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RESEARCH ARTICLE

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A topology control algorithm for preserving minimum-energy paths in wireless ad hoc networks

Abstract In this paper, a distributed topology control algorithm is proposed. By adjusting the transmission power of each node, this algorithm constructs a wireless network topology with minimum-energy property, i.e., it preserves a minimum-energy path between every pair of nodes. Moreover, the proposed algorithm can be used in both homogenous and heterogeneous wireless networks, and it can also work without an explicit propagation channel model or the position information of nodes. Simulation results show that the proposed algorithm has advantages over the topology control algorithm based on direct-transmission region in terms of average node degree and power efficiency.

Keywords wireless ad hoc networks, topology control, minimum-energy path, minimum-energy property

1 Introduction

A wireless ad hoc network is a collection of wireless nodes that are self-configured to form a network without the aid of any established infrastructure. Wireless ad hoc networks are used in situations where temporary network connectivity is needed, such as in disaster relief or battlefield [1].

The topology of a wireless ad hoc network has a significant impact on the network performance. Since the basic components of wireless ad hoc networks are mostly battery-operated devices, power conservation is one of the key issues of such networks. It is not energy efficient to use the communication network where each node transmits with its maximum power. The topology of this network is referred to as the maximum-power topology, denoted by a graph $G_{\max} = (V(G_{\max}), E(G_{\max}))$, where $V(G_{\max})$ is the set of nodes in the network, $(u, v) \in E(G_{\max})$ if u and v can communicate with each other using their maximum transmission power. Topology control deals with assigning the transmission power of each node, so that the generated topology satisfies some specified properties. The importance of topology control lies in the fact that it can conserve battery energy, reduce radio interference, and increase spatial reuse of wireless bandwidth. In this paper, we propose a distributed topology control algorithm, called minimum-energy property spanning subgraph (MPSS), for constructing a topology that preserves both connectivity and minimum-energy property [2]. A topology is said to have the minimum-energy property if it maintains a minimum-energy path between every pair (u, v) of nodes, one that allows messages to be transmitted with a minimum use of energy among all the paths between u and v in $G_{\max}$.

Several topology-control algorithms [2–4] have been proposed to construct a subgraph that has minimum-energy property. Rodoplu et al. [3] introduced the notion of relay region for the purpose of power control. The relay region of the transmit—relay node pair (u, v) is the physical region $R_{u\to v}$, where the process of relaying from v to any point in $R_{u\to v}$ takes less power than that of direct transmission. On the basis of the relay region, they proposed a distributed algorithm for searching the neighbors of each node for power-efficient transmission. Subsequently, improved algorithms have been proposed in Refs. [2,4]. The work most related to our work is that of Li et al. [2]. They proposed a distributed algorithm called a Small Minimum-Energy Communication Network (SMECN), which outperforms the algorithm proposed by Rodoplu et al. in power efficiency and time efficiency. The main idea of SMECN is to search a direct-transmission region $R_F(u)$ for each node u such that it transmits directly to any point in the region taking less power than relaying through any node within its maximum transmission range. Formally, $R_F(u) = \bigcap_{w\in N_F} (F(u, p_{\max}) - R_{u\to w})$, where $p_{\max}$ is the maximum transmission power, $F(u, p)$ is the region covered by u with power p, N_F denotes all the nodes in region F and $R_{u\to w}$ is the relay region of the node pair (u, w). Then, the final transmission power $p(u)$ of node u should cover $R_F(u)$, i.e., $p(u) = \min\{p : F(u, p) \supseteq R_F(u)\}$. However, the power $p(u)$ determined by SMECN may not be efficient. In SMECN,

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nodes can be grouped into two categories: full-enclosed and half-enclosed. Node u is said to be full-enclosed if there exists a power p, $p_{\max} > p > 0$, with the result $F(u, p) \supseteq R_F(u)$, otherwise node u is half-enclosed. Figure 1 illustrates the two types of nodes in SMECN, where the region $R_F(u)$ is represented by the grey area. As is shown in Fig. 1(b), if a node is half-enclosed, it has to use its maximum transmission-power, which will soon drain out its battery power. When compared with SMECN, MPSS can achieve significantly better performance in terms of average node degree and power efficiency.

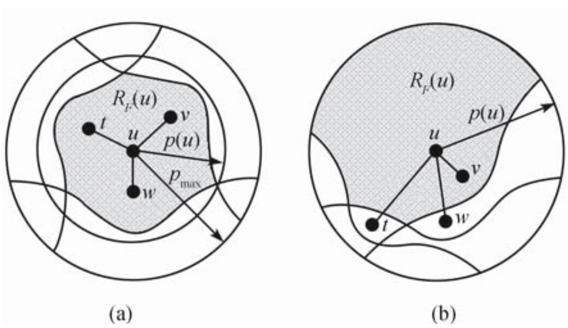


Fig. 1 Two types of nodes in SMECN
(a) Node u is full-enclosed; (b) Node u is half-enclosed

2 Network model

It is assumed that all the nodes are distributed in a two-dimensional plane. Each node has a unique ID and an omni-directional antenna. To facilitate description of the proposed algorithm, we also assume that each node has a low-power Global Position System (GPS) receiver for acquiring its own location information. Furthermore, each node u is able to adjust its transmission power $p(u)$ within the range $0 \leqslant p(u) \leqslant p_u^{\max}$, where $p_u^{\max}$ is the maximum transmission power of node u. For homogenous wireless networks, the maximum transmission powers of all nodes are the same. A transmission between node u and v takes power $p(u,v) = td(u,v)^{\beta}$ [5] for some appropriate constant t, where $\beta(\beta \geqslant 2)$ is the exponent in the path loss model, and $d(u,v)$ is the distance between u and v. In addition, a reception at the receiver takes power c, which is referred to as the receiver power [3]. Therefore, the total energy used in transmission along path $\text{PATH}(u,v) = (u = u_0, u_1, \ldots, u_{n-1}, u_n = v)$ is $C(\text{PATH}(u,v)) = \sum_{i=0}^{n-1} p(u_i, u_{i+1}) + nc$. The minimum-energy path between u and v, denoted by $\text{LP}(u,v)$, is the path along which the total energy used is the minimum among all the possible paths between u and v, i.e., $C(\text{LP}(u,v)) \leqslant C(\text{PATH}(u,v))$. Since a connected network constructed by bidirectional links is desirable not only for the acknowledgments at the link layer but also for the transmission and retransmission of packets, so unidirectional links will not be considered in our algorithm and the maximum-power topology formed by bidirectional links is assumed to be connected.

3 Minimum-energy property spanning subgraph (MPSS) topology control algorithm

The proposed algorithm consists of the following two phases: information collection and topology control. To facilitate the discussion of the proposed algorithm, we first define a few terms.

Definition 1 (reachable neighborhood): The reachable neighborhood of node u (denoted as $\text{NBR}(u)$) is the set of nodes that are adjacent to node u in $G_{\max}$, i.e., $\text{NBR}(u) = \{v \mid p_u^{\max} \geqslant p(u,v) \text{ and } p_v^{\max} \geqslant p(u,v)\}$. The weight of node v in $\text{NBR}(u)$ is a pair $w_u(v) = \langle p(u,v), \text{ID}(v)\rangle$. Further, $\forall v_1, v_2 \in \text{NBR}(u)$, $w_u(v_1) < w_u(v_2) \Leftrightarrow p(u,v_1) < p(u,v_2)$, or $(p(u,v_1) = p(u,v_2)$ and $\text{ID}(v_1) < \text{ID}(v_2))$.

Definition 2 (precede): Given $m, n \in \text{NBR}(u)$, we say node m precedes node n (denoted as $m \xrightarrow[u]{\text{precede}} n$) if $p_m^{\max} \geqslant p(m,n)$, $p_n^{\max} \geqslant p(m,n)$, and $p(u,m) + p(m,n) + c < p(u,n)$.

$m \xrightarrow[u]{\text{precede}} n$ means that for node u, relaying through m to n takes less power than that of direct transmission to n.

3.1 Information collection

Initially, each node u broadcasts a HELLO message using $p_u^{\max}$ and collects all ACK messages from $\text{NBR}(u)$. The information contained in a HELLO or ACK message includes the ID, the position, and the maximum power of the sending node. Once node u receives the ACK message from node v, it adds node v into its reachable neighborhood $\text{NBR}(u)$.

3.2 Topology control

After obtaining the information of $\text{NBR}(u)$, node u sorts all the nodes in $\text{NBR}(u)$ in the increasing order by weight. Then, node u determines its set of neighbors $\text{Nbrs}(u)$ to make sure that first, for any node n that is not in $\text{Nbrs}(u)$, there exists a node m in $\text{Nbrs}(u)$ that precedes n; and second, on the basis of the above-mentioned condition, maximum weight in $\text{Nbrs}(u)$ is minimized. Note that the neighbor relation may not be symmetric. For instance, $v \in \text{Nbrs}(u)$ but $u \notin \text{Nbrs}(v)$, which may result in a unidirectional link from u to v. To make the link bidirectional, node u sends a symmetry request (SR) message that contains its ID and position to each node v in $\text{Nbrs}(u)$. Upon receiving the SR from u, v will guarantee that its final power $p(v)$ is no less than $p(u,v)$. Finally, node u sets its transmission power $p(u)$ required to reach its neighbor of maximum weight. The detailed implementation of MPSS is shown in Table 1.

3.2 Correctness verification

Let graph $G_{\max}$ denote the maximum-power topology, and graph G the topology derived by MPSS. The following theorem holds.

Theorem 1 Given $G_{\max}$ is connected; and G is connected and has the minimum-energy property.

Table 1 MPSS topology control algorithm

Algorithm MPSS
For each node u:
$\quad$ $\text{Nbrs}(u) = \text{NonNbrs}(u) = \emptyset$;
$\quad$ Find the bidirectional reachable neighborhood $\text{NBR}(u)$;
$\quad$ Sort all nodes in $\text{NBR}(u)$ in increasing order by weight;
$\quad$ for each v, $v \in \text{NBR}(u)$, in sorted order do
$\quad\quad$ if $\exists a \in \text{Nbrs}(u)$, $a \xrightarrow[u]{\text{precede}} v$ then
$\quad\quad\quad$ $\text{NonNbrs}(u) = \text{NonNbrs}(u) \cup \{v\}$;
$\quad\quad\quad$ else if $\text{NP}(v) \neq \emptyset$, $\text{NP}(v) = \{w \mid w \in \text{NonNbrs}(u)$ and $w \xrightarrow[u]{\text{precede}} v\}$ then
$\quad\quad\quad$ Find the node d that has the minimum weight in $\text{NP}(v)$;
$\quad\quad\quad$ $\text{NonNbrs}(u) = \text{NonNbrs}(u) - \{d\}$;
$\quad\quad\quad$ $\text{Nbrs}(u) = \text{Nbrs}(u) \cup \{d\}$;
$\quad\quad\quad$ $\text{NonNbrs}(u) = \text{NonNbrs}(u) \cup \{v\}$;
$\quad\quad$ else
$\quad\quad\quad$ $\text{Nbrs}(u) \cup \{v\}$;
$\quad$ for each w in $\text{Nbrs}(u)$, send a symmetry request message;
$\quad$ $p(u) = \max_{a \in \text{Nbrs}(u)} p(u,a)$;

Proof By the above definition, the topology that has the minimum-energy property must be connected. Thus, all we need to do is prove that G has the minimum-energy property. That is, $\forall u, v \in V(G_{\max})$, let path $\text{LP}(u, v) = (u{=}u_0, u_1, \ldots, u_{n-1}, u_n = v)$ be the minimum-energy path from u to v in $G_{\max}$, and we need to prove that $(u_i,u_{i+1}) \in E(G)$, $i = 0, 1, \ldots, n-1$. Consider any node u_i on $\text{LP}(u, v)$, since $(u_i,u_{i+1}) \in E(G_{\max})$, so it follows that $u_{i+1} \in \text{NBR}(u_i)$. If $u_{i+1} \in \text{Nbrs}(u_i)$, we have $(u_i, u_{i+1}) \in E(G)$;otherwise,$u_{i+1} \in \text{NonNbrs}(u_i)$,i.e.,$\exists\, a \in \text{Nbrs}(u_i)$, $a \xrightarrow[u_i]{\text{Precede}} u_{i+1}$. By Definition 2, it follows (u_i,a), $(a,u_{i+1}) \in E(G_{\max})$ and $C(u_i,a,u_{i+1}) < C(u_i,u_{i+1})$. Therefore, replacing edge (u_i,u_{i+1}) with edge (u_i,a) and edge (a,u_{i+1}) and keeping all other edges unchanged on $\text{LP}(u,v)$ results in another path, along which the total energy used is less than that of $\text{LP}(u,v)$.This contradicts with the fact that $\text{LP}(u,v)$ is the minimum-energy path from u to v in $G_{\max}$.

3.3 Discussions

(1) For each node u running MPSS, let $V = |\text{NBR}(u)|$, it takes u the time $O(V\log V)$ to sort and with most of the time $O(V^2)$ to determine its neighbors. Therefore, the time complexity of MPSS is $O(V^2)$, which is the same as that of SMECN; whereas the time complexity of the algorithm proposed in Ref. [3], called Minimum-Energy Communication Network (MECN), is $O(V^3)$.

(2) Using Definition 2, the neighbor set of u identified by MECN satisfies the following conditions: (a) $\forall w \in \text{NonNbrs}(u)$, there exists $v \in \text{Nbrs}(u)$, $v \xrightarrow[u]{\text{Precede}} w$; (b) $\forall m$, $n \in \text{Nbrs}(u)$, there does not exist $m \xrightarrow[u]{\text{Precede}} n$ or $n \xrightarrow[u]{\text{Precede}} m$.

Since MPSS does not have the latter constraint, it is easy to show that the transmission power computed by MPSS does not exceed that of MECN. Although the neighbor set identified by SMECN is a subset of the neighbor set of MECN, SMECN has the problem of half-enclosed nodes. We give a simple example to show the results of these three algorithms. In Fig. 1(b), we have $v \xrightarrow[u]{\text{Precede}} w$ and $w \xrightarrow[u]{\text{Precede}} t$, thus for MECN, $\text{Nbrs}(u) = \{v,\ t\}$ and $p(u) = p(u,\ t)$; for SMECN, $\text{Nbrs}(u) = \{v\}$ and $p(u) = p_u^{\max}$; for MPSS, $\text{Nbrs}(u) = \{v,w\}$ and $p(u) = p(u,w)$. It is obvious that in this case, the power determined by MPSS outperforms the powers by the other two algorithms.

(3) As both MECN and SMECN are based on the computation of the relay region, they require an explicit propagation channel model and the position information of nodes. However, these requirements do not always hold in practice. Conversely, MPSS can relax these restrictions. For instance, for any neighbor v, node u can estimate the $p(u,v)$ based on the attenuation incurred in the transmission of the HELLO (or ACK) message from node v. From the above description of MPSS, it can be seen that MPSS can also work in heterogeneous wireless networks where nodes have different maximum power. As long as the maximum-power topology is connected, the topology derived by MPSS is connected.

4 Simulation results

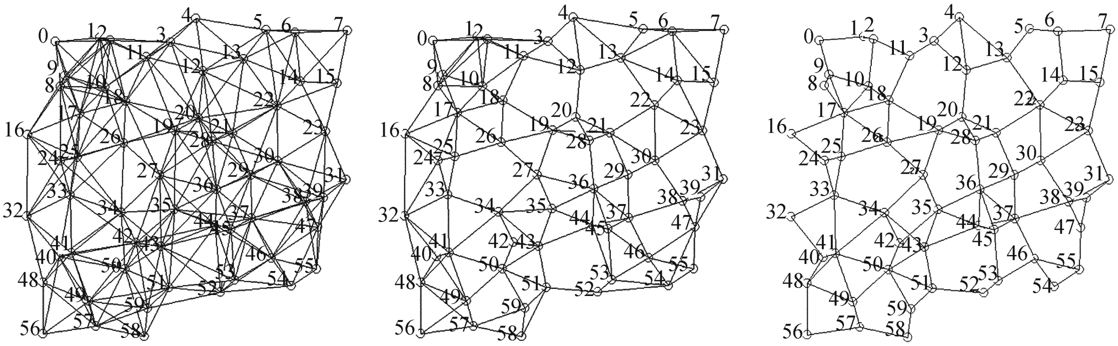
The performance of MPSS has been evaluated through simulations. In the simulations, n nodes are uniformly distributed in a 2,000 m × 2,000 m region. We vary the number of nodes n from 30 to 210 in increments of 30. The maximum transmission range of a node is 500 m and the power attenuation constant β is set at 4. Each node has an antenna with 0 dB gain and 1.5 m height. The receiving threshold is −94 dBW, and the receiver power c is set at 0 and 20 mW in each simulation, respectively. These parameters are the same with those used in Ref. [2]. For a given number of nodes (e.g., 30), we run the simulation for 20 scenarios.

To verify that MPSS and SMECN can maintain the minimum-energy property, we compute the power stretch factor [6] of MPSS and SMECN using Floyd algorithm [7] after completion of each simulation. The power stretch factor of graph G with respect to $G_{\max}$ is defined as

$$\rho_G = \max_{u,\, v \in V(G)} \left(\frac{C(\text{LP}_G(u,v))}{C(\text{LP}_{G_{\max}}(u,v))} \right) \tag{1}$$

where G is the topology derived by MPSS or SMECN, $G_{\max}$ the maximum-power topology, and $\text{LP}_G(u, v)$ (or $\text{LP}_{G_{\max}}(u, v)$) the minimum-energy path between u and v in G (or $G_{\max}$). Since $G \subseteq G_{\max}$, it follows that G has minimum-energy property if $\rho_G = 1$. Simulation results show that both SMECN and MPSS can preserve the minimum-energy property.

Figure 2 illustrates the topologies derived using the maximum transmission power, SMECN, and MPSS. For clarity, the unidirectional links are not shown here. As is shown in



(a) (b) (c)

Fig. 2 Network topologies derived under different algorithms with unidirectional links removed
(a) The maximum power topology; (b) The topology derived under SMECN; (c) The topology derived under MPSS

Fig. 2(b), the nodes located at the edge of the network are likely to be half-enclosed nodes. As a result, no power optimization has been done for those nodes.

Table 2 gives the average degrees in the derived topologies using the maximum power, SMECN, and MPSS, and it also provides the average number of neighbors determined by SMECN and MPSS. A smaller average node degree usually implies less contention and better spatial reuse. When there is no topology control, the average node degree increases almost linearly with the number of nodes. Comparatively, the average node degrees in the topologies under SMECN and MPSS are much lower especially when the network density is high. Another inference from our study is that the average number of neighbors determined by SMECN is not consistent with that derived by the assigned power. Specifically, the difference between the average node degree of the topology and the average number of neighbors derived by SMECN is at least 2 when the number of nodes exceeds 90. This is mainly due to the fact that some of the nodes in the network are half-enclosed nodes. Compared with SMECN, the average node degree of the topology constructed by MPSS is only 70%–75% of SMECN.

Table 2 Average degrees of the maximum power topology (G_{max}), SMECN algorithm, the topology derived by SMECN (G_{SMECN}), MPSS algorithm, and the topology derived by MPSS (G_{MPSS})

Nodes	G_{max}	$c=0$				$c=20$ mW			
		SMECN	G_{SMECN}	MPSS	G_{MPSS}	SMECN	G_{SMECN}	MPSS	G_{MPSS}
30	4.56	2.57	4.04	2.67	2.95	2.59	4.05	2.68	2.96
60	8.91	2.93	5.22	3.21	3.85	3.00	5.29	3.26	3.87
90	14.39	3.00	5.62	3.33	4.06	3.20	5.75	3.46	4.11
120	17.98	3.05	5.67	3.40	4.20	3.36	5.88	3.60	4.29
150	25.09	3.07	5.79	3.45	4.24	3.63	6.19	3.83	4.41
180	29.45	3.11	5.80	3.49	4.30	3.86	6.31	4.02	4.54
210	33.78	3.13	5.90	3.48	4.36	4.06	6.52	4.20	4.69

Figure 3 shows the percentage of half-enclosed nodes in SMECN. The results are almost the same, with or without considering the receiver power, which imply that the receiver power has a slight effect on the number of half-enclosed nodes. For SMECN, whether a node is half-enclosed or not is mainly dependent on its location and the distribution of its reachable neighborhood.

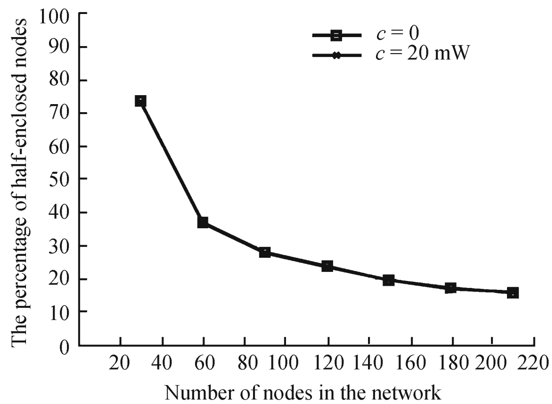


Fig. 3 The percentage of half-enclosed nodes

Figure 5 shows the average expended-energy ratio (EER) of SMECN and MPSS, where EER is defined in Ref. [8] as

$$\mathrm{EER} = \frac{\text{Average transmission power}}{\text{Maximum transmission power}} \times 100 \qquad (2)$$

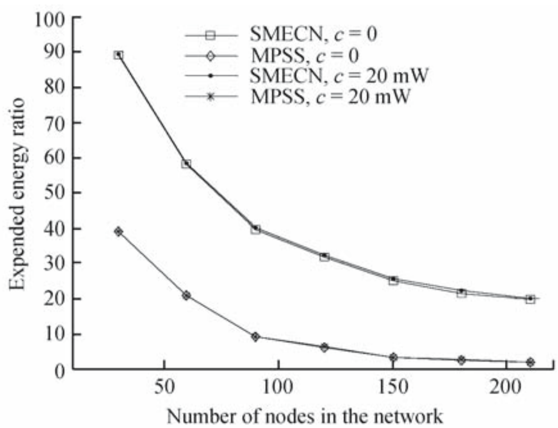


Fig. 4 Comparison of MPSS and SMECN with respect to the expended energy ratio

The average transmission power determined by MPSS is much less than that determined by SMECN. The EER of MPSS is from 2 to 40, whereas the EER of SMECN is from 20 to 90 with $c = 20$ mW. When the receiver power is not taken into consideration, there is a slight decrease in EER of both MPSS and SMECN. During the collection of necessary topology information, although SMECN intends to save the node energy by increasing the searching power gradually, we believe that the final transmission power is more important in saving the node energy.

5 Conclusions

Power conservation is one of the design challenges for wireless Ad hoc networks. Topology control deals with assigning the transmission power of each node, so that the resulted topology satisfies some specified properties. By adjusting transmission powers of the nodes, topology control is capable of optimizing network performance, reducing energy consumption, and increasing network lifetime. In this paper, we present a distributed topology control algorithm for constructing a topology with minimum-energy property. Simulation results show that the proposed algorithm outperforms the topology control algorithm based on direct-transmission region in terms of average node degree and power efficiency.

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