Front. Electr. Electron. Eng. China 2007, 2(3): 277–281
DOI 10.1007/s11460-007-0051-6

RESEARCH ARTICLE

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Adaptive fuzzy sliding mode control of Lorenz chaotic system

Abstract By using the exponential reaching law technology, a sliding mode controller was designed for Lorenz chaotic system subject to an unknown external disturbance. On this basis, considering the unknown disturbance, an adaptive law was introduced to adaptively estimate the parameters of the disturbance bounds. Furthermore, to eliminate the chattering resulting from the discontinuous switch controller and to guarantee system transient performance, a new adaptive fuzzy sliding mode controller was designed. The results of the simulation show the effectiveness of the proposed control scheme.

Keywords Lorenz chaotic system, reaching law, sliding mode control, adaptive law, fuzzy logic

1 Introduction

Chaos is a very interesting nonlinear phenomenon and has been intensively studied in the last three decades [1,2]. A chaotic system has a fundamental characteristic. Its extreme sensitivity to initial conditions, that is, small differences in the initial state can lead to extraordinary differences in the system state [1]. Chaotic behavior is irregular, complex, and in mechanical systems is generally undesirable. Therefore, it is often a source of poor system performance or even instability. One of the goals of the scientific research is to design suitable controllers that are easy to implement and eliminate chaos.

Since the1990s chaos control has received considerable attention, among which the most popular case is Lorenz chaos system control. From the recently published literature on Lorenz control, the methods applied include: feedback linearization [2], bang-bang control [3], nonlinear feedback control [4], neural network control [5], adaptive control [6], and sliding mode control [1,7–9].

Motivated by Yau and Yan [9], in this study, further investigations on the sliding mode control for the Lorenz system problem is made. By using the exponential reaching law technology, a sliding mode controller is designed for Lorenz chaotic system subject to an unknown external disturbance. On that basis, considering the unknown disturbance, an adaptive law is introduced to adaptively estimate the parameters of the disturbance bounds. Furthermore, to eliminate the chattering resulting from the discontinuous switch controller and to guarantee system transient performance, a new adaptive fuzzy sliding mode controller is designed. The results of the simulation show the effectiveness of the proposed control scheme.

2 System description

Lorenz chaotic system can be formulated by

$$\begin{cases} \dot{x}_1 = -\sigma x_1 + \sigma x_2 \\ \dot{x}_2 = r x_1 - x_2 - x_1 x_3 \\ \dot{x}_3 = x_1 x_2 - b x_3 \end{cases} \tag{1}$$

where x_1, x_2, x_3 are state components of the system, σ, r, b are positive real constants that decide the dynamics of the system. When $\sigma = 10$, $b = 8/3$, $r = 28$, the system shows the chaotic behavior [1]. To control the chaos and regulate the system states, a control in the second equation of Eq. (1) was introduced. Moreover, considering the exogenous disturbance, Lorenz system can be reformulated as

$$\begin{cases} \dot{x}_1 = -\sigma x_1 + \sigma x_2 \\ \dot{x}_2 = r x_1 - x_2 - x_1 x_3 + d(t) + u(t) \\ \dot{x}_3 = x_1 x_2 - b x_3 \end{cases} \tag{2}$$

The purpose of this study was to regulate the system states to a predefined point (x_{1r}, x_{2r}, x_{3r}) by designing an effective controller. It is seen that if $x_1(t) = x_{1r}$, then as $0 = \dot{x}_1(t) = -\sigma x_{1r} + \sigma x_{2r}$, then $x_2(t) = x_{2r} = x_{1r}$. Moreover, by solving the third equation of Eq. (1) the following equation is derived

$$x_3(t) = \mathrm{e}^{-bt} x_3(0) + \frac{x_{1r}^2 (1 - \mathrm{e}^{-bt})}{b} \tag{3}$$

Translated from *Journal of Harbin Institute of Technology*, 2006, 38(4): 499–502 [译自: 哈尔滨工业大学学报]

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Therefore, when $t \to \infty$ then $x_3(t) \to x_{3r} = x_{1r}^2/b$.

Defining $e_1 = x_1 - x_{1r}$, $e_2 = x_2 - x_{2r}$, $e_3 = x_3 - x_{3r}$ and $h(t) = d(t) - x_1 x_3$, it follows that

$$\begin{cases} \dot{e}_1 = -\sigma e_1 + \sigma e_2, \\ \dot{e}_2 = re_1 - e_2 + (r-1)x_{1r} + h(t) + u(t), \\ \dot{e}_3 = e_1 e_2 + x_{1r}(e_1 + e_2) - be_3, \end{cases} \tag{4}$$

By taking a model transformation for the first and the second equation in Eq. (4) by using

$$\hat{\boldsymbol{E}} = \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \end{bmatrix} = \boldsymbol{T} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 1/\sigma & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

then

$$\dot{\hat{e}}_1 = \hat{e}_2$$

$$\dot{\hat{e}}_2 = \sigma(r-1)\hat{e}_1 - \sigma(r+1)\hat{e}_2 + (r-1)x_{1r} + h(t) + u(t) \tag{5a}$$

$$\dot{e}_3 = -be_3 + f(\hat{e}_1, \hat{e}_2) \tag{5b}$$

where

$$f(\hat{e}_1, \hat{e}_2) = \sigma\hat{e}_1(\hat{e}_2 + \sigma\hat{e}_1) + x_{1r}(\hat{e}_2 + 2\sigma\hat{e}_1)$$

Here, the third Eq. (5b) can be considered as internal dynamics; so, when $\hat{e}_1 \to 0, \hat{e}_2 \to 0$ supply $e_3 \to 0$. From the analysis above, it can be seen that if the Eq. (5a) is stable on origin, then the Eqs. (5a) and (5b) are stable on origin too.

3 Adaptive sliding mode control

Design of the following linear switching function is

$$s = c\hat{e}_1 + \hat{e}_2 \tag{6}$$

where c is a real constant to be specified later.

According to the reaching condition where $s\dot{s} < 0$, the following exponential reaching law was chosen [10]

$$\dot{s} = -\varepsilon\,\mathrm{sign}(s) - ks \tag{7}$$

where ε, k are positive real constants. When $s > 0$ then

$$s(t) = \frac{\varepsilon}{k} + \left(s_0 - \frac{\varepsilon}{k}\right)\mathrm{e}^{-kt} \tag{8}$$

It is obvious that the convergent rate is higher than that with exponential reaching law especially when time t is large. Therefore, to decrease the chattering caused by sliding mode control, the convergent speed can be decreased when s is nearly convergent to $s = 0$. This can be achieved by decreasing ε and increasing k.

By differentiating Eq. (6) by time t and combining Eqs. (5) and (7), the following sliding mode controller is obtained

$$\begin{aligned} u(t) = {} & \sigma(r-1)\hat{e}_1 + (\sigma+1-c)\hat{e}_2 - (r-1)x_{1r} - h(t) \\ & - \varepsilon\,\mathrm{sign}(s) - ks \end{aligned} \tag{9}$$

Actually, the above controller Eq. (9) is not implemented because $h(t) = d(t) - x_1 x_3$ is unknown. The following assumption is used

$$\|h(t)\| = \|d(t) - x_1 x_3\| \leqslant l_0 + l_1\|\hat{e}_1\| + l_2\|\hat{e}_2\| + l_3\|e_3\| \tag{10}$$

where l_0, l_1, l_2, l_3 are unknown real positive constants.

Let $\hat{l}_0(t), \hat{l}_1(t), \hat{l}_2(t), \hat{l}_3(t)$ be the estimated values of l_0, l_1, l_2, l_3 respectively, then the corresponding estimation errors are given as

$$\tilde{l}_j(t) = \hat{l}_j(t) - l_j \quad (j = 0,\ 1,\ 2,\ 3) \tag{11}$$

Now, the following adaptive estimation laws are given as

$$\begin{cases} \dot{\hat{l}}_0(t) = r_0^{-1}\|s\| \\ \dot{\hat{l}}_1(t) = r_1^{-1}\|\hat{e}_1\|\|s\| \\ \dot{\hat{l}}_2(t) = r_2^{-1}\|\hat{e}_2\|\|s\| \\ \dot{\hat{l}}_3(t) = r_3^{-1}\|e_3\|\|s\| \end{cases} \tag{12}$$

where r_0, r_1, r_2, r_3 are real positive constants representing the adaptive gains.

Combining Eqs. (9)–(12), the following adaptive sliding mode controller is obtained

$$\begin{aligned} u(t) = {} & -\sigma(r-1)\hat{e}_1 + (\sigma+1-c)\hat{e}_2 - (r-1)x_{1r} - ks \\ & - \left[\hat{l}_0(t) + \hat{l}_1(t)\|\hat{e}_1\| + \hat{l}_2(t)\|\hat{e}_2\| \right. \\ & \left. + \hat{l}_3(t)\|e_3\| + \varepsilon\right]\mathrm{sign}(s) \end{aligned} \tag{13}$$

Theorem 1 When the subsystem of Eq. (5a) is considered, the system state can converge to the predefined sliding surface $s = 0$ under the control of Eq. (13).

Proof Choose the following Lyapunov function

$$V = \frac{1}{2}\left[s^2 + r_0\tilde{l}_0^2(t) + r_1\tilde{l}_1^2(t) + r_2\tilde{l}_2^2(t) + r_3\tilde{l}_3^2(t)\right] \tag{14}$$

When Eq. (14) is differentiated the following is derived

$$\begin{aligned} \dot{V} = {} & s\left[\sigma(r-1)\hat{e}_1 - (\sigma+1-c)\hat{e}_1 + (r-1)x_{1r}\right] \\ & + sh(t) + su(t) + r_0\tilde{l}_0(t)\dot{\tilde{l}}_0(t) + r_1\tilde{l}_1(t)\dot{\tilde{l}}_1(t) \\ & + r_2\tilde{l}_2(t)\dot{\tilde{l}}_2(t) + r_3\tilde{l}_3(t)\dot{\tilde{l}}_3(t) \leqslant s\left[\sigma(r-1)\hat{e}_1\right. \\ & \left. - (\sigma+1-c)\hat{e}_1 + (r-1)x_{1r}\right] + \|s\|\|h(t)\| + su(t) \\ & + r_0\tilde{l}_0(t)\dot{\hat{l}}_0(t) + r_1\tilde{l}_1(t)\dot{\hat{l}}_1(t) + r_2\tilde{l}_2(t)\dot{\hat{l}}_2(t) + r_3\tilde{l}_3(t)\dot{\hat{l}}_3(t) \end{aligned} \tag{15}$$

Substituting Eqs. (10)–(13) for (15), yields

$$\dot{V} \leqslant -\varepsilon\|s\| - ks^2 < 0 \tag{16}$$

On the other hand, from Eq. (12) it is known that

$$\dot{\tilde{l}}_j(t) = \dot{\hat{l}}_j(t) > 0, \ (j = 0,\ 1,\ 2,\ 3)$$

thus there exists $t_j\,(j=0,\ 1,\ 2,\ 3)$ such that when $t>t_j$, then

$$\tilde{l}_j(t)=\hat{l}_j(t)-l_j>0,\ (j=0,\ 1,\ 2,\ 3)$$

respectively. Therefore, when $t>t^{*}=\max\left\{t_j, j=0,1,2,3\right\}$ then

$$r_j\tilde{l}_j(t)\dot{\tilde{l}}_j(t)\geqslant 0,\ \ (j=0,\ 1,\ 2,\ 3) \tag{17}$$

Note Eqs. (15) and (17). It is clear that when $t>t^{*}$, the reaching condition $s\dot{s}<0$ holds, which completes the proof.

When the system state of Eq. (5a) arrives onto the sliding surface $s=0$, there is a sliding mode dynamics, which can be formulated by $\dot{\hat{e}}_1+c\hat{e}_1=0$. The stability of the sliding mode dynamics can be guaranteed by selecting parameter $c>0$.

4 Adaptive fuzzy sliding mode control

In the last section, we design an adaptive sliding mode controller, which guarantees the stability of the error system as given in Eqs. (5a) and (5b). It is known that the sliding mode control may cause chattering problem because there exists a sign function in it. The chattering is a key problem to be tackled; otherwise, it may affect the system performance, even making the system unstable.

From Eqs. (7) and (8) one can see that the parameters ε, k affect the convergent speed of switching function s. Generally speaking, when the speed of the system state is convergent to the sliding surface $s=0$ that is, $\dot{s}=\pm\varepsilon$ decreases with the decrease of ε, k, the chattering also decreases. However, the convergent time will be longer when the convergent speed is decreasing, which affects the transient performance of the error system. Therefore, there is a contradiction between the chattering problem and the transient performance of the system. This contradiction can be tackled by using the following law. On one hand, increase the values of ε, k when the system state is far away from the sliding surface $s=0$ by which the convergent speed can be increased and thus the transient performance is improved; on the other hand, decrease the values of ε, k when the system state is close to the sliding surface $s=0$, by which the convergent speed can be decreased, and thus the chattering can also be decreased.

In this study, the fuzzy logic theory is applied to realize the above logic. First, the fuzzy input variable s was chosen, the output variable as ε and k. Also, it was determined that the values of s could be *ZE*, *PS*, *PM*, *PB*, which represent negative big, negative middle, negative small, zero, positive small, positive middle, and positive big respectively. The output ε, k can be valued by *VS*, *S*, *B*, *VB*, which represent very small, small, big, and very big respectively. The memberships for input and output and the fuzzy rules were chosen as given Figs. 1 and 2, and relationship between input and output is shown in Fig. 3.

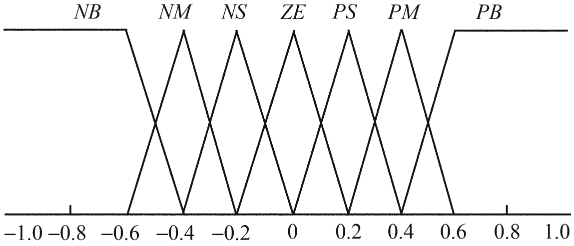

Fig. 1 The membership of input s

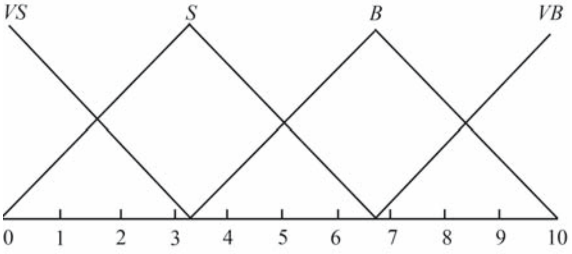

Fig. 2 The membership of outputs ε, k

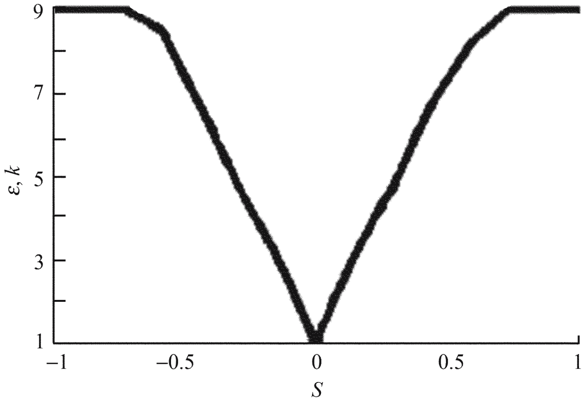

Fig. 3 The relationship between input and output

The language rules of the fuzzy control are shown in Table 1.

Table 1 Rule table of fuzzy sliding surface

s	*NB*	*NM*	*NS*	*ZE*	*PS*	*PM*	*PB*
ε	*VB*	*B*	*S*	*VS*	*S*	*B*	*VB*
k	*VB*	*B*	*S*	*VS*	*S*	*B*	*VB*

5 Simulation

Consider the following Lorenz chaotic system

$$\begin{cases}\dot{x}_1=-10x_1+10x_2\\ \dot{x}_2=28x_1-x_2-x_1x_3+d(t)+u(t)\\ \dot{x}_3=x_1x_2-2.67x_3\end{cases} \tag{18}$$

Let the initial condition be (12, 2, 9), and let $x_{1r}=x_{2r}=8$ and $x_{3r}=24$. Figure 4 shows the state of Eq. (18) without imposing controller. Obviously, it is a chaotic system. First, the adaptive sliding mode controller in Eq. (13) was used to regulate the system state by the setting $r_0 = r_1 = r_2 = r_3 = 1$, $c = 6$, $\varepsilon = 4$, $k = 10$, and then the following simulation results were derived

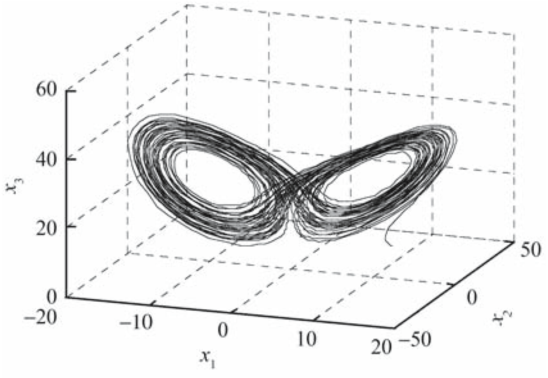

Fig. 4 Relationship of states x_1, x_2, x_3 before control action

After imposing the controller Eq. (13) into the system in Eq. (18), the state responses are shown in Fig. 5. It can be seen that the system state is regulated to a predefined point. Figure 6 gives the response of the switching function under the adaptive sliding mode controller, which shows that obviously there exists chattering. Figure 7 shows the estimation of parameters l_0, l_1, l_2, l_3.

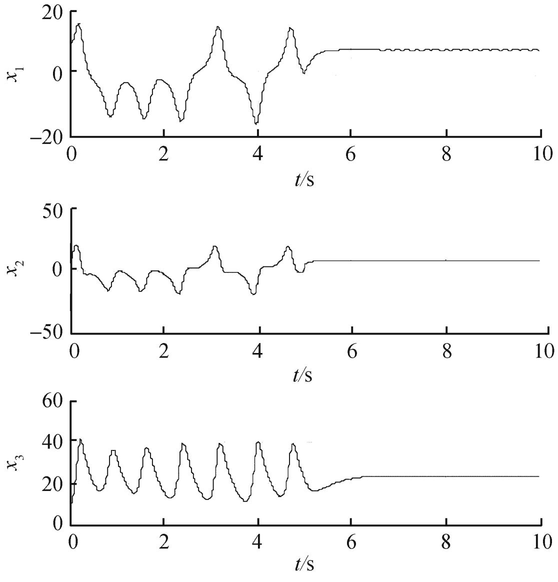

Fig. 5 Response curves of states x_1, x_2, x_3

Now, to solve the chattering problem, a fuzzy logic law was introduced into the adaptive sliding mode controller Eq. (13) to regulate the parameters ε, k. Then an adaptive fuzzy sliding mode controller is obtained. When it is imposed into Eq. (18), the chattering is clearly decreased. The switching function response is shown in Fig. 8.

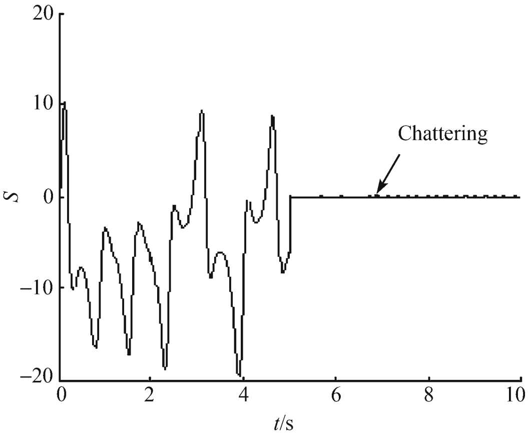

Fig. 6 Response curve of sliding surface function S

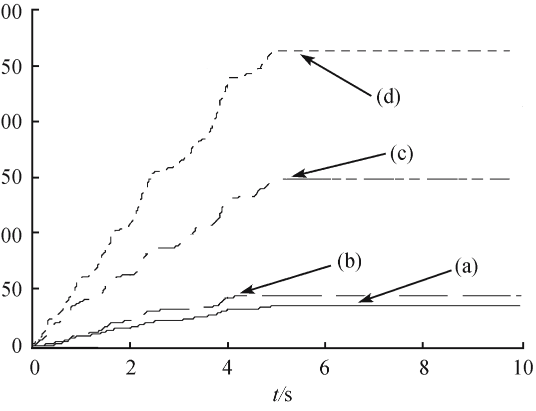

Fig. 7 Estimate curves of the parameters l_0, l_1, l_2, l_3

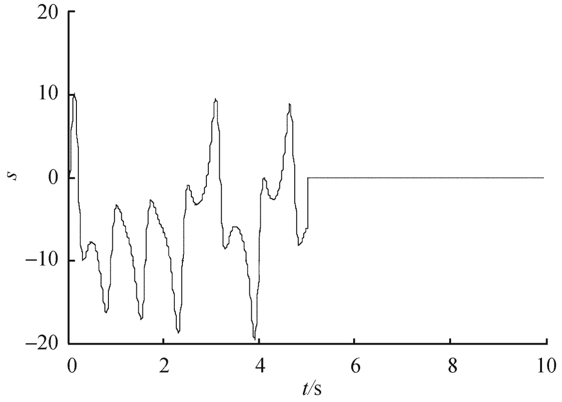

Fig. 8 Response curve of sliding surface function s

6 Conclusions

In this study, the Lorenz chaotic control problem was investigated by using adaptive fuzzy sliding mode control. By using the exponential reaching law technology, a sliding mode controller was designed for Lorenz chaotic system subject to an unknown external disturbance. On this basis, after considering the unknown disturbance, an adaptive law was introduced to adaptively estimate the parameters of the disturbance bounds. Moreover, to eliminate the chattering resulting from the discontinuous switch controller and to guarantee system transient performance, a new adaptive fuzzy sliding mode controller was designed. Finally, the results of the simulation show the effectiveness of the proposed control scheme.

Acknowledgements This work is supported by the National Natural Science Foundation of China (Grant No. 69874008).

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