mathrm{d}}(\rho,\phi)=\exp\left[-\mathrm{i}k_{xy}\rho\cos(\phi-\phi')\right]F_{+}\left(\sqrt{k_{xy}\rho[1+\cos(\phi-\phi')]}\right)$$

It is evident that Bilow's basis function stems from the transition function as follows

$$F[k_{xy}\rho(1+\cos(\phi-\phi')] \tag{5}$$

Based on the classical UTD, the dyadic diffraction coefficients that are valid both within and outside the transition regions (i.e., the regions of rapid field change adjacent to the shadow boundaries) contain four transition functions. In fact, the four transition functions take three forms

$$F[k_{xy}\rho(1+\cos(\phi\pm\phi')],\ F[k_{xy}\rho(1+\cos(\phi+\phi'-2n\pi)]$$

In the neighborhood of the shadow boundary, the dominant contribution to the diffracted field results from these terms in which the cotangent functions

$$\cot\left[\frac{\pi+(\phi\pm\phi')}{2n}\right]\ \text{and}\ \cot\left[\frac{\pi-(\phi\pm\phi')}{2n}\right]$$

are singular. Provided that the wedge face is close to a shadow boundary, the relational transition functions are

$$F[k_{xy}\rho(1+\cos(\phi\pm\phi')]\ \text{for}\ \phi'=\pi \text{ and the face } \phi=0,$$
$$F[k_{xy}\rho(1+\cos(\phi-\phi')],\ F[k_{xy}\rho(1+\cos(\phi+\phi'-2n\pi)]$$
$$\text{for}\ \phi'=\pi \text{ and the face } \phi=n\pi$$

Obviously, the above expressions are equivalent to Eq. (5).

If the wedge face is not close to a shadow boundary and ρ is sufficiently large, according to Burnside, the diffracted current basis function is of the form $\exp(\mathrm{i}k_{xy}\rho)/\sqrt{\rho}$. Samples of numerical results in the context will prove that one can replace $\exp(\mathrm{i}k_{xy}\rho)/\sqrt{\rho}$ with f_{d} for this case.

Thus, it is reasonable to derive a basis function from Eq. (5). The tangential components of the diffracted current on the wedge surface employed in Ref. [13] can be reduced to

$$J_{\rho}^{\mathrm{d}}=C_1 f_{\mathrm{d}},\ J_z^{\mathrm{d}}=C_2 f_{\mathrm{d}} \tag{6}$$

where C_1 and C_2 are unknown coefficients. According to Bilow, by introducing two additional matching points in the PO current region, one can obtain two additional equations to solve the two expanded coefficients.

Samples of numerical results for a perfectly conducting right-angled wedge are presented in order to validate the two types of basis functions and discuss its limits of applicability. In all of these figures the electric current ρ-component magnitude is normalized with respect to $|H^i|$, whereas z-component magnitude is normalized with respect to $|E^i/\eta|$ (η is the free-space intrinsic impedance). Positive coordinate values in these figures indicate distance from the edge on the face $\phi' = 0$, while negative coordinate values indicate the distance on the face $\phi' = n\pi$. The results presented in Figs. 3 and 4 are respectively those for normal incidence directions of $\phi' = 70°$, $120°$ and for H-polarization (i.e. for the incident H-field coplanar with the edge). The exact solution for this case is

$$\boldsymbol{J} = \boldsymbol{n}\times\boldsymbol{z}\frac{2}{n}H_z^{\mathrm{i}}\sum_{v}\mathrm{i}^{v}\mathrm{J}_v(k\rho)\cos(v\phi')\cos(v\phi)$$

Where J_v is v order Bessel function of the first kind, n is the surface normal, $v = m/n$, $m = 0,1,2,\cdots$. Suppose m is sufficiently large, such as 200, the above series will be well convergent.

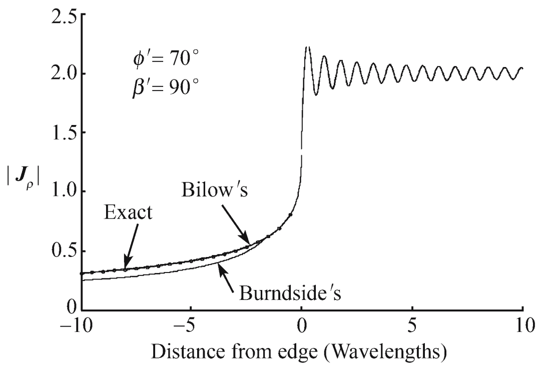

Fig. 3 Normalized current magnitude for H-polarized plane wave, $\phi'=70°$

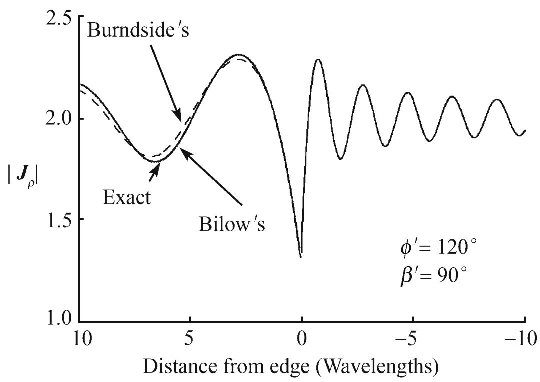

Fig. 4 Normalized current magnitude for H-polarized plane wave, $\phi'=120°$

The results obtained by Bilow's basis function are in better agreement with the analytic solutions.

3 Modification of Bilow's DCBF

Figures 5 and 6 respectively show the results for E-polarization when $\phi'=120°$ and $\phi'=180°$ (i.e. the face $\phi=0$ is at a shadow boundary). The exact solution for the E-polarization case is

$$\boldsymbol{J} = \boldsymbol{n}\times\boldsymbol{\rho}\left[-\frac{4\mathrm{i}\omega\varepsilon}{nk^2\rho}E_z^{\mathrm{i}}\sum_{v}v\mathrm{i}^{v}\mathrm{J}_v(k\rho)\sin(v\phi')\cos(v\phi)\right]$$

The data in the two figures depict that Bilow's diffracted current basis function also yields error for E-polarization, especially in the shadow boundary regions.

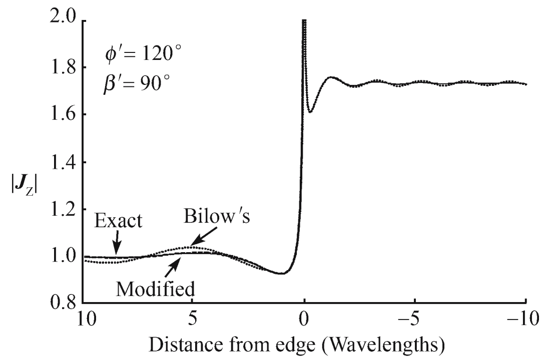

Fig. 5 Normalized current magnitude for E-polarized plane wave, $\phi'=120°$

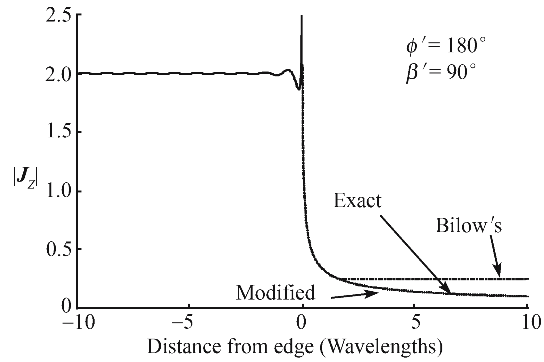

Fig. 6 Normalized current magnitude for E-polarized plane wave, $\phi'=180°$

These relations in Eq. (6) show that along the wedge face

$$H_z^{\mathrm{d}} \propto f_{\mathrm{d}},\ H_\rho^{\mathrm{d}} \propto f_{\mathrm{d}}$$

In some cases, the above expressions do not satisfy the relationships of the diffracted field. Based on the classical theory of diffraction and the fundamental theory of electromagnetism, the relations in Eq. (6) are modified.

The symmetry of the problem in the z-direction implies that the diffracted field must have the same z dependence factor exp ($\mathrm{i}k_z z$) as the incident field and the fields can be represented by their z components alone. Considering the contributions to the equivalent electric current on the wedge surface, here we write the ρ component of the diffracted field by its z components as

$$H_\rho^{\mathrm{d}} = \frac{\mathrm{i}k_z\partial H_z^{\mathrm{d}}}{k_{xy}^2\partial\rho} - \frac{\mathrm{i}\omega\varepsilon\ \partial E_z^{\mathrm{d}}}{k_{xy}^2\rho\ \partial\phi} \tag{7}$$

If the wedge face is not close to a shadow boundary and if ρ is sufficiently large, the z components of the diffracted field will take this form

$$E_z^{\mathrm{d}} = D_{\mathrm{s}}(\phi)\frac{\mathrm{e}^{\mathrm{i}k_{xy}\rho}}{\sqrt{\rho}} \quad H_z^{\mathrm{d}} = D_{\mathrm{h}}(\phi)\frac{\mathrm{e}^{\mathrm{i}k_{xy}\rho}}{\sqrt{\rho}}$$

where D_{s} and D_{h} are the scalar diffraction coefficients independent of ρ. Using the above formulae, one has

$$H_\rho^{\mathrm{d}} = -D_{\mathrm{h}}\frac{k_z}{k_{xy}}\frac{\mathrm{e}^{\mathrm{i}k_{xy}\rho}}{\sqrt{\rho}} - \frac{\mathrm{i}(0.5D_{\mathrm{h}}k_z + \omega\varepsilon D_{\mathrm{s}}')}{k_{xy}^2\rho}\frac{\mathrm{e}^{\mathrm{i}k_{xy}\rho}}{\sqrt{\rho}} \tag{8}$$

Based on the conclusion that f_{d} can substitute for $\mathrm{e}^{\mathrm{i}k_{xy}\rho}/\sqrt{\rho}$ in the current case, the diffracted current density along the wedge walls can now be written explicitly as

$$\boldsymbol{J}_\rho^{\mathrm{d}} = \boldsymbol{\rho}\bullet(\boldsymbol{n}\times\boldsymbol{z})C_1 f_{\mathrm{d}} \tag{9a}$$

$$\boldsymbol{J}_z^{\mathrm{d}} = \boldsymbol{z}\bullet(\boldsymbol{n}\times\boldsymbol{\rho})[-C_1\frac{k_z}{k_{xy}}f_{\mathrm{d}} + C_2\frac{f_{\mathrm{d}}}{\rho}] \tag{9b}$$

where C_1 and C_2 are unknown coefficients. The results of the modified basis functions from Eq. (9) plotted in Fig. 5 are in perfect agreement with the analytic solutions. However, when Eq. (9) is similar to Eq. (6), then the solution obtained by Eq. (9) also yields grave errors when the wedge face is inside the shadow boundary transition region, as Bilow's solution shown in Fig. 6.

The expressions in Eq. (6) are also modified from Eq. (7). If the wedge face is in the vicinity of a shadow boundary and ρ is sufficiently large

$$E_z^{\mathrm{d}} = f_{\mathrm{s}}(\phi)f_{\mathrm{d}},\ H_z^{\mathrm{d}} = f_{\mathrm{h}}(\phi)f_{\mathrm{d}} \tag{10}$$

where f_{s} and f_{h} are the diffraction coefficients independent of ρ. Thus

$$H_\rho^{\mathrm{d}} = \frac{\mathrm{i}k_z f_{\mathrm{h}}(\phi)\partial f_{\mathrm{d}}}{k_{xy}^2\partial\rho} - \frac{\mathrm{i}\omega\varepsilon\ \partial\ [f_{\mathrm{s}}(\phi)f_{\mathrm{d}}]}{k_{xy}^2\rho\ \partial\phi}$$

The first partial derivative

$$\frac{\partial f_{\mathrm{d}}}{\partial\rho} = -\mathrm{i}k_{xy}\cos(\phi-\phi')f_{\mathrm{d}} - 0.5\sqrt{k_{xy}[1+\cos(\phi-\phi')]}\frac{\mathrm{e}^{\mathrm{i}k_{xy}\rho}}{\sqrt{\rho}}$$

Making use of $\cos(\phi-\phi') \to -1$, the above equation can be reduced to

$$\frac{\partial f_{\mathrm{d}}}{\partial\rho} \approx \mathrm{i}k_{xy}f_{\mathrm{d}}$$

The other partial derivative can be represented as

$$\frac{\partial[f_{\mathrm{s}}(\phi)f_{\mathrm{d}}]}{\partial\phi} = f'_{\mathrm{s}}(\phi)f_{\mathrm{d}} + f_{\mathrm{s}}(\phi)\mathrm{e}^{\mathrm{i}k_{xy}\rho}\sqrt{0.5k_{xy}\rho} + \mathrm{i}\sin(\phi-\phi')k_{xy}\rho f_{\mathrm{s}}(\phi)f_{\mathrm{d}}$$

In the neighborhood of the shadow boundary, $f_{\mathrm{s}} \to$ a nonzero constant, and $\sin(\phi-\phi') \to 0$ while the value of $f_{\mathrm{s}}f_{\mathrm{d}}$ is finite. Then

$$\frac{\partial[f_{\mathrm{s}}(\phi)f_{\mathrm{d}}]}{\rho\ \partial\phi} \approx f_{\mathrm{s}}(\phi)\sqrt{0.5k_{xy}}\frac{\mathrm{e}^{\mathrm{i}k_{xy}\rho}}{\sqrt{\rho}}$$

Now, the diffracted current for the current case can be written as

$$\boldsymbol{J}_\rho^{\mathrm{d}} = \boldsymbol{\rho}\bullet(\boldsymbol{n}\times\boldsymbol{z})C_1 f_{\mathrm{d}} \tag{11a}$$

$$\boldsymbol{J}_z^{\mathrm{d}} = \boldsymbol{z}\bullet(\boldsymbol{n}\times\boldsymbol{\rho})\left[-C_1\frac{k_z}{k_{xy}}f_{\mathrm{d}} + C_2\frac{\mathrm{e}^{\mathrm{i}k_{xy}\rho}}{\sqrt{\rho}}\right] \tag{11b}$$

The relations in Eq. (11) can also be derived from Eq. (34) in Ref. [11]. When the wedge face is close to a shadow boundary, $f_{\mathrm{d}}(\rho,\phi) \propto \mathrm{e}^{\mathrm{i}k_{xy}\rho}$. Consequently, one cannot replace $\mathrm{e}^{\mathrm{i}k_{xy}\rho}/\sqrt{\rho}$ with f_{d} in Eq. (11). The relations between Eqs. (9) and (11) indicate that although the number of basis functions rises, the number of unknown coefficients does not increase.

The difference between Bilow's basis function and the modified one is that there is no second term of J_z^{d} in Bilow's formulations. For a perfectly conducting wedge excited by E-polarized plane wave, $C_1 = 0$, Eqs. (9b) and (11b) are different from the expression of J_z^{d} in Eq. (6). From Eqs. (9) to (11), one can conclude

$$\begin{cases} \boldsymbol{J}_\rho^{\mathrm{d}} = \boldsymbol{\rho}\bullet(\boldsymbol{n}\times\boldsymbol{z})C_1 f_{\mathrm{d}} \\ \boldsymbol{J}_z^{\mathrm{d}} = \begin{cases} \boldsymbol{z}\bullet(\boldsymbol{n}\times\boldsymbol{\rho})\left[-C_1\frac{k_z}{k_{xy}}f_{\mathrm{d}} + C_2\frac{f_{\mathrm{d}}}{\rho}\right], \\ \text{when the wedge face is} \\ \text{outside the transition regions} \\ \boldsymbol{z}\bullet(\boldsymbol{n}\times\boldsymbol{\rho})\left[-C_1\frac{k_z}{k_{xy}}f_{\mathrm{d}} + C_2\frac{\mathrm{e}^{\mathrm{i}k_{xy}\rho}}{\sqrt{\rho}}\right], \\ \text{when the wedge face is} \\ \text{within the transition regions} \end{cases} \end{cases}$$

4 Conclusion

This paper analyzed and compared the diffracted current basis functions for electromagnetic Scattering of Plane wave by an infinite wedge, and presented new diffracted current basis functions. Numerical results demonstrate the accuracy and flexibility of the current approach, which does offer a more useful procedure for engineering applications.

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