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A novel autofocus algorithm based on maximum total variation criteria for SAR images

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Abstract A novel autofocus algorithm for synthetic aperture radar (SAR) based on total variation is presented in this paper. The method, which starts with a complex phase-degraded SAR image, after the phase errors model is introduced into the range-compressed phase-history domain, carries out phase errors correction by changing the focus till the total variation of the azimuth profile is maximized. Compared with the minimum entropy autofocus algorithm, the autofocus algorithm has less computational complexity and is easier to implement. The simulation and the processing results of the measured data show the validity of the proposed method.

Keywords synthetic aperture radar, total variation, autofocus, phase errors

1 Introduction

Synthetic aperture radar (SAR) forms a long synthetic array by the uniform motion of its carrier along the ideal flight course. Accordingly, it may achieve high azimuth resolution. The motion of airborne SAR, especially when flying at medium or low altitudes, is rolling due to the instability of air current. Motion compensation is required by using inertial navigation system (INS) and global positioning system (GPS), when the carrier departs from the course. To gain a better focused SAR image, high resolution SAR imaging generally needs the residual phase error to be auto estimated and compensated after traditional motion compensation. The above process is called autofocus.

There are several important autofocus algorithms, such as map drift (MD) [1], phase gradient autofocus (PGA) [2,3], and minimum entropy algorithm [4]. The MD algorithm

divides the whole aperture into two sub-apertures, and then estimates the phase error according to the correlation between the two sub-images. One of its disadvantages is that it can only estimate quadratic phase error, which makes it unsuitable for high resolution imaging. Furthermore, when the azimuth beam width is narrow, the weak correlation between the two sub-apertures will lead to an enlarged estimation of phase error. The PGA algorithm compensates the phase errors by using an iterative method, but it needs strong isolated scatterers in the SAR image, which means it will be invalid in the situation where there are no such scatterers. Based on the blind convolution using minimum entropy criteria, the minimum entropy algorithm, which starts right with complex phase-degraded SAR images, carries out phase error correction via multidimensional search. However, it cannot meet the requirement of real-time imaging due to the large computation in calculating the entropy of the SAR image.

This paper proposes a method that introduces a phase error model into the range-compressed phase-history domain and carries out phase error correction by using the total variation of the image as the cost function of focus.

2 Total variation

Total variation measures the holistic amplitude of signal vibration. If function f is differentiable, its total variation [5] is defined as

$$\|f\|_v = \int_{-\infty}^{+\infty} |f'(t)| dt \quad (1)$$

It measures the holistic amplitude of function f . For example, if $f(t) = \exp(-t^2)$, then $\|f\|_v = 2$.

For discrete signal, suppose that $f_N[n] = f(n/N)$ is the uniformly sampled discrete signal, N^{-1} is the sampling interval. The total variation of f_N is defined as

$$\|f_N\|_v = \sum_n |f_N[n] - f_N[n-1]| \quad (2)$$

The total variation of function f depends not only on the holistic amplitude, but also on the domain of variation length.

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For a SAR image $\{x_{m,n}\}$, its size is $M \times N$, where m is the number of range cell, n is the number of azimuth cell. For a certain range cell, its total variation is defined as

$$\|x_m\|_v = \sum_n \|x_m[n] - |x_m[n-1]|\| \quad (3)$$

When the contrast of the SAR image is low, for better searching precision, Eq. (3) can be transformed into

$$\|x_m\|_v = \sum_n \|x_m[n] - |x_m[n-1]|\|^2 \quad (4)$$

This transformation magnifies the variation between each two sampling points. In simulation experiment, calculating the transformed total variation by using Eq. (4) can improve the searching accuracy in the case of low contrast.

Because the total variation depends on the length of the signal, signal-to-noise ratio (SNR) can be improved by adding the total variation of several range cells.

It can be seen easily from Eqs. (3) and (4) that the algorithm based on total variation has less computational complexity compared with the algorithm based on entropy. In order to compare the two algorithms, the range-compressed echo from only one range cell is considered. Suppose there are N sampling points in the azimuth profile. To calculate the entropy of this signal, at least N modular operations, $N+1$ logarithmic operations, $2N$ multiplications, N additions, and one division are needed. Meanwhile, calculating the total variation only needs N modular operations and $2N-3$ additions. If Eq. (4) is adopted, there will be $N-1$ multiplications in excess. Thus, it can be seen that the algorithm presented in this paper avoids the logarithmic operation and division effectively and it is easier to implement.

3 Phase error model

The phase error of a SAR echo may be classified as low-frequency phase error, high-frequency phase error or wide-band phase error [6]. Among these, the low-frequency phase error is mainly created by the approximation of the imaging algorithm and the metrical error of velocity and acceleration. If expanded by Taylor series, it consists of linear phase error, quadratic phase error and high order phase error. The low-frequency phase error leads to displaced and unfocused images, thus reducing the resolution of SAR images. The high-frequency phase error created by no compensation of the vibration of antenna phase center mainly affects the side lobes. The wide-band phase error created by the random metrical motion error affects the contrast of the image.

The motion compensation based on instrument measurement, which can compensate the high-frequency part in phase error perfectly, depends on INS and GPS. Most of the autofocus algorithms mainly deal with no compensation of the low-frequency phase error.

The linear component in low-frequency phase error only makes the holistic displacement of the image, and does not

affect the image construction. Therefore, the phase error model of the echo may be supposed as

$$\varphi_e(t) = \alpha t^2 + \beta t^3 + \gamma t^4 + \dots, \quad (5)$$

4 Maximum total variation algorithm

The maximum total variation algorithm starts with the azimuth-compressed waveform. The basic idea of the method is that for a certain range cell, the more precisely the azimuth matches, the sharper the spectrum peak will be, and the bigger the vibration amplitude will be. The signal will get maximum total variation when totally matched.

The range-compressed signal may be expressed as follows

$$s(\hat{t}, t_m; r) = A \sin c_r \left(\hat{t} - \frac{2r}{c} \right) a_a(t_m, Q) \cdot \exp[j\varphi_n(t_m; r) + j\varphi_e(t_m; r)] \quad (6)$$

Here, \hat{t} is the range fast time, t_m is the azimuth slow time, $a_a(\cdot)$ is the window function, $\varphi_n(t_m; r)$ is the ideal phase, and $\varphi_e(t_m; r)$ is the phase error.

The phase error $\hat{\varphi}_e(t_m; r)$ is introduced into the phase of a certain range cell after the range compression. If $\hat{\varphi}_e(t_m; r) = \varphi_e(t_m; r)$, then the total variation of the range-compressed signal waveform will be maximized. Thus, the maximum total variation algorithm is to search the phase error corresponding to maximum total variation in certain area. The flow chart of this method is shown in Fig. 1.

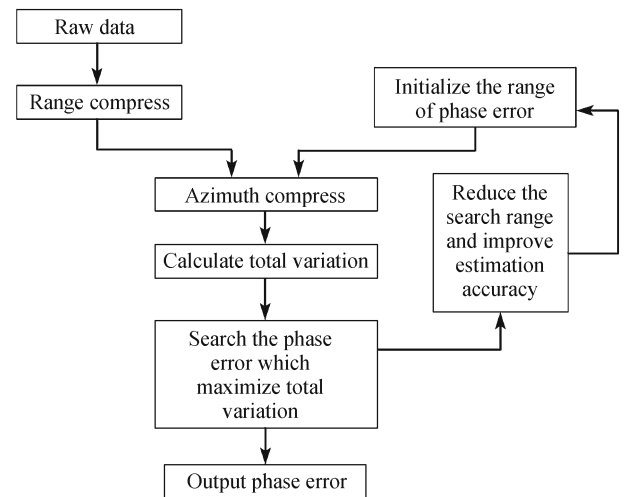


Fig. 1 Flow chart of the maximum total variation algorithm

5 Verification of simulated data

To explain the method presented in this paper, it is supposed that the phase error only contains the quadratic and the third order phase error, viz.

$$\varphi_e(t_m; r) = \alpha t^2 + \beta t^3 \quad (7)$$

The high order components in low-frequency phase error are relatively small, so their influence on the main lobe is not remarkable.

A complex image is produced, the size of which is $1\ 024 \times 1\ 024$. The complex reflection factor of clutter in every range cell conforms to Gaussian distribution, and Gaussian white noise is added in each pixel. The azimuth profile of a certain range cell is shown in Fig. 2. Figure 2(a) corresponds to a fully focused point target. After the introduction of the phase error in Eq. (5), range-compressed phase-history domain and image domain assume a Fourier transform relation. Thus, the point target influenced by spread function is unfocused, which means the widened main lobe, the dropped peak value, and the raised side-lobe level as shown in Fig. 2(b).

The relationship between the phase error coefficient and the total variation of a certain range cell in the image above is shown in Fig. 3(a). Because the high order components in low-frequency phase error are relatively small, their impact on the main lobe is also quite small when the squinting angle is small. It seems that the total variation is mainly determined by the quadratic phase error but not by the third order phase

error. This is related to the impact of each order phase error on the SAR image.

The phase error curve estimated by maximum total variation algorithm is shown in Fig. 3(b) when SNR is 20 dB. It is evident that the proposed method can overcome the influence of noise.

It is noted that the total variation may be taken for negative in the process of estimating phase error. The total variation is a convex function shown in Fig. 3(a), but most of the searching methods are based on the concave function.

6 Verification of measured data

Some raw data of an airborne radar is used to verify the maximum total variation algorithm in this chapter. This radar works in X band, records data in stripmap mode and works in a side-looking way. Its resolution is $3\text{ m} \times 3\text{ m}$. The phase error is unknown, and the size of the image is 512×256 .

Figure 4(a) shows the unfocused image without motion compensation obtained by using R-D algorithm. The image

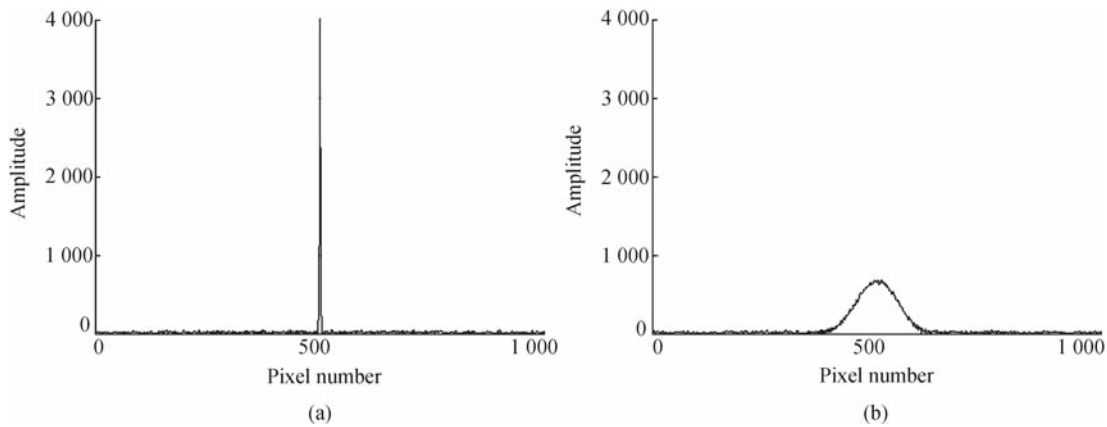


Fig. 2 Azimuth profile of point target

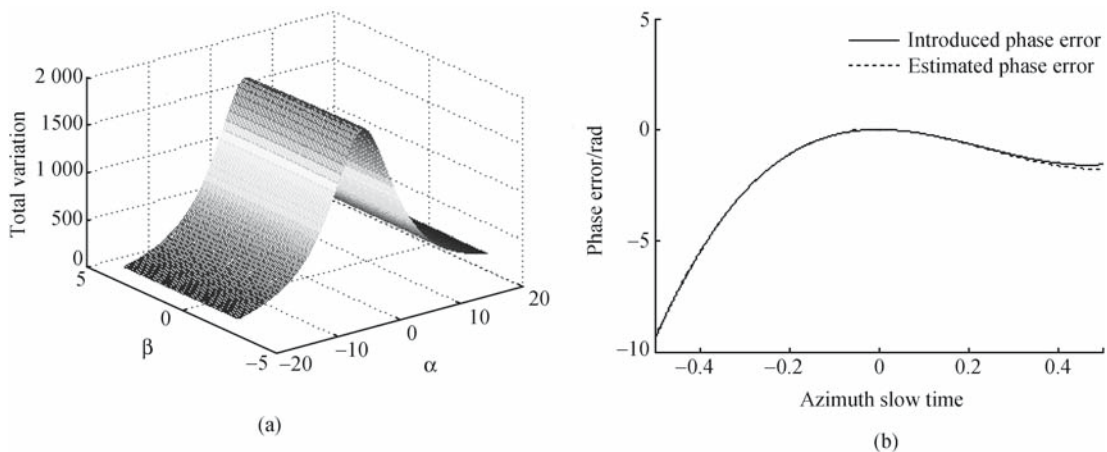


Fig. 3 Relationship between total variation and phase error

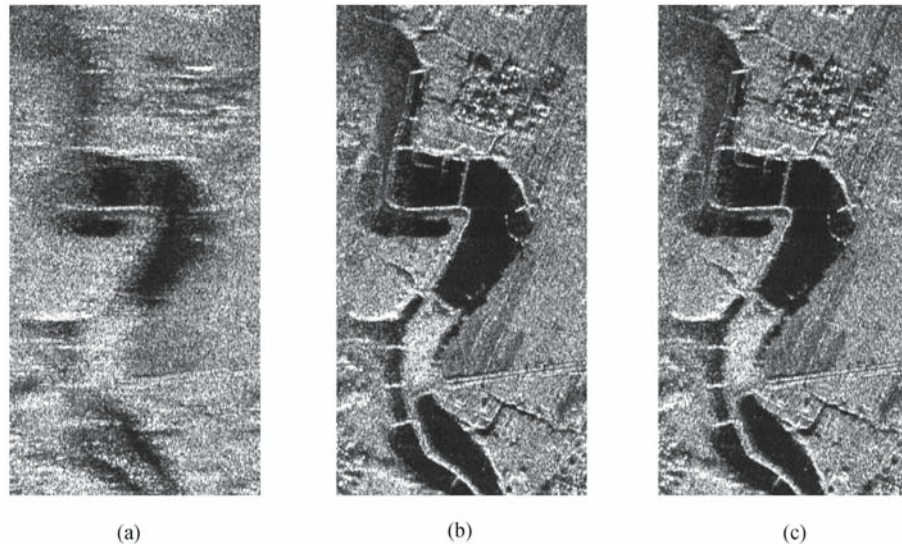


Fig. 4 Processing result of raw data

obtained by using the method presented in this paper to estimate the motion parameter is shown in Fig. 4(b). It is evident that the phase error brought by motion error is calibrated, so that the image is focused perfectly. In order to compare the proposed method with the minimum entropy algorithm, Fig. 4(c) shows the imaging result obtained by using the minimum entropy algorithm. The estimated phase error curve by interpolation is shown in Fig. 5. From the imaging results, there is no evident difference in focusing results between these two images. However, from the point of computational complexity, the maximum total variation algorithm needs less imaging time, and is easier to implement.

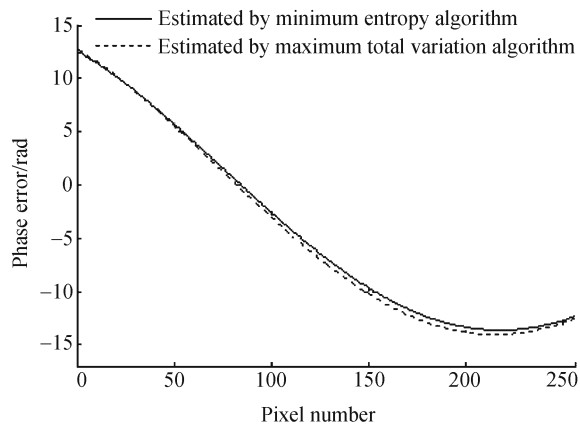


Fig. 5 Phase error of measured data

7 Conclusion

This paper presents a novel SAR imaging autofocus algorithm, which estimates the phase error using the total variation of signal. Compared with the minimum entropy algorithm, this method can not only estimate any order phase error by using multidimensional search but also needs less computational complexity.

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References

1. Li F K, Held D N, Curlander J C, et al. Doppler parameter estimation for spaceborne synthetic aperture radar. *IEEE Trans. on GRS*, 1985, 23(1): 47–56
2. Wahl D E, Eichel P H, Ghiglia D C, et al. Phase gradient autofocus—a robust tool for high resolution SAR phase correction. *IEEE Trans. on AES*, 1994, 30(3): 827–835
3. Zheng Yi-ming, Bao Zheng. An improved phase error estimation algorithm. *Journal of Xidian University*, 2001, 28(4): 472–477 (in Chinese)
4. Wu Xin-wei, Zhu Zhao-da. A novel autofocus algorithm based on minimum entropy criteria for SAR images. *Systems Engineering and Electronics*, 2003, 25(7): 865–869 (in Chinese)
5. Stéphane Mallat. *A wavelet tour of signal processing*. Beijing: China Machine Press, 2003, 24–28 (in Chinese)
6. Zhang Cheng-bo. *Synthetic aperture radar theory, system analysis and applications*. Beijing: Science Press, 1989, 163–178 (in Chinese)