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RESEARCH ARTICLE

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Sensitivity analysis of the channel estimation deviation to the MAP decoding algorithm

Abstract As a necessary input parameter for maximum a-posteriori(MAP) decoding algorithm, SNR is normally obtained from the channel estimation unit. Corresponding research indicated that SNR estimation deviation degraded the performance of Turbo decoding significantly. In this paper, MAP decoding algorithm with SNR estimation deviation was investigated in detail, and the degradation mechanism of Turbo decoding was explained analytically. The theoretical analysis and computer simulation disclosed the specific reasons for the performance degradation when SNR estimation was less than the actual value, and for the higher sensitivity of SNR estimation to long-frame Turbo codes.

Keywords Turbo codes, MAP decoding, SNR estimation, wireless communications

1 Introduction

Turbo codes were first proposed by C. Berrou etc. in 1993 [1], which offered near Shannon limits error correction protection for digital transmission. Because of the out performance, Turbo codes are suitable for wireless transmission and other low signal-to-noise scenarios. MAP algorithm and soft output viterbi algorithm (SOVA) are the effective and common algorithms for Turbo decoding, and MAP is the optimal a-posteriori probabilities algorithms. Some MAP-like algorithm including Log-MAP and Max-Log-MAP sub-optimal algorithms are calculated in log domain

Different from SOVA algorithms, MAP algorithm takes E_b/N_0 as the decoding parameter, which offered by channel estimation unit. However, because of the imperfect channel estimation, the Turbo decoding performance decreased with estimation deviation [2–4]. It has been disclosed that with high signal-to-noise ratio under estimation deviation and longer the interleaver depth was, performance decreased more noticeably. Recently, for Max-Log-MAP decoding algorithm, it has been proved that in practice the estimation of E_b/N_0 can be ignored, but for MAP and Log-MAP algorithm, the correct estimation of E_b/N_0 is needed [5].

In this paper, sensitivity of channel estimation deviation to Turbo decoding has been investigated, and the principlion, decoding performance with higher estimation value is better than the lessr ones, is recommended for practical systems. The Turbo encoder here means the conventional parallel concatenated convolutional (PCCC) structure, which consisted of two parallel recursive systematic convolutional (RSC) encoders. The information bits and its interleaved version are sent into two RSC encodes, after puncture unit; Turbo codes with 1/2 code rate can be achieved. MAP algorithm with parallel iterative decoding structure was used to decode Turbo codes, and corresponding interleaver and deinterleaver were used. The two component decoders were used to process the component code of the Turbo codes, and in each component decoder, the log likelihood ratio (LLR) of component code was calculated, and it can be called external information.

2 Analysis of Log-MAP decoding algorithm

In MAP decoding algorithm, the LLR of the received information d_k can be obtained in each SISO decoder. Based on the symbol system of [6], the LLR can be written as:

$$\Lambda(d_k) = \ln \frac{\sum_{S_k}\sum_{S_{k-1}} \gamma_1(y_k, S_{k-1}, S_k)\alpha_{k-1}(S_{k-1})\beta_k(S_k)}{\sum_{S_k}\sum_{S_{k-1}} \gamma_0(y_k, S_{k-1}, S_k)\alpha_{k-1}(S_{k-1})\beta_k(S_k)} \quad (1)$$

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In the practice system, calculation in log domain can reduce computation complexity, which can convert multiplication into addition. Log state transfer probability can be calculated based on SNR, received information, and check bits.

$$\ln \gamma_i((y_k^s, y_k^p), S_{k-1}, S_k) = \frac{2E_s}{N_0} y_k^s x_k^s(i) + \frac{2E_s}{N_0} y_k^p x_k^p(i, S_k, S_{k-1}) + \ln \Pr\{S_k \mid S_{k-1}\} + C, \quad i = 1, 0 \tag{2}$$

where, y_k^s is the information bits, y_k^p is the check bits, and $\Pr\{S_k|S_{k-1}\}$ is the external information. The channel SNR, E_s/N_0, has linear relationship with E_b/N_0 based on modulation mode. C is constant, which can be removed in the calculation. $\ln \alpha$ and $\ln \beta$ can be calculated by

$$\ln \alpha_k(S_k) = \ln \Big(\sum_{S_{k-1}} \sum_{i=0}^{1} \mathrm{e}^{\ln \gamma_i((y_k^s, y_k^p), S_{k-1}, S_k) + \ln \alpha_{k-1}(S_{k-1})}\Big) - \ln \Big(\sum_{S_k} \sum_{S_{k-1}} \sum_{i=0}^{1} \mathrm{e}^{\ln \hat{\gamma}_i((y_k^s, y_k^p), S_{k-1}, S_k) + \ln \alpha_{k-1}(S_{k-1})}\Big) \tag{3}$$

$$\ln \beta_k(S_k) = \ln \Big(\sum_{S_{k+1}} \sum_{i=0}^{1} \mathrm{e}^{\ln \gamma_i((y_{k+1}^s, y_{k+1}^p), S_k, S_{k+1}) + \ln \beta_{k+1}(S_{k+1})}\Big) - \ln \Big(\sum_{S_k} \sum_{S_{k+1}} \sum_{i=0}^{1} \mathrm{e}^{\ln \hat{\gamma}_i((y_{k+1}^s, y_{k+1}^p), S_k, S_{k+1}) + \ln \hat{\alpha}_{k+1}(S_{k+1})}\Big) \tag{4}$$

Substituting $\ln \alpha$ and $\ln \beta$ for Eq. (1), LLR can be obtained.

In iterative decoding structure, external information transfer between every component decoder can be written as:

$$\Lambda(d_k) = \ln \Big(\sum_{S_k} \sum_{S_{k-1}} (\ln \gamma_1(y_k^p, S_{k-1}, S_k) + \ln \alpha_{k-1}(S_{k-1}) + \ln \beta_k(S_k)) - \sum_{S_k} \sum_{S_{k-1}} \ln \gamma_0(y_k^p, S_{k-1}, S_k) + \ln \alpha_{k-1}(S_{k-1}) + \ln \beta_k(S_k)\Big) + \frac{4E_s y_k^s}{N_0} + L(d_k) \tag{5}$$

The former two items of Eq. (5) can be taken as the external information for the next iterative calculation.

$$L(d_{k+1}) = \Lambda(d_k) - \frac{4E_s y_k^s}{N_0} - L(d_k) \tag{6}$$

3 Theory analysis of estimation deviation affection

In Eq. (2), E_s/N_0 is used as trusts weight of received bits, therefore calculation in Eqs. (3) and (4) are also affected by E_s/N_0. Although E_s/N_0 is involved in Eq. (5), in Eq. (6) the effect of $4E_s y_k^s/N_0$ to external information can be ignored.

Assuming that the actual channel SNR E_s/N_0 is E_n, estimated value is $\hat{E}_n$, and after computation the estimated α, β, γ can be written as $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$, Eq.(2) can be written as:

$$\begin{aligned}\ln \hat{\gamma}_i((y_k^s, y_k^p), S_{k-1}, S_k) &= 2\hat{E}_n y_k^s x_k^s(i) + 2\hat{E}_n y_k^p x_k^p(i, S_k, S_{k-1}) + \ln \Pr\{S_k \mid S_{k-1}\} \\ &= 2E_n y_k^s x_k^s(i) + 2E_n y_k^p x_k^p(i, S_k, S_{k-1}) + \ln \Pr\{S_k \mid S_{k-1}\} + 2(\hat{E}_n - E_n)(y_k^s x_k^s(i) + y_k^p x_k^p(i, S_k, S_{k-1})) \\ &= \ln \gamma_i((y_k^s, y_k^p), S_{k-1}, S_k) + (\hat{E}_n - E_n) \cdot (2y_k^s x_k^s(i) + 2y_k^p x_k^p(i, S_k, S_{k-1})) \\ &= \ln \gamma_i((y_k^s, y_k^p), S_{k-1}, S_k) + \delta \varepsilon_k\end{aligned} \tag{7}$$

Here $\delta = \hat{E}_n - E_n$, $\varepsilon_k = 2y_k^s x_k^s(i) + 2y_k^p x_k^p(i, S_k, \cdots, S_{k-1})$. To simplify analysis, the approximation formula can be written as:

$$\ln \Big(\sum_i \mathrm{e}^{x_i}\Big) \approx \max_i (\ln \mathrm{e}^{x_i}) = \max_i (x_i) \tag{8}$$

$$\ln \Big(\sum_{S_k} \sum_{S_{k-1}} \sum_{i=0}^{1} \mathrm{e}^{\ln \gamma_i((y_k^s, y_k^p), S_{k-1}, S_k) + \delta\varepsilon_k + \ln \alpha_{k-1}(S_{k-1})}\Big) \approx \max \Big(\ln \sum_{S_{k-1}} \sum_{i=0}^{1} \mathrm{e}^{\ln \gamma_i((y_k^s, y_k^p), S_{k-1}, S_k) + \delta\varepsilon_k + \ln \alpha_{k-1}(S_{k-1})}\Big) \tag{9}$$

Substituting Eqs. (8) and (9) for Eq. (2)

$$\begin{aligned}\ln \hat{\alpha}_k(S_k) &= \ln \Big(\sum_{S_{k-1}} \sum_{i=0}^{1} \mathrm{e}^{\ln \hat{\gamma}_i((y_k^s, y_k^p), S_{k-1}, S_k) + \ln \hat{\alpha}_{k-1}(S_{k-1})}\Big) - \ln \Big(\sum_{S_k} \sum_{S_{k-1}} \sum_{i=0}^{1} \mathrm{e}^{\ln \hat{\gamma}_i((y_k^s, y_k^p), S_{k-1}, S_k) + \ln \hat{\alpha}_{k-1}(S_{k-1})}\Big) \\ &\approx \ln \Big(\sum_{S_{k-1}} \sum_{i=0}^{1} \mathrm{e}^{\ln \gamma_i((y_k^s, y_k^p), S_{k-1}, S_k) + \delta\varepsilon_k + \ln \hat{\alpha}_{k-1}(S_{k-1})}\Big) - \max \Big(\ln \sum_{S_{k-1}} \sum_{i=0}^{1} \mathrm{e}^{\ln \gamma_i((y_k^s, y_k^p), S_{k-1}, S_k) + \delta\varepsilon_k + \ln \alpha_{k-1}(S_{k-1})}\Big)\end{aligned} \tag{10}$$

Here max represents calculating the maximum value of $\gamma_i(\cdot) + \delta\varepsilon + \alpha_{k-1}(\cdot)$ for all available states of convoluational encoder of the kth bit

Similarly,

$$\ln \hat{\beta}_k(S_k) \approx \ln \Big(\sum_{S_{k+1}} \sum_{i=0}^{1} \mathrm{e}^{\ln \gamma_i((y_{k+1}^s, y_{k+1}^p), S_k, S_{k+1}) + \delta\varepsilon_k + \ln \hat{\beta}_{k+1}(S_{k+1})}\Big) - \max \Big(\ln \sum_{S_{k-1}} \sum_{i=0}^{1} \mathrm{e}^{\ln \gamma_i((y_k^s, y_k^p), S_{k-1}, S_k) + \delta\varepsilon_k + \ln \hat{\alpha}_{k+1}(S_{k+1})}\Big) \tag{11}$$

α and β are effected by the estimation deviation thought to be δ and ε_k factors, and by increasing δ, more reduction of decoding performance will be seen. Because of the iterative calculation of α and β, the long-frame Turbo codes have more sensitivity to estimation deviation. From Eqs. (10) and (11), the following conclusions can be made:

1) When $\delta < 0$, the effect of ε_k is reduced. In MAP algorithm, the more difference between α and β, higher decoding performance can be achieved; because $\delta < 0$ reduces the state

difference between α_k and β_k. The illegibility can be transferred by iterative calculation, α_k and β_k cannot be calculated precisely, which leads to unreliable hard decision output. When δ is small, the effect of ε_k can be ignored, and more decoding performance is lost.

2) When δ>0, the effect of ε_k is enhanced, and the state difference between α_k and β_k is enlarged, which reduces the performance of Turbo codes slightly.

4 Simulation results

Simulation has been carried out to investigate the performance of MAP decoding algorithm under imperfect estimation [5, 7, 8] Turbo codes with 1/2 code rates are used. Frame length equalling 400 and 1 200 bits, Log-MAP algorithm, 5 iteration times, and BPSK modulation ($E_b/N_0 = E_s/N_0$) are used in the simulation.

In Fig. 1, the BER performance vs. channel estimation deviation is given. It's not hard to discover that Turbo codes are sensitive to the channel estimation deviation apparently. When channel estimation result is over the actual value (positive estimation deviation), the BER performance is reduced slightly, but when channel estimation result is less than the actual value, from some point, the decoding performance will be reduced dramatically, especially the Turbo codes with long frame length. From observation, the point meets the inequation $\delta/E_n<0.5$, which means that $\varepsilon_k = 2y_k^s x_k^s(i) + 2y_k^p x_k^p\ (i, S_k, S_{k-1})$ is less than half of actual value in calculation of γ, α and β.

In Fig. 2 and Fig. 3, the distribution of ln α_k and ln β_k in first decoding iteration of 400 bits-length frame vs. −6 dB, −4 dB, −2 dB, 0 dB, 2 dB, 4 dB, 6 dB channel estimation deviation are given. It can be disclosed that the more channel estimation deviation, the more difference to actual value. When channel estimation result is less than the actual value, ln α_k and ln β_k are near the hard decision threshold 0, which will lead to more decoding errors.

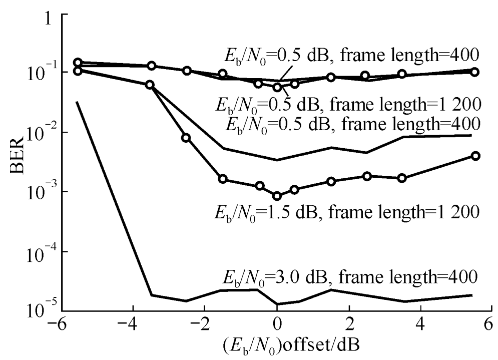

Fig. 1 Turbo decoding performance vs. channel estimation deviation with 400 bit and 1 200 bit frame length under different E_b/N_0

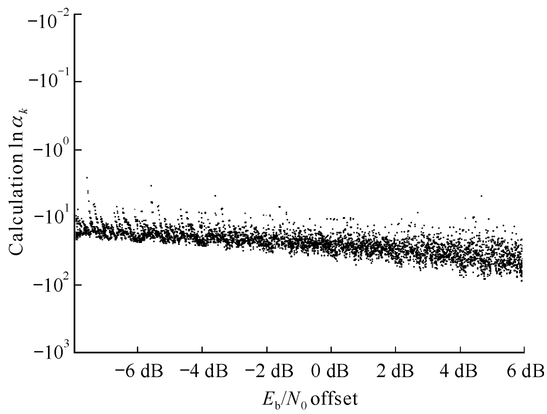

Fig. 2 Calculation α_k of a frame vs. channel estimation deviation (E_b/N_0 = 2.0 dB)

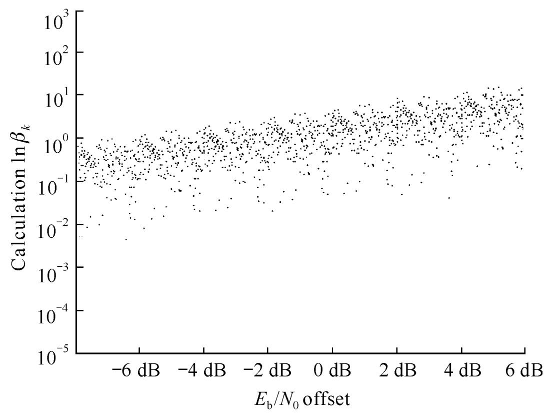

Fig. 3 Calculation β_k of a frame vs. channel estimation deviation (E_b/N_0 = 2.0 dB)

5 Conclusions

In this paper the sensitivity of channel estimation deviation to MAP decoding algorithm of Turbo codes has been investigated. The detailed analysis of the main factor to performance decrease has been given. The theoretical analysis and computer simulation have disclosed the specific reasons for the higher sensitivity of SNR estimation to long frame Turbo codes. The effect of channel estimation deviation can be reduced by the iterative decoding, but the performance of MAP decoding may degrade significantly when SNR estimation is far less than the actual value. In practice application, it is better to use the higher channel estimation than the lower one.

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