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Robust controller for a class of uncertain switched fuzzy systems

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Abstract A robustness control of uncertain switched fuzzy systems is presented. Using the switching technique and the Lyapunov function method, a continuous state feedback controller is built to ensure that for all allowable uncertainties the relevant closed-loop system is asymptotically stable. Furthermore, a switching strategy that achieves system global asymptotic stability of the uncertain switched fuzzy system is given. In this model, each subsystem of the switched system is an uncertain fuzzy system, and a common parallel distributed compensation controller is presented. The main condition is given in the form of convex combinations which are more solvable. This method transforms a certain switched system and has strong robustness for various system parameters. Simulations show the feasibility and the effectiveness of this method.

Keywords switched systems, fuzzy systems, robust control, asymptotic stability, global model, switching law

1 Introduction

A switched system is an important hybrid system, which consists of a family of continuous-time or discrete-time subsystems and a rule that specifies the switching among them. In recent years, the stability analysis of switched systems has acquired many results [1–5]. For uncertain switched systems, the results of robust control are relatively few. In Ref. [6], the stability of a class of nonlinear switched systems having

disturbances is addressed. In Ref. [7], a robust controller is designed for a class of uncertain linear switched systems using a common Lyapunov function of nominal systems.

In addition, the fuzzy logical control has emerged as one of the most active and fruitful areas. Recently, some useful stability analysis techniques have come forth. Linear matrix inequality (LMI)-based designs for Takagi-Sugeno (T-S) fuzzy systems have sparked a move towards the fuzzy control technique [8]. In Ref. [9], the uncertain T-S fuzzy model is represented as a set of uncertain linear systems and a controller design algorithm is proposed. Specifically, a switching static output-feedback fuzzy-model-based controller is designed by solving some LMIs, which are derived from the developed local representations.

A switched system is called a switched fuzzy system if all the subsystems are fuzzy systems. Comparing the stability of switched systems and those of fuzzy control systems, the results on switched fuzzy systems are very few. In Ref. [10], the combination of hybrid systems and fuzzy multiple model systems is described, and a fuzzy switched hybrid controller is put forward. In Refs. [11,12], a switching fuzzy model is studied. Such a switching fuzzy system model has two levels of structure, the first level is a region rule level and the second level is a local fuzzy rule level. This model is switching in the local fuzzy rule level of the second level according to the premise variable in the region rule level of the first level. In fact, it is switching according to the same premise variable. Stability conditions are given. References [13–15] give some extensions based on Refs. [11,12].

A new model of a class of uncertain switched fuzzy systems is proposed in this paper. A system of this class is a switched system whose subsystems are all fuzzy systems with uncertainties. Continuous controllers for all switched subsystems and a switching law are designed to give a robust asymptotic stability. In contrast with the existing results, we study switched fuzzy systems without the levels of structure. The method provides a different premise variable switching directly. We design both continuous controllers for subsystems and the switching law while only fixed state-dependent switching is considered in Refs. [11–15].

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2 System model

Consider the continuous uncertain switched fuzzy model $R_{\sigma(t)}^l$: if ξ_1 is $M_{\sigma(t)1}^l \dots$ and ξ_p is $M_{\sigma(t)p}^l$, then

$$\dot{\mathbf{x}}(t) = (\mathbf{A}_{\sigma(t)l} + \Delta\mathbf{A}_{\sigma(t)l})\mathbf{x}(t) + (\mathbf{B}_{\sigma(t)l} + \Delta\mathbf{B}_{\sigma(t)l})\mathbf{u}_{\sigma(t)}(t), \quad l = 1, 2, \dots, N_{\sigma(t)} \quad (1)$$

where $\xi_1, \xi_2, \dots, \xi_p$ are the premise, $\sigma(t): R_+ \rightarrow M = \{1, 2, \dots, m\}$ variables, is a piecewise constant function called a switching signal. $R_{\sigma(t)}^l$ denotes the l th fuzzy inference rule in the σ th switched subsystem. $N_{\sigma(t)}$ is the number of inference rules in the σ th switched subsystem. $\mathbf{x}(t) = [\mathbf{x}_1(t) \ \mathbf{x}_2(t) \ \dots \ \mathbf{x}_n(t)]^T \in \mathbb{R}^n$ is the state variable vector. $\mathbf{A}_{\sigma(t)l}$ and $\mathbf{B}_{\sigma(t)l}$ are known constant matrices of the appropriate dimensions of the σ th switched subsystem, $\Delta\mathbf{A}_{\sigma(t)l}$ and $\Delta\mathbf{B}_{\sigma(t)l}$ are real-valued matrix functions representing the norm-bounded parameter uncertainty of the σ th switched subsystem.

The global model of the i th switched subsystem is described by

$$\dot{\mathbf{x}}(t) = \sum_{l=1}^{N_i} \eta_{il}(\xi(t)) [(\mathbf{A}_{il} + \Delta\mathbf{A}_{il})\mathbf{x}(t) + (\mathbf{B}_{il} + \Delta\mathbf{B}_{il})\mathbf{u}_i(t)], \quad l = 1, 2, \dots, N_i, \quad i = 1, 2, \dots, m \quad (2)$$

where $0 \leq \eta_{il}(\xi(t)) \leq 1$, $\sum_{l=1}^{N_i} \eta_{il}(\xi(t)) = 1$

$$w_{il}(\xi(t)) = \prod_{p=1}^p M_{ip}^l(\xi_p(t)), \quad \eta_{il}(\xi(t)) = \frac{w_{il}(\xi(t))}{\sum_{l=1}^{N_i} w_{il}(\xi(t))}$$

where $M_{ip}^l(\xi_p(t))$ denotes the membership function and $\xi_p(t)$ belongs to the fuzzy set M_{ip}^l .

3 Main results

Assumption The admissible parameter uncertainties are of the norm-bounded form

$$[\Delta\mathbf{A}_{il} \ \Delta\mathbf{B}_{il}] = \mathbf{D}_{il}\mathbf{F}_{il}(t)[\mathbf{E}_{il1} \ \mathbf{E}_{il2}], \quad l = 1, 2, \dots, N_i \quad (3)$$

where \mathbf{D}_{il} and \mathbf{E}_{il1} , \mathbf{E}_{il2} are known real constant matrices of proper dimensions, which represent the structure of the uncertainties, and $\mathbf{F}_{il}(t)$ is an unknown time-varying matrix satisfying $\mathbf{F}_{il}^T(t)\mathbf{F}_{il}(t) \leq \mathbf{I}$, $l = 1, 2, \dots, N_i$.

Here, the parallel distributed compensation (PDC) fuzzy controller design method is used for every sub fuzzy system—the global control

$$\mathbf{u}_i(t) = \sum_{l=1}^{N_i} \eta_{il}\mathbf{K}_{il}\mathbf{x}(t) \quad (4)$$

Then the global model of the i th sub fuzzy system is described by

$$\dot{\mathbf{x}}(t) = \sum_{l=1}^{N_i} \eta_{il} \sum_{r=1}^{N_i} \eta_{ir} \{ \mathbf{A}_{il} + \Delta\mathbf{A}_{il}(t) + [\mathbf{B}_{il} + \Delta\mathbf{B}_{il}(t)]\mathbf{K}_{ir} \} \mathbf{x}(t) \quad (5)$$

Theorem 1 Suppose there exists a positive definite matrix \mathbf{P} and constants $\lambda_{j_i} > 0$ ($i = 1, 2, \dots, m$, $j_i = 1, 2, \dots, N_i$) such that

$$\sum_{i=1}^{N_i} \lambda_{j_i} \left[\mathbf{H}_{ij_i, \vartheta_i}^T \mathbf{P} + \mathbf{P}\mathbf{H}_{ij_i, \vartheta_i} + 2\mathbf{P}\mathbf{D}_{ij_i} \mathbf{D}_{ij_i}^T \mathbf{P} + \mathbf{E}_{ij_i, 1}^T \mathbf{E}_{ij_i, 1} + \mathbf{K}_{i\vartheta_i}^T \mathbf{E}_{ij_i, 2}^T \mathbf{E}_{ij_i, 2} \mathbf{K}_{i\vartheta_i} \right] < 0, \quad j_i, \vartheta_i = 1, 2, \dots, N_i \quad (6)$$

where $\mathbf{H}_{ij_i, \vartheta_i} = \mathbf{A}_{ij_i} + \mathbf{B}_{ij_i} \mathbf{K}_{i\vartheta_i}$, then Eq. (1) is stable under the switching law

$$\begin{aligned} \sigma &= \arg \min \bar{V}_i(\mathbf{x}) \\ &\triangleq \max_{j_i, \vartheta_i} \left\{ \mathbf{x}^T \left[\mathbf{H}_{ij_i, \vartheta_i}^T \mathbf{P} + \mathbf{P}\mathbf{H}_{ij_i, \vartheta_i} + 2\mathbf{P}\mathbf{D}_{ij_i} \mathbf{D}_{ij_i}^T \mathbf{P} + \mathbf{E}_{ij_i, 1}^T \mathbf{E}_{ij_i, 1} + \mathbf{K}_{i\vartheta_i}^T \mathbf{E}_{ij_i, 2}^T \mathbf{E}_{ij_i, 2} \mathbf{K}_{i\vartheta_i} \right] \mathbf{x} < 0, \quad j_i, \vartheta_i = 1, 2, \dots, N_i \right\} \end{aligned} \quad (7)$$

Proof From Eq. (6) we know that for any $\mathbf{x} \neq 0$, it holds that

$$\sum_{i=1}^{N_i} \lambda_{j_i} \mathbf{x}^T \left[\mathbf{H}_{ij_i, \vartheta_i}^T \mathbf{P} + \mathbf{P}\mathbf{H}_{ij_i, \vartheta_i} + 2\mathbf{P}\mathbf{D}_{ij_i} \mathbf{D}_{ij_i}^T \mathbf{P} + \mathbf{E}_{ij_i, 1}^T \mathbf{E}_{ij_i, 1} + \mathbf{K}_{i\vartheta_i}^T \mathbf{E}_{ij_i, 2}^T \mathbf{E}_{ij_i, 2} \mathbf{K}_{i\vartheta_i} \right] \mathbf{x} < 0 \quad (8)$$

Note that Eq. (8) holds for any $j_i, \vartheta_i \in \{1, 2, \dots, N_i\}$ and $\lambda_{j_i} > 0$, then there exists at least an i such that for any j_i, ϑ_i

$$\mathbf{x}^T \left[\mathbf{H}_{ij_i, \vartheta_i}^T \mathbf{P} + \mathbf{P}\mathbf{H}_{ij_i, \vartheta_i} + 2\mathbf{P}\mathbf{D}_{ij_i} \mathbf{D}_{ij_i}^T \mathbf{P} + \mathbf{E}_{ij_i, 1}^T \mathbf{E}_{ij_i, 1} + \mathbf{K}_{i\vartheta_i}^T \mathbf{E}_{ij_i, 2}^T \mathbf{E}_{ij_i, 2} \mathbf{K}_{i\vartheta_i} \right] \mathbf{x} < 0 \quad (9)$$

Thus, the switching law is well-defined by Eq. (7). We now compute the time derivative of $V(t) = \mathbf{x}^T(t)\mathbf{P}\mathbf{x}(t)$

$$\begin{aligned} \dot{V} &= \dot{\mathbf{x}}^T \mathbf{P}\mathbf{x} + \mathbf{x}^T \mathbf{P}\dot{\mathbf{x}} \\ &= \sum_{l=1}^{N_i} \sum_{r=1}^{N_i} \eta_{il}\eta_{ir} \mathbf{x}^T \left\{ [(\mathbf{A}_{il} + \mathbf{B}_{il}\mathbf{K}_{ir}) + (\Delta\mathbf{A}_{il} + \Delta\mathbf{B}_{il}\mathbf{K}_{ir})]^T \mathbf{P} \right. \\ &\quad \left. + \mathbf{P}[(\mathbf{A}_{il} + \mathbf{B}_{il}\mathbf{K}_{ir}) + (\Delta\mathbf{A}_{il} + \Delta\mathbf{B}_{il}\mathbf{K}_{ir})] \right\} \mathbf{x} \end{aligned} \quad (10)$$

Take $[\Delta\mathbf{A}_{il} \ \Delta\mathbf{B}_{il}] = \mathbf{D}_{il}\mathbf{F}_{il}[\mathbf{E}_{il1} \ \mathbf{E}_{il2}]$ into Eq. (10), and define $\mathbf{H}_{ilr} = \mathbf{A}_{il} + \mathbf{B}_{il}\mathbf{K}_{ir}$. We obtain

$$\begin{aligned} \dot{V} &= \sum_{l=1}^{N_i} \sum_{r=1}^{N_i} \eta_{il}\eta_{ir} \mathbf{x}^T \left\{ (\mathbf{H}_{ilr}^T \mathbf{P} + \mathbf{P}\mathbf{H}_{ilr}) + [(\mathbf{E}_{il1}^T \mathbf{F}_{il}^T \mathbf{D}_{il}^T \mathbf{P} + \mathbf{P}\mathbf{D}_{il} \mathbf{F}_{il} \mathbf{E}_{il1}) + (\mathbf{K}_{ir}^T \mathbf{E}_{il2}^T \mathbf{F}_{il}^T \mathbf{D}_{il}^T \mathbf{P} + \mathbf{P}\mathbf{D}_{il} \mathbf{F}_{il} \mathbf{E}_{il2} \mathbf{K}_{ir})] \right\} \mathbf{x} \\ &\leq \sum_{l=1}^{N_i} \sum_{r=1}^{N_i} \eta_{il}\eta_{ir} \mathbf{x}^T \left\{ (\mathbf{H}_{ilr}^T \mathbf{P} + \mathbf{P}\mathbf{H}_{ilr}) + [(\mathbf{P}\mathbf{D}_{il} \mathbf{D}_{il}^T \mathbf{P} + \mathbf{E}_{il1}^T \mathbf{E}_{il1}) + (\mathbf{P}\mathbf{D}_{il} \mathbf{D}_{il}^T \mathbf{P} + \mathbf{K}_{ir}^T \mathbf{E}_{il2}^T \mathbf{E}_{il2} \mathbf{K}_{ir})] \right\} \mathbf{x} \\ &= \sum_{l=1}^{N_i} \sum_{r=1}^{N_i} \eta_{il}\eta_{ir} \mathbf{x}^T \left\{ (\mathbf{H}_{ilr}^T \mathbf{P} + \mathbf{P}\mathbf{H}_{ilr}) + [2\mathbf{P}\mathbf{D}_{il} \mathbf{D}_{il}^T \mathbf{P} + \mathbf{E}_{il1}^T \mathbf{E}_{il1} + \mathbf{K}_{ir}^T \mathbf{E}_{il2}^T \mathbf{E}_{il2} \mathbf{K}_{ir}] \right\} \mathbf{x} \end{aligned}$$

Take Eqs. (3) and (9) into account and we deduce that $\dot{V} < 0$. Therefore, system (1) is stable under the switching law defined by Eq. (7).

4 Simulations

Example Consider a continuous switched fuzzy system as follows

- R_1^1 : IF x is Ω_{11}^1 , then $\dot{x} = (A_{11} + \Delta A_{11})x + (B_{11} + \Delta B_{11})u_1$
- R_1^2 : IF x is Ω_{11}^2 , then $\dot{x} = (A_{12} + \Delta A_{12})x + (B_{12} + \Delta B_{12})u_1$
- R_2^1 : IF y is Ω_{21}^1 , then $\dot{x} = (A_{21} + \Delta A_{21})x + (B_{21} + \Delta B_{21})u_2$
- R_2^2 : IF y is Ω_{21}^2 , then $\dot{x} = (A_{22} + \Delta A_{22})x + (B_{22} + \Delta B_{22})u_2$

where

$$A_{11} = \begin{bmatrix} -10 & 0.01 \\ -9.3 & -1.0493 \end{bmatrix}, B_{11} = \begin{bmatrix} 0 \\ 1 \end{bmatrix};$$

$$A_{12} = \begin{bmatrix} 0 & 0.1 \\ -32 & -4.529 \end{bmatrix}, B_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A_{21} = \begin{bmatrix} -10 & 0.1 \\ 10 & -0.1 \end{bmatrix}, B_{21} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; A_{22} = \begin{bmatrix} 0 & 0.8 \\ -8 & -0.9 \end{bmatrix}, B_{22} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$D_{11} = D_{12} = \begin{bmatrix} -0.1125 & 1 \\ 1 & 0 \end{bmatrix}, D_{21} = D_{22} = \begin{bmatrix} 0.01 & 1 \\ 1 & 0 \end{bmatrix},$$

$$E_{111} = E_{121} = \begin{bmatrix} 1 & 0.2 \\ 0 & 0 \end{bmatrix}$$

$$E_{211} = E_{221} = \begin{bmatrix} 0.5 & 1 \\ 0 & 0 \end{bmatrix}, E_{112} = E_{122} = \begin{bmatrix} 0 \\ 0.6 \end{bmatrix},$$

$$E_{212} = E_{222} = \begin{bmatrix} 0 \\ 0.8 \end{bmatrix}$$

$$F_{11}(t) = F_{12}(t) = F_{21}(t) = F_{22}(t) = \begin{bmatrix} \sin t & 0 \\ 0 & \cos t \end{bmatrix}$$

The state feedback gains of subsystems are obtained as

$$K_{11} = [-0.131 \quad -0.1148], K_{12} = [-0.0623 \quad -2.302]$$

$$K_{21} = [-1.8 \quad -1.9], K_{22} = [-0.7 \quad -1.3]$$

The fuzzy sets of “ $\Omega_{11}^1, \Omega_{11}^2, \Omega_{21}^1, \Omega_{21}^2$ ” are represented by the following membership functions respectively

$$\mu_{11}^1(x) = 1 - \frac{1}{1 + e^{-2x}}, \mu_{11}^2(x) = \frac{1}{1 + e^{-2x}},$$

$$\mu_{21}^1(y) = 1 - \frac{1}{1 + e^{-(2(y-0.3))}}, \mu_{21}^2(y) = \frac{1}{1 + e^{-(2(y-0.3))}}$$

For

$$\sum_{i=1}^2 \lambda_{\vartheta_i} \left[H_{ij,\vartheta_i}^T P + PH_{ij,\vartheta_i} + 2PD_{ij} D_{ij}^T P + E_{ij,1}^T E_{ij,1} + K_{i\vartheta_i}^T E_{ij,2}^T E_{ij,2} K_{i\vartheta_i} \right] < 0, j_i, \vartheta_i = 1, 2$$

where $H_{ij,\vartheta_i} = A_{ij} + B_{ij} K_{i\vartheta_i}$. Choosing $\lambda_{\vartheta_i} = 1$, we can have the matrix $P = \begin{bmatrix} 2.027 & 7 & 0.359 & 7 \\ 0.359 & 7 & 0.240 & 4 \end{bmatrix}$.

Then, the system is asymptotically stable under the following switching law

$$\sigma = \arg \min \left\{ \bar{V}_i(x) \triangleq \max_{j_i, \vartheta_i} \left\{ x^T \left[H_{ij,\vartheta_i}^T P + PH_{ij,\vartheta_i} + 2PD_{ij} D_{ij}^T P + E_{ij,1}^T E_{ij,1} + K_{i\vartheta_i}^T E_{ij,2}^T E_{ij,2} K_{i\vartheta_i} \right] x < 0, j_i, \vartheta_i = 1, 2 \right\} \right\}$$

The simulation results with the initial condition $[1 \quad 1]^T$ are shown in Fig. 1.

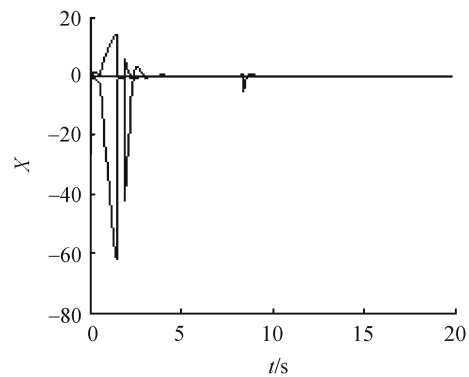


Fig. 1 The state response of the system according to PDC fuzzy controller

5 Conclusions

A model of uncertain switched fuzzy systems has been presented. Sufficient conditions for stability are given. According to these conditions, in order to check the stability, we only need to check the stability of a certain combination of subsystem matrices, which is easy to realize. The stabilizing switching laws of the state-dependent form are also designed.

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