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RESEARCH ARTICLE

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PRF-ambiguity resolution for SAR by contrast minimization in range-Doppler domain

Abstract An algorithm was developed to accurately estimate the Doppler centroid, which is needed for high-quality synthetic aperture radar (SAR) image formation by resolving the SAR pulse repetition frequency (PRF) ambiguity. The algorithm uses the SAR range migration to resolve the PRF-ambiguity by searching for a PRF-ambiguity number that minimizes the intensity contrast in the range-Doppler domain. Experimental results show that the approach, compared with traditional methods for resolving the SAR PRF ambiguity, is more suitable for both high contrast scenes such as urban areas and low contrast scenes such as mountains. Moreover, the approach is more computationally efficient for there are no time-consuming correlations or fast Fourier transform (FFT) operations needed in the range-Doppler domain and only part of the range cells are used in the calculation.

Keywords SAR, PRF-ambiguity resolution, range migration, range-Doppler domain, scene contrast

1 Introduction

The high azimuth resolution for SAR is accomplished by making use of the echo phase modulation due to the relative movement between the radar and the target. The phase modulation can be described with two important parameters: one is the Doppler centroid f_{D}, which is defined as the Doppler frequency at the moment the centre of the antenna beam hits the point target, and the other is the Doppler rate. In practice, it is necessary to estimate f_{D} from the received raw data for high-quality image formation. However, because the azimuth spectrum is periodically repeated with the radar PRF f_{P}, only the f_{DB}, the mapped value of f_{D} in the main period, can be estimated with traditional clutter-lock methods [1,2]. Therefore, when $|f_{\mathrm{D}}| > f_{\mathrm{P}}/2$, the Doppler centroid ambiguity occurs and subsequently leads to the deterioration in image quality [3].

So far, the well-known SAR PRF-ambiguity resolvers are: the range look cross-correlation technique [4], the wavelength diversity [5], the multi-look cross correlation (MLCC), and the multi-look beat frequency (MLBF) [6]. The range look cross-correlation technique and the MLBF require scene contrast, while the wavelength diversity and the MLCC work best with low contrast scene.

In this paper, according to the property of SAR range migration (RM), a method, which is based on the signal intensity contrast minimization in the range-Doppler (R-D) domain, is proposed to resolve SAR PRF-ambiguity.

2 The property of RM and signal model in the R-D domain

2.1 The property of RM

For SAR, the distance between the radar and the point target, i.e. the RM trajectory, varies according to

$$R(t;r_{\mathrm{c}}) = \sqrt{r_{\mathrm{c}}^2 + (v_{\mathrm{s}}t)^2 - 2r_{\mathrm{c}}v_{\mathrm{s}}t\sin\theta} \approx r_{\mathrm{c}} - \frac{\lambda}{2}f_{\mathrm{D}}t - \frac{\lambda}{4}f_{\mathrm{r}}t^2 \quad (1)$$

where r_{c} is the slant range between the radar and the target at the time when the antenna beam-centre hits the target, λ is the transmitted radar wavelength, v_{s} is the sensor velocity, θ is the squint angle, i.e. the angle between the side-looking direction and the line of sight of the target when the antenna beam-centre hits it, t is the azimuth slow time, and the Doppler centroid $f_{\mathrm{D}} = 2v_{\mathrm{s}}\sin\theta/\lambda$, the Doppler rate $f_{\mathrm{r}} = -2v_{\mathrm{P}}^2/(\lambda r_{\mathrm{c}})$. In Eq. (1), the second equation is the Fresnel approximation to the first one.

From Eq. (1), the amount of RM relative to r_{c} is

$$\Delta R(t;r_{\mathrm{c}}) = R(t;r_{\mathrm{c}}) - r_{\mathrm{c}} = -\frac{\lambda}{2}f_{\mathrm{D}}t - \frac{\lambda}{4}f_{\mathrm{r}}t^2 \quad (2)$$

After range compression and azimuth FFTs, the received raw data is transformed to the R-D domain, in which from

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the principle of stationary phase (PSP), the RM trajectory is

$$R(f_{\mathrm{t}};r_{\mathrm{c}}) = r_{\mathrm{c}} - \frac{\lambda}{4f_{\mathrm{r}}}\left[(nf_{\mathrm{P}}+f_{\mathrm{t}})^2 - (nf_{\mathrm{P}}+f_{\mathrm{DB}})^2\right] \tag{3}$$

where f_{t} ($-f_{\mathrm{p}}/2 < f_{\mathrm{t}} < f_{\mathrm{p}}/2$) is the azimuth base-band frequency.

The RM amount is

$$\Delta R(f_{\mathrm{t}};r_{\mathrm{c}}) = -\frac{\lambda}{4f_{\mathrm{r}}}\left[(nf_{\mathrm{p}}+f_{\mathrm{t}})^2 - (nf_{\mathrm{p}}+f_{\mathrm{DB}})^2\right] \tag{4}$$

In practical processing, f_{t} is discretely evaluated, i.e.

$$f_{\mathrm{t}} = l\frac{f_{\mathrm{p}}}{N_{\mathrm{a}}} - 0.5f_{\mathrm{p}} \tag{5}$$

where N_{a} is the number of the azimuth frequency bins, l ($0 \leqslant l \leqslant N_{\mathrm{a}}-1$) is the azimuth frequency bin index.

From Eq. (3), in the R-D domain, the signals at the same RM trajectory are the vector superposition of the returns from all of the targets with the same slant range. Therefore, it is reasonable to think that the signals from the same RM trajectory are independently and identically distributed, while the signals from different RM trajectory are of congeneric distribution with different parameters of the probability distribution functions (PDFs).

According to the central limit theorem, the target return signals and the receive noise are independent of each other and are stationary complex Gaussian stochastic processes. As a result, the intensity of the return signals plus noise is exponentially distributed [2].

2.2 Signal model in the R-D domain

In the discrete R-D domain, let us denote the RM amount at the lth azimuth frequency bin by

$$N(l) = \mathrm{round}\ \frac{\Delta R\left(l\dfrac{f_{\mathrm{p}}}{N_{\mathrm{a}}} - 0.5f_{\mathrm{p}};r_{\mathrm{c}}\right)}{R_{\mathrm{u}}} \tag{6}$$

where round (●) is integer operator, R_{u} is one range cell length. The complex sample at the RM trajectory corresponding to the kth range bin can be written as

$$\boldsymbol{x}(k+N(l),l) = \boldsymbol{u}_k + \boldsymbol{\eta} \tag{7}$$

where $\boldsymbol{\eta}$ is the complex Gaussian-distributed thermal noise with zero-mean and unit variance, $\boldsymbol{u}_k$ is the vector superposition of the return signals from the targets at the kth range bin, following the Gaussian distributed with zero-mean and its variance assumed to be P_k, which is diverse for different k. Due to the unit variance of the noise, P_k is just the signal-to-noise ratio (SNR) at the kth range bin.

Because both $\boldsymbol{u}_k$ and $\boldsymbol{\eta}$ are Gaussian-distributed and orthogonal to each other, the distribution of $\boldsymbol{x}(k+N(l),l)$ is also Gaussian-distributed with zero-mean and variance $(1+P_k)$. According to the probability theory, the PDF of the intensity of $\boldsymbol{x}(k+N(l),l)$, i.e. $|\boldsymbol{x}(k+N(l),l)|^2$, is given by Exponential distribution

$$\mathrm{PDF}(|\boldsymbol{x}(k+N(l),l)|^2) = \frac{1}{2(1+P_k)}\exp\left(-\frac{|\boldsymbol{x}(k+N(l),l)|^2}{2(1+P_k)}\right) \tag{8}$$

From Eq. (8), $|\boldsymbol{x}(k+N(l),l)|^2$ has the typical speckle statistics. Therefore, if the scene to be imaged is uneven, the dark or bright stripes, which represent RM trajectory, can be recognized on the azimuth unfocused intensity image of the R-D domain.

3 Description of the algorithm

To begin with, an intuitive interpretation of the proposed algorithm is presented. As we mentioned earlier, in the intensity image of the R-D domain, there are bright or dark stripes representing the RM trajectory. Obviously, the sequence made up of the intensity of the signals at the same RM trajectory has the minimal sample variance, i.e. contrast. In other words, with different values assigned to n in Eq. (3), only when n equates the correct PRF-ambiguity number, the contrast of the signal intensity sequence along the RM trajectory determined by Eq. (3) is the lowest.

For an MN-long sequence composed of M groups, each of which has N elements, $\{z(i,j),\ i\in[1,M], j\in[1,N]\}$, its contrast is defined as

$$C_{\mathrm{M}} = \frac{\dfrac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N} z(i,j)^2}{\left[\dfrac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N} z(i,j)\right]^2} \tag{9}$$

Assume that all the elements of $\{z(i,j),\ i\in[1,M], j\in[1,N]\}$ follow an exponential distribution, but their distribution parameters are different among the groups with their sample mean supposed to be ζ_i, $i\in[1,M]$. According to the property of exponential distribution, its sample second-order original moment is $2\zeta_i^2, i\in[1,M]$. For large N, we have

$$C_M \approx \frac{\dfrac{2}{M}\sum_{i=1}^{M}\zeta_i^2}{\left[\dfrac{1}{M}\sum_{i=1}^{M}\zeta_i\right]^2} \geqslant 2 \tag{10}$$

Apparently, C_M reaches its minimum only when $M=1$ or ζ_i is constant against i, and the two cases are equivalent, which suggests that an exponentially distributed sequence has the minimal contrast when all its elements are identically distributed.

From Eq. (8), in the R-D domain, the signal intensity is exponentially distributed, and has the identical PDF parameters at the same RM trajectory. Hence, the sequence made up of the signal intensity at the same RM trajectory has the lowest contrast. This is consistent with the qualitative interpretation above.

The algorithm to resolve SAR PRF-ambiguity based on R-D domain contrast minimization is described as follows. From Eq. (4), let

$$\Delta R^m(f_t;r_c) = -\frac{\lambda}{4f_r}\left[(mf_P + f_t)^2 - (mf_P + f_{DB})^2\right] \quad (11)$$

$$N^m(l) = \text{round}\frac{\Delta R^m\left(l\dfrac{f_P}{N_a} - 0.5f_P; r_c\right)}{R_u} \quad (12)$$

where m is a trial PRF-ambiguity number. Then, only when m is equal to the correct PRF-ambiguity number n, the signal intensity sequence at the RM trajectory corresponding to the kth range bin, $|\boldsymbol{x}(k+N^m(l),l)|^2$, $0 \leqslant l \leqslant N_a-1$ has the identical PDF parameters. Suppose

$$C_k^m = \frac{\dfrac{1}{N_a}\sum_{l=0}^{N_a-1}|\boldsymbol{x}(k+N^m(l),l)|^4}{\left(\dfrac{1}{N_a}\sum_{l=0}^{N_a-1}|\boldsymbol{x}(k+N^m(l),l)|^2\right)^2} \quad (13)$$

C_k^m reaches its minimum only when $m = n$. In consideration of the receiver noise, it is necessary to take the average of Eq. (13) over many range bins

$$\bar{C}^m = \frac{1}{N_r}\sum_{k=0}^{N_r-1} C_k^m \quad (14)$$

where N_r is the number of the used range bins. The final goal of the algorithm is to search for an integer minimizing $\bar{C}^m$, and the integer is just the correct PRF-ambiguity number.

The proposed method consists of the following steps:

1) range compression;

2) estimate f_{DB} by using the correlation Doppler estimator (CDE) [1], and transfer the zero frequency of each azimuth spectrum to its f_{DB};

3) transform the range compressed data to azimuth frequency domain by azimuth FFTs;

4) according to Eqs. (11)–(14), by assigning m different values, and subsequently search for an integer that minimizes $\bar{C}^m$. Then the integer is the correct PRF-ambiguity number.

Notably, the proposed algorithm is performed within the azimuth Doppler bandwidth; therefore, if the data length in the azimuth direction is far less than one synthesis aperture, which often occurs with air-borne SAR, it is required to compute and use the valid Doppler bandwidth.

4 Performance analysis

Because the Doppler rate f_r is inversely proportional to λ, from the RM amount in Eq. (3), therefore, the method accuracy is proportional to λ^2.

Also from Eq. (3), if f_D changes with one PRF, i.e. PRF-ambiguity n alters by ± 1, the variation of the RM is

$$D(f_t) = \left|\frac{\lambda}{2f_r} f_P(f_t - f_{DB})\right| \quad (15)$$

Assuming the Doppler bandwidth is B_D. The maximum of $D(f_t)$ is

$$D_m = \left|\frac{\lambda}{4f_r} f_P B_D\right| \quad (16)$$

Only when $D_m > R_u/2$, the method correctly detects the variation of the PRF-ambiguity. For space-borne SAR Radarsat and ERS-1, their D_m/R_u are about 2, and accordingly, the algorithm is sensitive to the change of their PRF-ambiguity number.

Except for the FFTs being used to range compression and transform to azimuth frequency domain, which is necessary for some image formation processing such as range-Doppler imaging algorithm, the proposed method merely does simple operations in the R-D domain without the requirement of any extra FFTs or cross-correlation. In addition, what is more important is that both of the theoretical analysis and the experimental results show that the algorithm does not require all the range bins to be used. Therefore, the algorithm is computationally efficient.

The robustness of the algorithm includes two aspects: one is the SNR, and the other is scene contrast. Let

$$q^m = \bar{C}^m - \bar{C}^n; \quad m \neq n. \quad (17)$$

And only when $q^m > 0$, the PRF-ambiguity can be correctly resolved, and the larger q^m is, the better the performance of the algorithm will be. In the R-D domain, according to the property of RM, the contrast in the range direction is

$$C_r = \frac{\dfrac{1}{N_r}\sum_k P_k^2}{\left(\dfrac{1}{N_r}\sum_k P_k\right)^2} \quad (18)$$

and the average SNR is

$$\gamma = \frac{1}{N_r}\sum_k P_k \quad (19)$$

The mean of q^m is

$$\bar{q}^m = \frac{2(C_r - 1)\gamma^2}{(1+\gamma)^2} \quad (20)$$

From Eq. (20), whether the algorithm works or not is dependent on two factors: C_r and γ. If C_r is 1, i.e. P_k is constant against k, or γ is zero, all the signal intensity in the R-D domain has the same PDF parameters, and the method fails. This is consistent with the analysis above. There exists a complementary relation between the SNR and scene contrast, i.e. the increase of SNR can compensate the decrease of scene contrast and vice versa.

From the Chebyshev inequation in the probability theory, the probability that the proposed approach works is determined merely by the ration of $\overline{q}^m$ to the standard deviation of q^m. However, due to the complexity of Eq. (13), it is difficult to obtain the closed-form expression of the standard deviation of q^m. Nevertheless, from Eq. (14) the standard deviation of q^m is inversely proportional to $\sqrt{N_r}$, therefore, we can enhance the correct probability of the proposed algorithm by increasing N_r.

For Gaussian scene ($C_r = 2$), with different N_r, the relation curve of the correct PRF-ambiguity resolving probability versus the SNR γ, which is obtained via simulation with Radarsat's parameters, is shown in Fig. 1. The simulation parameters are: $\lambda = 0.056\ 6$ m, $f_r = -1\ 800$ Hz/s, $f_p = B_D = 1\ 257$ Hz, $N_a = 1\ 000$. The search range for PRF-ambiguity number is from −20 to +20.

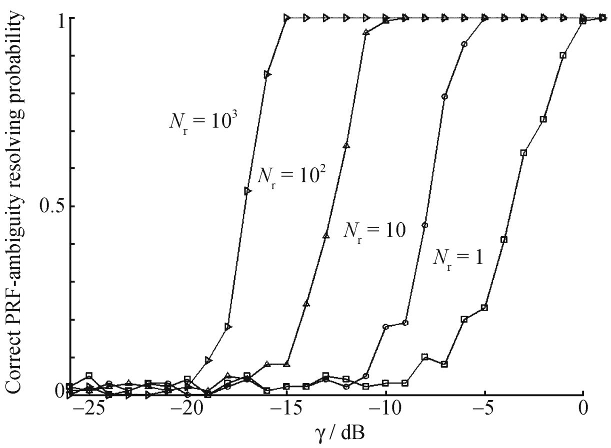

Fig. 1 Correct PRF-ambiguity resolving probability versus γ

From Fig. 1, it is easily found that when $N_r = 1\ 000$, even if the average SNR is simply −16 dB, the proposed algorithm can resolve the PRF-ambiguity with probability 1. In practice, the SNR in the range compressed R-D domain is far larger than −16 dB, and consequently the algorithm is robust against scene contrast and SNR.

5 Experimental results with real data

We have validated the proposed method by the experiments with real space-borne SAR data of Radarsat and ERS-1. Transformed into the R-D domain, the used data have 4 096 azimuth frequency bins and 4 096 range time bins. To improve the computational efficiency, N_r are 500 and 1 000, much smaller than 4 096. Radarsat data of the PRF-ambiguity number −7 is from an urban area of high contrast, while ERS-1 data of the PRF-ambiguity number 0 is from mountains with low contrast. The experimental results are shown in Fig. 2, which shows that the proposed algorithm is valid and robust against scene contrast.

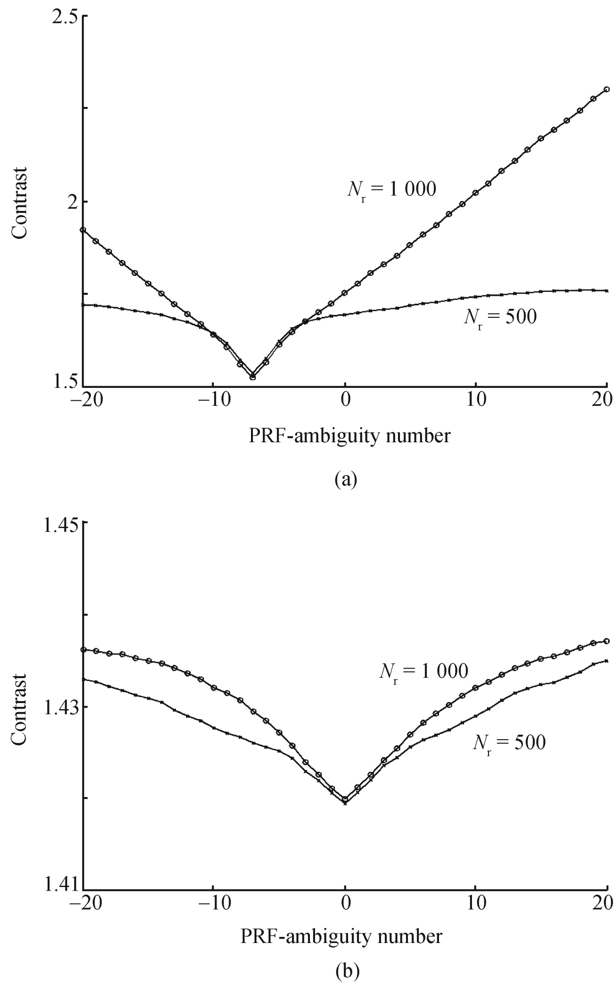

Fig. 2 Relation curve of contrast versus PRF-ambiguity number. (a) Radarsat data; (b) ERS-1 data

6 Conclusion

The property of RM in the R-D domain is analyzed, the signal model constructed, and furthermore, a method to resolve SAR PRF-ambiguity is proposed in this paper. The experimental results show that the method is computationally efficient, robust against scene contrast, and can be naturally incorporated into the range-Doppler imaging algorithm.

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