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RESEARCH ARTICLE

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Yield modeling of elliptical defect

Abstract Physical defects have always played an important role in integrated circuit (IC) yields, and the design sensitivity to these physical elements has continued to increase in today's nanometer technologies. The modeling of defect outlines that exhibit a great variety of defect shapes is usually modeled as a circle, which causes the errors of critical area estimation. Since the outlines of 70% defects approximate to elliptical shapes, a novel yield model associated with elliptical outlines of defects is presented. This model is more general than the circular defects model as the latter is only an instance of the proposed model. Comparisons of the new and circular models in the experiment show that the new model can predict yield caused by real defects more accurately than what the circular model does, which is of significance for the prediction and improvement of the yield.

Keywords real defect, elliptical defect model, critical area, yield modeling

1 Introduction

The yield estimation can be used in production control, material management, timely product delivery, operation planning tool, and also in determining the cost of new devices and their resource implications. It is also the focus of the industry that evolves from production of standard products, such as memories, to the production of application of specific integrated circuit (ASIC) and system on chip (SOC) products [1]. With yield estimation, measures designing rules and conditions for processes, including changing layout shapes, are adopted to obtain the maximum yields for mature processes [2].

Physical defects have always played an important role in integrated circuit (IC) yields, and the design sensitivity to these physical elements has continued to increase in today's nanometer technologies [3]. Among typical CMOS processes, it is one of the greatest loss factors of the yield that the defect causes the electrical faults. The modeling of real defect outlines that exhibit a great variety of defect shapes is usually modeled as circular available [4–6]. Hess and Weiland [7] pointed out that the outlines of defects can be classified into three types: type-0, type-1, and type-2, and of all the inspected defects yields, 23% are type-0 defects, 67% are type-1 defects, and just 10% are type-2 defects. Our investigation results show that 25.73% of all the inspected defects belong to type-0, 50.88% to type-1, and 23.39% to type-2. Their types are shown in Fig. 1.

Obviously, the outlines of type-1 defects are similar to elliptical shape and their equivalent ellipses can be obtained by using image process technology [8]. More than one and a half of outlines of defects resemble ellipse, which is usually modeled as a circle in the yield model, which causes the errors estimation for IC [9,10]. This paper discusses the yield model and its correlative critical area model for an elliptical defect. At the same time, the verification of the models is confirmed in our experiments.

2 Yield modeling for elliptical defect outline

In general, based on the circular defect outlines, the Poisson yield model and the Negative binomial model are often used to predict the yield loss from defects.

Poisson model $$Y_F = \prod_{i=1}^{M} \mathrm{e}^{-A_i^{(c)} D_i} \quad (1)$$

Negative binomial model $$Y_F = \prod_{i=1}^{M} \left(1 + \frac{D_i A_i^{(c)}}{\alpha_i}\right)^{-\alpha_i} \quad (2)$$

Here, Y_F denotes the yield, which is the product of all layer of yields. D_i is the defect density for layer i. The Poisson yield has been observed to give pessimistic yield-predictions for

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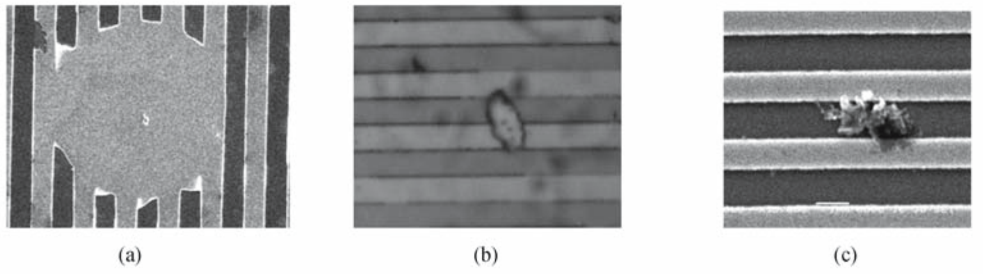

Fig. 1 Real defect
(a) Type-0; (b) Type-1; (c) Type-2

large devices. In the better negative binomial model, α_i is the clustering parameter. Both in Poisson yield model and Negative binomial model, $A_i^{(c)}$ is the critical area related to a layout for layer i, and $A_i^{(c)}$ can be written as

$$A_i^{(c)} = \int_{R_{\min}}^{R_{\max}} A(R)h(R)\mathrm{d}R \tag{3}$$

where $A(R)$ is the critical area with diameter R, $h(R)$ is the distribution of defect sizes, $R_{\min}$ is the minimal diameter, and $R_{\max}$ is the maximal diameter based on the circular model.

Let an elliptical defect be $d(a,b,\theta)$. Its parametric equation will be $x = \alpha_1\cos t - \beta_1\sin t$, $y = \alpha_2\cos t + \beta_2\sin t$, where $\alpha_1 = a\cos\theta$, $\alpha_2 = a\sin\theta$, $\beta_1 = b\sin\theta$, $\beta_2 = b\cos\theta$. Its major axis is the length a, and its minor axis is the length b. The angle θ is formed by its major axis and the positive x-axis. On the basis of the elliptical outline model of a real defect, the critical area associated with an elliptical defect, and an IC layout for ith process in IC, Eq. (3) can be expressed by the Stieltjes integral as

$$A_i^{(c)} = \int_{A_{\min}}^{A_{\max}}\int_{B_{\min}}^{B_{\max}}\int_{\theta_{\min}}^{\theta_{\max}} A(a,b,\theta)\mathrm{d}t_1(a)\mathrm{d}t_2(b)\mathrm{d}t_3(\theta) \tag{4}$$

where $A_{\min}$ and $A_{\max}$ are the minimal and maximal lengths of the major axis, respectively; $B_{\min}$ and $B_{\max}$ are the minimal and maximal lengths of the minor axis, respectively; $\theta_{\min}$ and $\theta_{\max}$ are the minimal and maximal angles of the major axis with the positive x-axis, respectively. The distribution function for the defects in x-axis direction is $t_1(a)$, that is, $\int_{A_{\min}}^{A_{\max}} \mathrm{d}t_1(a) = 1$, $t_2(b)$ is the distribution function for the defects in y-axis direction, that is $\int_{B_{\min}}^{B_{\max}} \mathrm{d}t_2(b) = 1$, and $t_3(\theta)$ is the distribution function in the positive angle of the major axis with the x-axis positive direction for the defects, namely $\int_{\theta_{\min}}^{\theta_{\max}} \mathrm{d}t_3(\theta) = 1$. The critical area caused by the defect $d(a, b, \theta)$ is $A(a, b, \theta)$. In general, if $F(a, b, \theta)$ is the joint distribution function about a, b, θ, Eq. (4) is also expressed as

$$A_i^{(c)} = \int_{A_{\min}}^{A_{\max}}\int_{B_{\min}}^{B_{\max}}\int_{\theta_{\min}}^{\theta_{\max}} A(a, b, \theta)\mathrm{d}F(a, b, \theta) \tag{5}$$

If a, b, θ are mutually independent, then $F(a, b, \theta) = t_1(a)t_2(b)t_3(\theta)$, that is, $A_i^{(c)}$ is equal to Eq. (4). For a mature process, b and θ are mutually independent, so the yield model associated with the elliptical defect is equal to Eq. (4). Since $t_1(a)$, $t_2(b)$, and $t_3(\theta)$ are fixed and cannot be changed in Eq. (4), it is important for us to solve Eq. (4) for $A(a, b, \theta)$, which is determined commonly by the layout routing and the defect equation, Eq. (1). It is well known that IC interconnect is eventually going to be the dominant contributor to signal performance and that the largest contributors to yield loss is metal-to-metal shorted lines, which can occur when a particle defect falls between two lines. The short critical area that is the representative of a group is as follows.

Let $d(a, b, \theta)$ be an ellipse, R be its circumrectangle, and R be tangent to $d(a, b, \theta)$ at $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$, and $D(x_4, y_4)$, respectively, as is shown in Fig. 2(a). L and W are the length and width of a metal conductor, respectively. S is the space between two metal conductors (see Fig. 2(b)). When $S < y_1 - y_3 < W + 2S$, for two horizontal metal conductors, the black area made of c_1, c_2, c_3 shown in Fig. 2(b) is

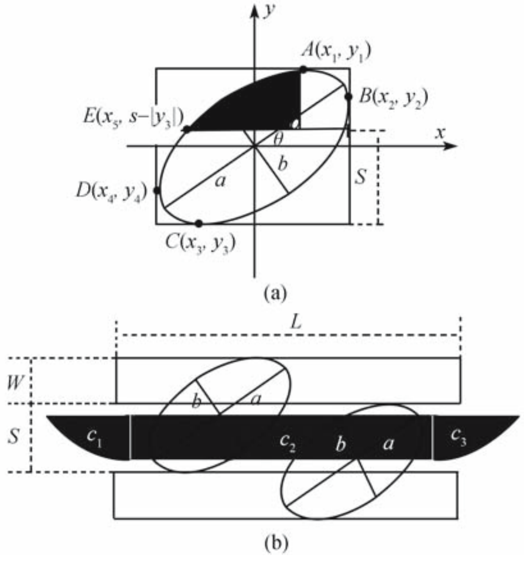

Fig. 2 Elliptical defect
(a) Elliptical defect and critical area c_1; (b) Short critical area for horizontal metal conductor caused by elliptical defect with angle θ

the short-circuit critical area within which the center of the elliptical defect must fall, and to result in a complete link between two metal conductors, where the shape of c_2 is a rectangle, c_1 is equal to c_3 in size, as the black area in Fig. 2(a) shows.

In order to characterize the short critical area, let t_1 denote the point $E(x_5, S-|y_3|)$ and t_2 denote the point $A(x_1, y_1)$ (see Fig. 2), then the short critical area $A_{\text{short}}^{\text{hc}}(a,b,\theta)$ for horizontal metal conductors caused by the elliptical defect with angle θ can be expressed as

$$A_{\text{short}}^{\text{hc}}(a,b,\theta)=\begin{cases}0, & 0\leqslant y_1-y_3\leqslant S\\ c_1+c_2+c_3, & S<y_1-y_3\leqslant W+2S\\ (S+W)L, & y_1-y_3>W+2S\end{cases}\quad(6)$$

where $c_2=(y_1-y_3-S)[L-2(x_2-x_4)]$ and

$$c_1=c_3=\frac{\cos(2t_2)-\cos(2t_1)}{4}(\alpha_1\alpha_2+\beta_1\beta_2)-\frac{t_2-t_1}{2}(\alpha_1\beta_2+\beta_1\alpha_2)+\frac{\sin(2t_2)-\sin(2t_1)}{4}(\alpha_1\beta_2+\beta_2\alpha_1)$$

Similarly, the short critical area $A_{\text{short}}^{\text{hc}}(a,b,\theta)$ for vertical metal conductors caused by the elliptical defect with angle θ can be expressed as

$$A_{\text{short}}^{\text{hc}}(a,b,\theta)=\begin{cases}0, & 0\leqslant x_1-x_3\leqslant S\\ c_1+c_2+c_3, & S<x_1-x_3\leqslant W+2S\\ (S+W)L, & x_1-x_3>W+2S\end{cases}\quad(7)$$

where $c_2=(x_1-x_3-S)[L-2(y_2-y_4)]$ and

$$c_1=c_3=\frac{\cos(2t_2)-\cos(2t_1)}{4}(\alpha_1\alpha_2+\beta_1\beta_2)-\frac{t_2-t_1}{2}(\alpha_1\beta_2+\beta_1\alpha_2)+\frac{\sin(2t_2)-\sin(2t_1)}{4}(\alpha_1\beta_2+\alpha_1\beta_1)$$

As a result, for an extra defect, based on the ellipse model of a real defect, the yield model consists of Eqs. (1), (4), (6), and (7) or consists of Eqs. (2), (4), (6), and (7).

3 Modeling verification

For yield loss estimation from the elliptical defect, the aim of modeling verification is to confirm that the yield estimation by the elliptical model of a real defect is more accurate than that by the circular model of a real defect. Figure 3 shows the layout of a defect test chip used for our experiments. The circular and elliptical outlines of these defects are derived by the segmentation method of color image processing, which was presented by the authors' previous papers [11–13]. To facilitate computation, it is assumed that the distribution of the defects in space is unified and the angle θ of an ellipse is fixed. For the layout shown in Fig. 3, the critical area and the yield are obtained by using the Eqs. (1), (4), (6), and (7), where $S = W = 3e$, $e = 0.3905$ μm, $L = 189$ μm, the number of lines is 30, and the density of defect D is 1.9×10^{-6} μm^2 by the real measurement value.

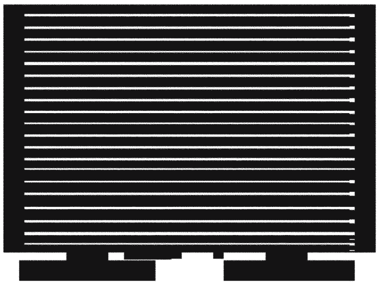

Fig. 3 Test chip layout

For the test chip layout in Fig. 3, comparisons of yields associated with ellipses, which are different in size, and that of the related maximal circle are summarized in Figs. 4 and 5. A comparison of the yield from a circular defect and from an elliptical defect is shown in Fig. 4, where x-axis denotes the maximal size of the circular defect, the elliptical parameter a is equal to the radius of a circular defect and the elliptical parameter $b = a/2$, and $\theta = 1.0$. In Fig. 4 for the analyzed layout, the maximal circle diameter varies between 0 and $4e$ and the yield is 1, because the electronic fault cannot occur if the maximal circle diameter of a defect is smaller than the minimal line space; when the maximal circle diameter is larger than $16e$ and θ of the elliptical model is not equal to 90°, the yield estimation by elliptical model is larger than that of circular model, for which the errors occur more often in the yield estimation. The estimation errors caused by maximal circular model are related to the parameter of an ellipse. Especially, when θ is 90° and if the critical areas c_1 and c_3 are not considered, then the critical area and yield from elliptical defect are equal to that from the circular defect. The above-result shows that the yield from circular defect is only

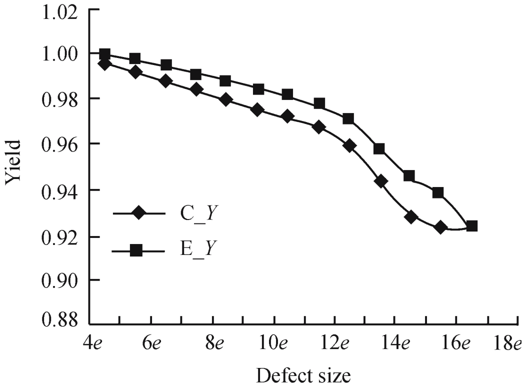

Fig. 4 Yields caused by circular and elliptical defect

the special case of yield from the elliptical defect. Noticeably for the defect, since the defect in our experiment is sampled at random, the yield difference between the circular model and elliptical model of random defect is smaller than that for the elliptical defect. That is to say, if the defect is elliptical, then the errors caused by the circular model and the elliptical model are larger than the errors caused by the random defect, which indicates significance of the elliptical model.

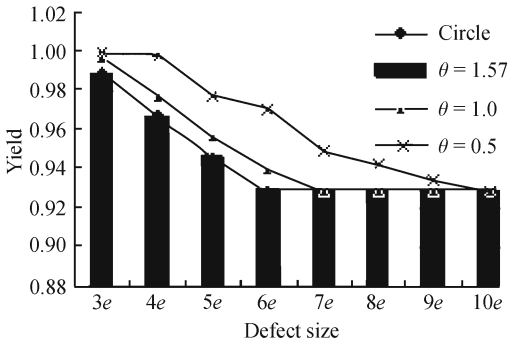

Fig. 5 Yield caused by different angle θ for elliptical and circular defect

4 Conclusions

The accurate yield model plays an important role in IC manufacturing because more than half of the test defects are elliptical shapes. This paper presents a new yield model for the elliptical defects. The yield-loss comparison obtained by the proposed model with available model in experiment reveals that the circular yield model is only the special case of the suggested model, because the elliptical yield model is more general than the circular yield model. The results obtained in this paper lay the foundation for enhancing the accuracy of predicted yield caused by elliptical defects and the accuracy of predicted IC critical area and yield.

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