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RESEARCH ARTICLE

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Analytical delay models for RLC interconnects under ramp input

Abstract Analytical delay models for Resistance Inductance Capacitance (RLC) interconnects with ramp input are presented for different situations, which include overdamped, underdamped and critical response cases. The errors of delay estimation using the analytical models proposed in this paper are less by 3% in comparison to the SPICE-computed delay. These models are meaningful for the delay analysis of actual circuits in which the input signal is ramp but not ideal step input.

Keywords ramp input, delay, RLC interconnect model, analytical formula

1 Introduction

With the development of super large-scale IC process, delay by interconnects in circuit is playing a more and more important role in the performance of the circuit. Simple but effective analytical delay models of interconnects are useful for IC designers to avoid some timing problems and to optimize the design, such as minimizing the delay [1–5]. So it is necessary to build accurate and effective delay estimation models for interconnects.

Elmore delay model [1], which is simple in form and easy to be used, has been widely adopted to estimate the interconnect delays in the performance-driven synthesis and layout of very large-scale integrate (VLSI) routing topologies. It is actually the first-order estimation of the interconnect delay with the input ideal step signal, that is, assuming rising time to be zero. With the speed of circuits becoming higher and higher, inductance effect of interconnects is becoming more and more important and can no longer be ignored. Under this circumstance, the Elmore model is not accurate any longer since the model only takes resistance and capacitance effects into account [5]. It is necessary to use a second-order model, which includes the effect of inductance. Therefore, the concept of equivalent Elmore delay based on the Resistance Inductance Capacitance (RLC) model has been considered [5,6].

However, for all the aforementioned models, a precondition exists that the input is assumed to be an ideal step signal that is not the same as the case in practice. There must be some rising time, that is, the input is a ramp signal. If we keep on using the delay model, which is obtained with an ideal step input, it seems likely we would get results with a large error. In a study by Hasegawa and Seki [4], the concept of delay models for interconnects under ramp input was brought forward. However, the paper only offered a brief discussion on overdamped cases irrespective of the underdamped and critical response cases. Moreover, generally speaking, Elmore model offers quite an accurate estimation of the delay in the case of overdamped response.

In this paper, analytical delay models for RLC interconnects under ramp input are presented for different situations, that is, overdamped, underdamped and critical response cases. With these analytical models, one can easily have a basic view of the delay during the circuit design.

2 Delay model for interconnects under step input

Here is a second-order analytical delay model for interconnects under step input. In the transform domain, output response can be obtained through $U_{\text{out}}(s) = U_{\text{in}}(s)H(s)$, where $H(s)$ is the transfer function of the system. By inverse Laplace transform, the output response in the time domain can also be obtained. With the time domain output response $u_{\text{out}}(s)$, one can easily obtain the delay under a certain output threshold.

Transfer function for the interconnect line of Fig. 1 is expressed by Ref. [2]

$$H(s) = \frac{U_2(s)}{U_0(s)} = \frac{1}{\left[A + \frac{Z_s}{Z_0}B\right] + \frac{1}{Z_{\text{T}}}\left[Z_0 B + Z_s A\right]} \tag{1}$$

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where $A=\cosh(\theta h)$, $B=\sinh(\theta h)$, $\theta=\sqrt{(r+sl)sc}$ is the transfer constant, $Z_0=\sqrt{(R+sL)/sC}$ is the characteristic impedance, $Z_{\mathrm{T}}=(sC_{\mathrm{T}})^{-1}$, $Z_s=R_s+sL_s$, $r=R/h$, $l=L/h$, $c=C/h$ are resistance, inductance and capacitance over unit length of interconnect, respectively, and h is the length of the interconnect.

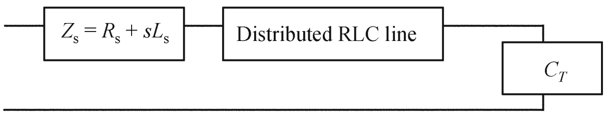

Fig. 1 Two-port model of a distributed RLC line with resistive and inductive source impedance and capacitive load impedance

By expanding Cosh and Sinh as infinite series and collecting terms up to the coefficient of s^2 in the denominator, the truncated transfer function can be obtained as follows

$$H(s)\approx\frac{1}{1+sb_1+s^2b_2} \tag{2}$$

with coefficients

$$b_1=R_sC+R_sC_{\mathrm{T}}+\frac{RC}{2}+RC_{\mathrm{T}}$$

$$b_2=\frac{R_sRC^2}{6}+\frac{R_sRCC_{\mathrm{T}}}{2}+\frac{(RC)^2}{24}+\frac{R^2CC_{\mathrm{T}}}{6}+L_sC+L_sC_{\mathrm{T}}+\frac{LC}{2}+LC_{\mathrm{T}}$$

When the input is an ideal step signal with the amplitude U_0

$$U_{\mathrm{out}}(s)=U_{\mathrm{in}}(s)H(s)=\frac{U_0}{s}\frac{1}{1+sb_1+s^2b_2}$$

$u_{\mathrm{out}}(t)$ can be obtained by inverse Laplace transform. Assuming $u_{\mathrm{out}}(t)=0.9U_0$, we can get the 90% delay $t_{0.9}$.

3 Analytical delay model for interconnects under ramp input

In practice, the input is impossible to be ideal. The rising or falling of the signal cannot be finished instantly. Instead, there must be a finite rise (or fall) time. Thus, using the delay model, which is obtained with the ideal step signal as the stimulus, will inevitably lead to a calculation error.

In the following section, a detailed discussion on the cases with ramp input is given.

The finite rising ramp input shown in Fig. 2 can be expressed in the time domain as [7]

$$\begin{aligned}u_{\mathrm{f\_in}}(t)&=u_{\mathrm{if\_in}}(t)-u_{\mathrm{if\_in}}(t-T_r)\\&=\frac{U_0}{T_r}\left[tu(t)-(t-T_r)u(t-T_r)\right]\quad t\geqslant 0\end{aligned} \tag{3}$$

where $u(t)$ denotes the step function. The finite ramp input in the transform domain is

$$U_{\mathrm{f\_in}}(s)=\frac{U_0}{T_r}\frac{1}{s^2}\left[1-\mathrm{e}^{-sT_r}\right] \tag{4}$$

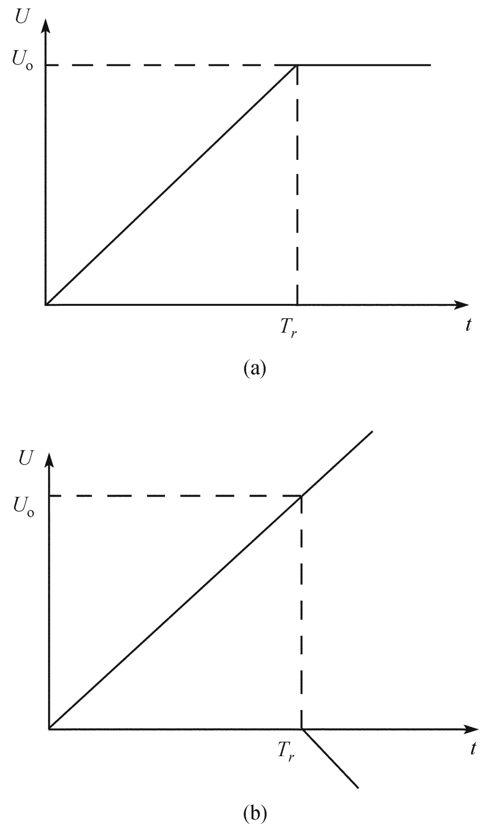

Fig. 2 A ramp input function
(a) Finite ramp with rise time T_r; (b) Finite ramp decomposed into two shifted infinite ramps

In the transform domain, the output response is

$$\begin{aligned}U_{\mathrm{f\_out}}(s)&=\frac{U_0}{T_r}\frac{1}{s^2}\left[1-\mathrm{e}^{-sT_r}\right]H(s)\\&=U_{\mathrm{if\_out}}(s)\left[1-\mathrm{e}^{-sT_r}\right]\end{aligned} \tag{5}$$

Taking Eq. (1) into Eq. (5), one can get the $u_{\mathrm{f\_out}}(t)$ after inverse Laplace transform.

Applying Elmore's original definition of delay for step input yields an analytical delay T_{AD} for ramp input [7], i.e.

$$T_{\mathrm{AD}}=\frac{T_r}{2}+b_1-a_1=\frac{T_r}{2}+T_{\mathrm{ED}} \tag{6}$$

where a_1, b_1 are the coefficients of s^2 in the numerator and denominator of the transfer function, respectively, and T_{ED} is the Elmore delay for a step input, that is, the first moment of the transfer function (Note: T_{ED} in Eq. (6) should correspond with the different output threshold; Elmore delay corresponds to $T_{0.632}$, that is, delay with the output threshold of $0.632U_0$ [8] while $T_{0.9} = 2.3T_{\rm ED}$, $T_{0.5} = 0.693T_{\rm ED}$ etc.).

In the following section, we discuss the two-pole methodology for interconnect response computation.

3.1 Real poles

In the transform domain, response of infinite ramp is

$$
\begin{aligned}
U_{\rm if\_out}(s) &= \frac{U_0}{T_r}\frac{1}{s^2}\frac{1}{1+sb_1+s^2b_2} \\
&= \frac{U_0}{T_r}\left(\frac{-b_1}{s}+\frac{1}{s^2}-\frac{1+b_1s_2}{s_1-s_2}\frac{1}{s-s_1}\right. \\
&\quad \left.+\frac{1+b_1s_1}{s_1-s_2}\frac{1}{s-s_2}\right)
\end{aligned}
\tag{7}
$$

where s_1, s_2 are the poles of the transfer function. The corresponding response in the time domain is

$$
u_{\rm if\_out}(t) = \frac{U_0}{T_r}\left(-b_1+t+\frac{1+b_1s_2}{s_2-s_1}{\rm e}^{s_1t}+\frac{1+b_1s_1}{s_1-s_2}{\rm e}^{s_2t}\right)u(t) \tag{8}
$$

Considering $s_{1,2}<0$ and $|s_2|>|s_1|$, $\frac{1+b_1s_1}{s_1-s_2}{\rm e}^{s_2t}$ decreases more rapidly compared with $\frac{1+b_1s_2}{s_2-s_1}{\rm e}^{s_1t}$. Hence, the two-pole response can be approximated (lower-bounded) as

$$
u_{\rm if\_out}(t) \approx \frac{U_0}{T_r}\left[-b_1+t+\frac{1+b_1s_2}{s_2-s_1}{\rm e}^{s_1t}\right]u(t) \tag{9}
$$

Then we can get the response of finite ramp:

$$
\begin{aligned}
u_{\rm f\_out}(t) &= u_{\rm if\_out}(t)-u_{\rm if\_out}(t-T_r) \\
&= \frac{U_0}{T_r}\left[T_r+\frac{(1+b_1s_2)\left({\rm e}^{s_1t}-{\rm e}^{s_1(t-T_r)}\right)}{(s_2-s_1)}\right. \\
&\quad \left.+\frac{(1+b_1s_1)\left({\rm e}^{s_2t}-{\rm e}^{s_2(t-T_r)}\right)}{(s_1-s_2)}\right]u(t) \\
&\approx \frac{U_0}{T_r}\left[T_r+\frac{(1+b_1s_2)\left({\rm e}^{s_1t}-{\rm e}^{s_1(t-T_r)}\right)}{(s_2-s_1)}\right]u(t)
\end{aligned}
\tag{10}
$$

Assuming that the delay corresponds to τ when the output reaches $u_{\rm th}$, that is, $u_{\rm f\_out}(\tau) = u_{\rm th}$ ($U_0 = 1$), then

$$
\tau = \frac{1}{|s_1|}\ln\left[\frac{\left({\rm e}^{|s_1|T_r}-1\right)(1+b_1s_2)}{T_r(1-u_{\rm th})(s_2-s_1)}\right] \tag{11}
$$

3.2 Complex poles

Assume that poles of the transfer function are $s_{1,2} = -\alpha \pm {\rm j}\beta$. Response of infinite ramp in the transform domain is

$$
U_{\rm if\_out}(s) = \frac{U_0}{T_r}\left[\frac{-b_1}{s}+\frac{1}{s^2}+\frac{\frac{2\alpha}{(\alpha^2+\beta^2)}s+\frac{3\alpha^2-\beta^2}{(\alpha^2+\beta^2)}}{(s+\alpha)^2+\beta^2}\right] \tag{12}
$$

whose response in time domain is

$$
u_{\rm if\_out}(t) = \frac{U_0}{T_r}\left[-b_1+t+\frac{1}{\beta}{\rm e}^{-\alpha t}\sin(\beta t+\theta)\right]u(t) \tag{13}
$$

where $\alpha = b_1/2b_2$, $\beta = \sqrt{4b_2-b_1^2}/2b_2$, $\theta = \arctan\ 2\alpha\beta/(\alpha^2-\beta^2)$.

Then we can get the response of finite ramp as follows

$$
\begin{aligned}
u_{\rm f\_out}(t) &= u_{\rm if\_out}(t)-u_{\rm if\_out}(t-T_r) \\
&= \frac{U_0}{T_r}\left[T_r+\frac{1}{\beta}{\rm e}^{-\alpha t}\sin\ (\beta t+\theta)\right. \\
&\quad \left.-\frac{1}{\beta}{\rm e}^{-\alpha(t-T_r)}\sin\ \left(\beta(t-T_r)+\theta\right)\right]u(t)
\end{aligned}
\tag{14}
$$

Assume that the delay corresponds to τ when the output reaches $u_{\rm th}$, that is

$$
\begin{aligned}
u_{\rm f\_out}(\tau) = u_{\rm th} &= \frac{1}{T_r}\left[T_r+\frac{1}{\beta}{\rm e}^{-\alpha\tau}\sin\ (\beta\tau+\theta)\right. \\
&\quad \left.-\frac{1}{\beta}{\rm e}^{-\alpha(\tau-T_r)}\sin\ \left(\beta(\tau-T_r)+\theta\right)\right]u(t),
\end{aligned}
$$

($U_0 = 1$). It is a transcendental equation that can be computed by recursively solving for time in the equation given immediately above. One way to solve the recursion is to approximate the time variable in the sine term by equivalent Elmore delay under ramp input, that is, substitute $T'_{\rm AD} = u_{\rm th}T_r$ for time t. Therefore

$$
\tau \approx -\frac{1}{\alpha}\ln\frac{(1-u_{\rm th})T_r}{m_2-m_1} \tag{15}
$$

where $m_1 = \frac{1}{\beta}\sin\ (\beta T'_{\rm AD}+\theta)$, $m_2 = \frac{1}{\beta}{\rm e}^{\alpha T_r}\sin[\beta(T'_{\rm AD}-T_r)+\theta]$

If $m_1 > m_2$, then substitute $T'_{\rm AD} = u_{\rm th}T_r$ for the time variable in the exponential term and expand sine as a Taylor series and only consider the first term. Thus, we can get

$$
\tau \approx \frac{\left((1-u_{\rm th})+{\rm e}^{-\alpha(T'_{\rm AD}-T_r)}\right)T_r}{{\rm e}^{-\alpha T'_{\rm AD}}\left({\rm e}^{\alpha T_r}-1\right)}-\frac{\theta}{\beta} \tag{16}
$$

3.3 Double poles

$$U_{\text{if\_out}}(s)=\frac{U_0}{T_r}\left[\frac{2}{s_1}\frac{1}{s}+\frac{1}{s^2}-\frac{2}{s_1}\frac{1}{s-s_1}+\frac{1}{\left(s-s_1\right)^2}\right] \tag{17}$$

where $s_1=-b_1/2b_2$. The response of finite ramp in transform domain is

$$\begin{aligned}u_{\text{f\_out}}(t)&=u_{\text{if\_out}}(t)-u_{\text{if\_out}}(t-T_r)\\&=\frac{U_0}{T_r}\left[T_r+\left(\frac{2}{s_1}\mathrm{e}^{s_1t}+t\mathrm{e}^{s_1t}\right)\left(\mathrm{e}^{-s_1T_r}-1\right)\right.\\&\left.\quad-T_r\mathrm{e}^{s_1(t-T_r)}\right]u(t)\end{aligned} \tag{18}$$

Assume that the delay corresponds to τ when the output reaches uth, that is

$$\begin{aligned}u_{\text{f\_out}}(\tau)=u_{\text{th}}&=\frac{1}{T_r}\left[T_r+\left(\frac{2}{s_1}\mathrm{e}^{-s_1T_r}-T_r\,\mathrm{e}^{-s_1T_r}-\frac{2}{s_1}\right)\mathrm{e}^{s_1\tau}\right.\\&\left.\quad+\left(\mathrm{e}^{-s_1T_r}-1\right)\tau\mathrm{e}^{s_1\tau}\right]u(t),\,(U_0=1).\end{aligned}$$

It is a transcendental equation. For simplification, we substitute $T'_{\text{AD}}\mathrm{e}^{s_1\tau}$ for $\tau\mathrm{e}^{s_1\tau}$ and get the result

$$\tau\approx\frac{1}{s_1}\ln\left[\frac{0.1T_r}{\left(T_r-T'_{\text{AD}}\right)\mathrm{e}^{-s_1T_r}+\dfrac{2}{s_1}\left(1-\mathrm{e}^{-s_1T_r}\right)+T'_{\text{AD}}}\right] \tag{19}$$

4 Experimental results

Case 1 Real poles. Assume that the length of the interconnect is $h=2\,000$ μm; rising time of the input signal is $T_r=100$ ps; interconnect parameters are $r=0.015$ Ω/μm; $l=0.246$ pH/μm; and $c=0.176$ pF/μm. For 90% delay $T_{0.9}$, the delay estimations using analytical model Eq. (11) is within 2% error of SPICE-computed delay (please refer to Table 1, where R_s and L_s are source resistance and source inductance, respectively).

Table 1 Real poles

Source		Load	SPICE/ps	Delay model	
R_s / Ω	L_s / pH	C_T /pF		Equivalent Elmore model /ps	Model of this paper /ps
50	2.46	0.176	139.00	135.01(2.87%)	138.60(0.29%)
100	2.46	0.176	197.10	195.73(0.70%)	195.52(0.80%)
1 000	2.46	0.176	1 273.10	1 288.70(1.23%)	1 286.60(1.05%)
25	2.46	1.760	293.20	305.02(4.03%)	293.15(0.02%)
100	2.46	1.760	661.60	619.34(6.39%)	661.56(0.01%)
1 000	2.46	1.760	4 970.00	4 991.20(0.43%)	5 040.40(1.42%)

Case 2 Complex poles. Assume that the length of the interconnect is $h=2\,000$ μm; rising time of the input signal is $T_r=500$ ps; interconnect parameters are $r=0.015$ Ω/μm; $l=0.246$ pH/μm; and $c=0.176$ pF/μm. For 90% delay $T_{0.9}$, the delay estimations using analytical models Eqs. (15) and (16) are within 3% error of SPICE-computed delay (please refer to Table 2).

Table 2 Complex poles

Source		Load	SPICE/ps	Delay model	
R_s / Ω	L_s / pH	C_T /pF		Equivalent Elmore model /ps	Model of this paper /ps
10	0.024 6	0.017 6	460.2	271.9(40.93%)	463.8(0.78%)
10	0.024 6	0.176	466.0	286.4(38.53%)	467.4(0.31%)
20	0.024 6	0.176	471.1	271.1(42.45%)	471.7(0.13%)
10	2.46	0.017 6	460.4	271.9(40.84%)	463.8(0.73%)
20	2.46	0.017 6	463.2	280.4(39.47%)	476.5(2.87%)
10	2.46	0.176	466.2	286.4(38.56%)	467.1(0.20%)
20	2.46	0.176	473.0	271.1(42.68%)	471.7(0.27%)
10	24.6	0.017 6	460.1	271.9(40.91%)	463.4(0.72%)
20	24.6	0.017 6	463.1	280.4(39.46%)	475.5(2.68%)
10	24.6	0.176	466.2	286.4(38.51%)	463.9(0.47%)
20	24.6	0.176	472.0	271.1(42.56%)	473.0(0.21%)

5 Conclusions

In this paper analytical delay models for RLC interconnects under ramp input are presented. Under different situations, that is, overdamped, underdamped and critical response cases, the delay estimations using the proposed analytical models are within 3% error of SPICE-computed delay.

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