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RESEARCH ARTICLE

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Multi-scale Kalman filters algorithm for GPS common-view observation data based on correlation structure of discrete wavelet coefficients

Abstract Global positioning system (GPS) common-view observation data were processed by using the multi-scale Kalman algorithm based on a correlative structure of the discrete wavelet coefficients. Suppose that the GPS common-view observation data has the $1/f$ fractal characteristic, the algorithm of wavelet transform was used to estimate the Hurst parameter H of GPS clock difference data. When $0 < H < 1$, the $1/f$ fractal characteristic of the GPS clock difference data is a Gaussian zero-mean and non-stationary stochastic process. Thus, the discrete wavelet coefficients can be discussed in the process of estimating multi-scale Kalman coefficients. Furthermore, the discrete clock difference can be estimated. The single-channel and multi-channel common-view observation data were processed respectively. Comparisons were made between the results obtained and the Circular T data. Simulation results show that the algorithm discussed in this paper is both feasible and effective.

Keywords communication, multi-scale Kalman filters, $1/f$ fractal characteristic, correlation structure, fractal increment

1 Introduction

It must be noted that the conclusions obtained in the paper are based upon the following hypotheses:

The GPS common-view observation data have the $1/f$ fractal characteristic after being pretreated.

Suppose that the GPS common-view clock difference data process is demonstrated by $y_H(t)$, $t \in R$, then the increment $G(t) = y_H(t+1) - y_H(t)$ is discrete fractal Gaussian noise. This discrete Gaussian noise is a zero-mean and stationary Gaussian random stochastic process.

According to the method in Ref. [1], the Hurst parameter of one parameter of GPS common-view clock differences data can be estimated. When $0 < H < 1$, this discrete data is a Gaussian zero-mean and non-stationary stochastic process, which can be considered to possess the $1/f$ fractal characteristic.

The wavelet transform and the least squares algorithm can be used to estimate the Hurst parameter of the GPS data [1]. Because the wavelet coefficients are correlative [2–4], a novel multi-scale Kalman filter algorithm is proposed in this paper to analyze the GPS common-view observation data. In this paper, a filter bank design based on orthonormal wavelets and equipped with a multi-scale Kalman filter from the point of view of state space model is used for the data restoration corrupted by external noise. Section 2 presents the wavelet transform algorithm of estimating the Hurst parameter. In Sect. 3, on the basis of the Hurst parameter, the correlation among the wavelet coefficients is discussed in every scale. Furthermore, in Sect. 4, we devise a multi-scale Kalman filter based on the optimal mean-square estimation. Finally, in Sect. 5, the filters are used to process the single-channel and multi-channel GPS common-view observation data respectively. The experimental results demonstrate the performance of our method. And we conclude the paper in Sect. 6.

2 The estimation of exponent *H* of GPS clock difference data

According to the above, we can figure out its exponent H. If $0 < H < 1$, the algorithm proposed in this paper can be applied to estimate the clock difference data. The Mallat algorithm can be utilized to analyze discrete fractal Gaussian noise of the discrete clock difference data [1]. Wavelet transform and

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the least squares algorithm are used to estimate the exponent H. According to Mallat

$$a_m[k] = \sum_{j=-\infty}^{\infty} h_{2k-j} a_{m-1}[k],$$

$$x_m[k] = \sum_{j=-\infty}^{\infty} g_{2k-j} a_{m-1}[k] \tag{1}$$

where g_k and h_k are the filter coefficients of the wavelet function $\phi(t)$ and scale function $\varphi(t)$ respectively. Moreover, $g_k = (-1)^k h_{1-k}$. As to the Haar wavelet, $h_0 = h_1 = g_0 = -g_1 = \sqrt{2}$, based on which Eq. (1) can be expressed as

$$a_{m+1}[k] = \frac{1}{\sqrt{2}}\left(a_m[2k] + a_m[2k-1]\right)$$

$$x_{m+1}[k] = \frac{1}{\sqrt{2}}\left(a_m[2k] - a_m[2k-1]\right) \tag{2}$$

According to the theorem in Ref. [5] (the detailed proving process is in Refs. [1] and [5]), we can obtain the "approximate" wavelet coefficients $a_m[k]$ and "detailed" wavelet coefficients $x_m[k]$ by transforming discrete fractal Gaussian noise on the basis of the orthonormal Haar discrete wavelet using the Mallat algorithm [6]. $R_m[j]$ represents the autocorrelation of the wavelet coefficients and V_m the variance of the "detailed" wavelet term. Then,

$$R_m[j] = 2^{(2H-1)m} R[j] \tag{3}$$

$$V_m = 2^{(2H-1)(m-1)} \delta_H^2 (2 - 2^{2H-1}) \tag{4}$$

where $R[j]$ is the autocorrelation function of discrete fractal Gaussian noise.

$$R[j] = \frac{\delta_H^2}{2} |\Delta x|^{2H} [|j+1|^{2H} + |j-1|^{2H} - 2|j|^{2H}]$$

Let $(\Delta x = 1, j \in z)$

here, the wavelet transform is considered to be a whitening filter for fractal discrete Gaussian noise, which greatly reduces the correlation of the original signal so that we can ignore the correlation of the wavelet coefficients obtained. Because the mean value of the fractal discrete Gaussian noise is zero and the mean value of the wavelet coefficients after wavelet transform is also zero, the variance var (x_m^k) of the "detailed" wavelet term resulting from transforming fractal discrete Gaussian on the basis of Haar can be estimated by the following equation

$$\hat{\delta}_m^2 = \frac{1}{N(m)} \sum_{k=1}^{N(m)} \left(x_m^k\right)^2 \tag{5}$$

where $N(m)$ is the number of the detailed wavelet coefficients in the mth scale. According to Eq. (4), we can obtain

$$\hat{\delta}_m^2 = 2^{r(m-1)} \delta_H^2 \left(2 - 2^r\right) \tag{6}$$

where $r = 2H - 1$, and the parameter to be estimated is (δ_H^2, r).

We get the new nonlinear equation by operating the logarithm on both sides of Eq. (6).

$$\alpha_m = \beta + r(m-1) + \log_2\left(2 - 2^r\right) \tag{7}$$

where $\alpha_m = \log_2 \hat{\delta}_m^2$, and $\beta_m = \log_2 \hat{\delta}_H^2$.

Let

$$s(\beta, r) = \sum_{m=1}^{M} \left(\alpha_m - \beta - r(m-1) - \log_2\left(2 - 2^r\right)\right)^2 \tag{8}$$

The least squares estimation of (β, r) can be obtained from $(\hat{\beta}, \hat{r})$ which makes $s(\beta, r)$ the least. Therefore

$$\frac{\partial s}{\partial \beta} = \sum_{m=1}^{M} \left(\alpha_m - \beta - r(m-1) - \log_2\left(2 - 2^r\right)\right) = 0$$

$$\frac{\partial s}{\partial r} = \sum_{m=1}^{M} \left(1 - m + \frac{\alpha_m}{2^{1-r} - 1}\right) \Big[\alpha_m - \beta - r(m-1) - \log_2\left(2 - 2^r\right) = 0\Big] \tag{9}$$

Equation (9) reveals that we can get

$$\hat{r} = \frac{12\sum_{m=1}^{M} m\alpha_m - 6(M+1)\sum_{m=1}^{M} \alpha_m}{M\left(M^2 - 1\right)} \tag{10}$$

And we can get H from

$$\hat{H} = \frac{(\hat{r} + 1)}{2} \tag{11}$$

Experimental results show that the estimation precision increases to some extent but not substantially with the scale of the wavelet decomposition. However, it does not mean that the greater the scale, the more precise the estimation. The precision of the estimation increases with the number of the experimental samples.

3 Correlation analysis

The Haar wavelet will be employed for further discussions in this paper because Haar wavelet outperforms other wavelets in correlation within scale and the dependency information between scales and within scales could be better utilized to improve the performance of processing signals.

Suppose that the discrete clock difference data are represented as $y_H(k)$, the wavelet series of the process $y_H(k)$ up to the scale j will be [6]

$$y_H(k) = 2^{-\frac{j}{2}} \sum_{n=0}^{\frac{N_0}{2^J}} a_J[n] \phi(2^{-J} k - n) + \sum_{j=1}^{J} 2^{-\frac{j}{2}} \sum_{n=0}^{\left(\frac{N_0}{2^j}\right)-1} d_j[n] \psi(2^{-j} k - n)$$

$$k = 0, \ldots, N_0 - 1 \tag{12}$$

Because the approximation term $a_J[n]$ of the wavelet expansion can be ignored [3,7], $y_H(k)$ can be approximately denoted by

$$\hat{y}_H(k) = \sum_{j=1}^{J} 2^{-\frac{j}{2}} \sum_{n=0}^{\left(\frac{N_0}{2^j}\right)-1} d_j[n]\psi\left(2^{-j}k - n\right) \tag{13}$$

Use the Haar wavelet $\psi(t) = \begin{cases} 1, & 0 \leqslant t < 1/2 \\ -1, & 1/2 \leqslant t < 1 \\ 0, & \text{otherwise} \end{cases}$ the orthonormal basis of which can be utilized for the multiresolution analysis. According to Ref. [3], it is possible after some calculation to obtain the autocorrelation function of the sequence of wavelet coefficients for $n \neq 0$

$$\begin{aligned} R_j(n) &= E\left[d_j(m+n)d_j(m)\right] \\ &= \frac{\delta^2}{2(2H+1)(2H+2)}\xi[n]\left(2^j\right)^{2H+1} \end{aligned} \tag{14}$$

where

$$\begin{aligned} \xi[n] &= \left[|n|-1\right]^{2H+2} - 4\left[|n|-\frac{1}{2}\right]^{2H+2} \\ &\quad + 6|n|^{2H+2} + \left[|n|+1\right]^{2H+2} - 4\left[|n|+\frac{1}{2}\right]^{2H+2} \end{aligned} \tag{15}$$

According to Ref. [3], from Eqs. (14) and (15), we conclude that when the Haar wavelet is 1/2, the sequences of wavelet coefficients in each scale j are uncorrelated, but when $H \neq 1/2$, $R_j(n) \neq 0$, $|n| > 1$, which demonstrates that the wavelet coefficients are correlated.

The variance of the wavelet coefficients is given by

$$R_j(0) = \operatorname{var}\left(d_j[n]\right) = \frac{\delta^2}{2}V_\psi(H)\left(2^j\right)^{2H+1} \tag{16}$$

where

$$V_\psi(H) = \frac{1-2^{-2H}}{(H+1)(2H+1)} \tag{17}$$

4 Bank of Kalman filters based on correlation structure of wavelet term series

Assume that the real value of clock difference at the time of k is expressed as χ_k, which constitutes the state variable X_k. That is

$$X_k = (\chi_k)$$

The observation data are

$$\boldsymbol{y}(k) = \boldsymbol{H}(k)\boldsymbol{X}(k) + w(k)$$

here, $\boldsymbol{H}(k)$ is observation matrix; $\boldsymbol{X}(k)$ is the data of clock difference to be estimated; and $\boldsymbol{w}(k)$ is the observation noise.

Then the sequences of wavelet coefficients of the observation data will be $\boldsymbol{y}_j[n] = \boldsymbol{d}_j[n] + \boldsymbol{w}_j[n]$ $j = 1, 2, \ldots, J$

where $\boldsymbol{d}_j[n]$ is the sequence of wavelet coefficients of the clock difference data and $\boldsymbol{w}_j[n]$ is the observation noise. The sequences $\{\boldsymbol{w}_j[n], n \in N\}$ are observation noises with the variance of δ_w^2. As the sequences $\boldsymbol{d}_j[n]$ are stationary processes for any scale [8], they can be estimated on the basis of the AR model in the time-scale domain.

$$\boldsymbol{d}_j[n] = \sum_{i=1}^{p}\phi_i^j\boldsymbol{d}_j[n-i] + \boldsymbol{e}_j[n] = \boldsymbol{\phi}^j\boldsymbol{\chi}_j[n-1] + \boldsymbol{e}_j[n] \tag{18}$$

where $\{\boldsymbol{e}_j[n], n \in N\}$ is a zero-mean model noise; p denotes the order of the AR model and the p-dimensional vectors are defined as

$$\begin{aligned} \boldsymbol{\chi}_j[n-1] &= \left(d_j[n-1], \ldots, d_j[n-p]\right)^{\mathrm{T}} \\ \boldsymbol{\phi}^j &= \left(\phi_1^j, \phi_2^j, \ldots, \phi_p^j\right) \end{aligned} \tag{19}$$

The optimal coefficients [9] of the Kalman filters are given by

$$\begin{cases} \boldsymbol{\phi}_j = \boldsymbol{h}_j\boldsymbol{R}_{x_j[n-1]}^{-1} \\ \delta_e^2 = R_j(0) - \boldsymbol{h}_j\boldsymbol{R}_{x_j[n-1]}^{-1}\boldsymbol{h}_j^{\mathrm{T}} \\ \boldsymbol{h}_j = \left(R_j(1),\ R_j(2), \ldots,\ R_j(p)\right) \\ \boldsymbol{R}_{x_j[n-1]} = \begin{bmatrix} R_j(0) & R_j(1) & \ldots & R_j(p-1) \\ R_j(1) & R_j(0) & \ldots & R_j(p-2) \\ \ldots & \ldots & \ldots & \ldots \\ R_j(p-1) & R_j(p-2) & \ldots & R_j(0) \end{bmatrix} \end{cases} \tag{20}$$

where $R_j(n)$ and $R_j(0)$ can be figured out by Eqs. (14)–(17).

Based on the definition of $\boldsymbol{X}_j[n]$,

$$\begin{aligned} \boldsymbol{\chi}_j[n] &= \boldsymbol{F}_j\boldsymbol{\chi}_j[n-1] + \boldsymbol{G}\boldsymbol{e}_j[n] \\ \boldsymbol{y}_j[n] &= H_j\boldsymbol{\chi}_j[n] + \boldsymbol{w}_j[n] \end{aligned} \tag{21}$$

The state space model can be derived from Eq. (18), where

$$\boldsymbol{F}_j = \begin{bmatrix} \phi_1^j & \phi_2^j & \ldots. & \phi_p^j \\ 1 & 0 & \ldots. & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & 1 & 0 \end{bmatrix}$$

$$\boldsymbol{G} = (1, 0, \ldots, 0)^{\mathrm{T}}\ \boldsymbol{H} = (1, 0, \ldots, 0) \tag{22}$$

Then, we can get the wavelet coefficients series $\boldsymbol{d}_j[n]$ by using Kalman filters [8],

$$\hat{\boldsymbol{\chi}}_j[n] = (\hat{d}_j[n,n], \hat{d}_j[n,n-1], \ldots, \hat{d}_j[n,n-p+1])^{\mathrm{T}} \tag{23}$$

5 Data processing

The CRL-NTSC in 2001 (MJD(51912–52001)) is used as the single-channel common-view data for pretreatment, where the second-degree interpolation polynomial is not used to insert absent data. The NTSC-NICT in 2005 (MJD(53367–53551)) is used as the experimental data during the pretreatment of multi-channel common-view data. With the common-view request strictly satisfied, we disregard the astral data of which elevation is less than 20 degrees. Data processing will be discussed in the following two steps.

5.1 The estimate of exponent *H*

The fractal brown motion is known to be a zero-mean Gaussian non-stationary random process, while its increment is a zero-mean stationary random process. Thus, we can figure out the exponent *H* of the fractal signal by transforming its increment with wavelet. Here, the pretreated common-view data are considered to be the raw fractal signals and we make their mean value zero without affecting the *H* parameter for their mean value is not zero. After some data processing, we can further obtain its increment. That is $G(t)=y_H(t+1)-y_H(t)$. Then, it could be processed with wavelet transformation according to Mallat algorithm. As for both the single-channel and the multi-channel data, we use five scales of Haar wavelet. Meanwhile, the *H* parameter could be estimated by using the method mentioned earlier in this paper. Figure 1 shows the increment of the single-channel data of CRL-NTSC(MJD (51912–52001)) in 2001, where *H* parameter is 0.647 3. Figure 2 indicates the increment of the multi-channel data of NTSC-NICT (MJD(53367–53551)) in 2005, with the *H* parameter being 0.672 6.

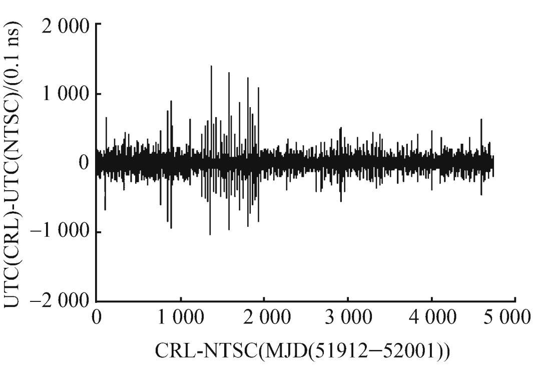

Fig. 1 CRL-NTSC (MJD(51912–52001)) the increment of single-channel clock difference data

5.2 The estimation of clock difference data

In this paper, the processing of two types of channel GPS common-view observation data is discussed. As to the white noise δ^2 inserted in Eqs. (14) and (16), we let it be 1, and for the variance δ_w^2 of the observation noise in the observation equation, we also let it be 1. The other coefficients can be obtained by the optimal coefficients in Eq. (20). According to Ref. [3], the original value of $\boldsymbol{d}_j[n]$ is given by $\boldsymbol{a}_0^*[n]=\boldsymbol{y}(n)$. That is to say, $\boldsymbol{y}(1), \boldsymbol{y}(2), \boldsymbol{y}(3)$ and $\boldsymbol{y}(4)$ are taken as the original value of $\boldsymbol{d}_j[n]$. However, in our processing, it is found that what is chosen as the original value of $\boldsymbol{d}_j[n]$ is not important in the algorithm of correlation structure. In the final stage of processing, it is discovered that five days later, at the time point of 389.5 s, the results, when compared with those of Circular T, are practically the same.

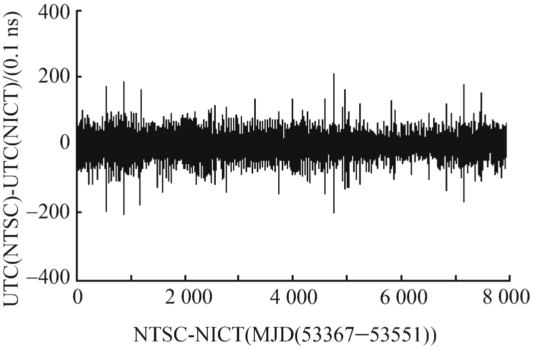

Fig. 2 NTSC-NICT (MJD(53367–53551)) the increment of multi-channel clock difference data

In this paper, $p=4$. For single-channel data, the filter bank corresponds to three scales of Haar wavelet. For multi-channel, six scales of Haar wavelet are used. Comparison is made between the results of CRL-NTSC (MJD(51912–52001)) (see Fig. 3) and Circular T, and the root-mean-square is 5.20 ns. The corresponding results between NTSC-NICT (MJD(53367–53459)) (see Fig. 5) and Circular T shows that its root-mean-square is 4.89 ns. Figures 3–6 show the data before and after the filter banks.

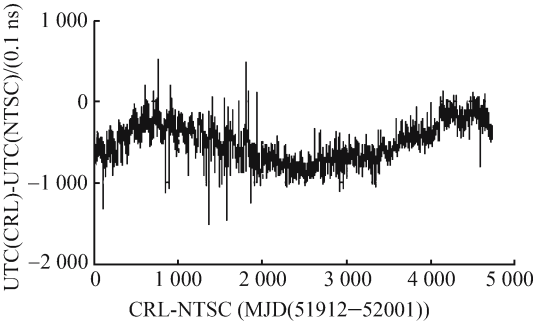

Fig. 3 CRL-NTSC (MJD(51912–52001)) raw single-channel common-view data

6 Conclusions

It must be made clear that the conclusions obtained in the paper are based on the hypothesis that the common-view clock difference data to be processed have the 1/*f* fractal characteristic. By using correlation, the dependency information among scales and within scales could be better utilized to

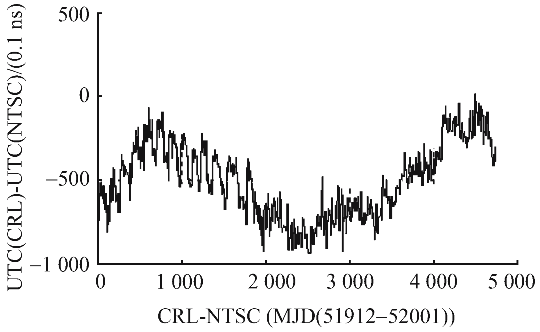

Fig. 4 CRL-NTSC (MJD(51912–52001)) processed data deriving from the multi-Kalman filters

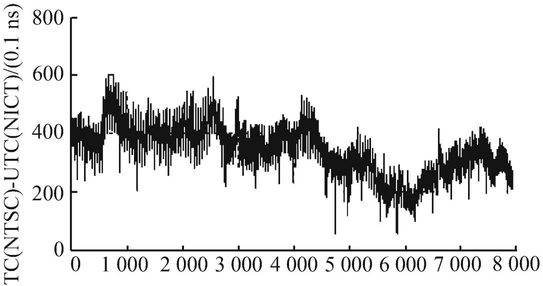

Fig. 5 NTSC-NICT MJD(53367–53551) raw multi-channel common-view data

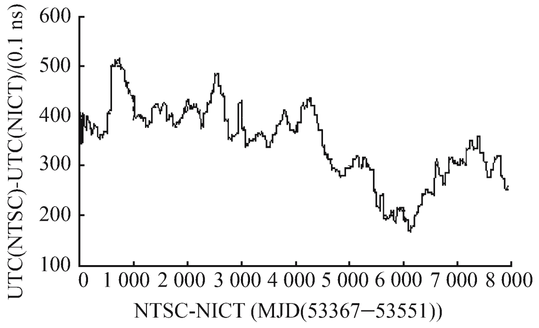

Fig. 6 NTSC-NICT MJD(53367–53551) processed data deriving from the multi-Kalman filters

improve the performance of processing signals. The superiority has more incarnated in the process of multi-channel data processing, as shown in Figs. 5 and 6. The algorithm can be used in any data estimation and signal processing of time series with $1/f$ fractal characteristic. Therefore, it gives us a new idea and method of estimating the time series data.

The algorithm proposed in this paper is also effective in that except that we let $\delta^2 = 1$ and $\delta_w^2 = 1$, other coefficients can be obtained by theory rather than by experience or transcendent methods, which can greatly reduce the man-made error in estimation. It is also a good approach to acquire the values of optimal coefficients and consequently conducive to make the final results precise.

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