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RESEARCH ARTICLE

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A software sampling frequency adaptive algorithm for reducing spectral leakage

Abstract Spectral leakage caused by synchronous error in a nonsynchronous sampling system is an important cause that reduces the accuracy of spectral analysis and harmonic measurement. This paper presents a software sampling frequency adaptive algorithm that can obtain the actual signal frequency more accurately, and then adjusts sampling interval base on the frequency calculated by software algorithm and modifies sampling frequency adaptively. It can reduce synchronous error and impact of spectral leakage; thereby improving the accuracy of spectral analysis and harmonic measurement for power system signal where frequency changes slowly. This algorithm has high precision just like the simulations show, and it can be a practical method in power system harmonic analysis since it can be implemented easily.

Keywords sampling frequency, spectral leakage, synchronous error, self-adaptive, harmonic analysis

1 Introduction

With the rapid development of electric and electronic techniques, more and more non-linear loads were applied in power system. Obviously, they affected the environment of the power supply greatly and polluted the system. The harmonic disturbance may cause power system network waveform distortion, more loss than normal running state, jamming with other electric equipments, and harm to the running stability, reliability and economy of the power system [1]. There are many research subjects about harmonics, such as harmonic detection, harmonic suppressant and compensation, harmonic source analysis, harmonic restricted standard, harmonic power flow calculation and so on. Among these subjects, harmonic detection and analysis are the premise and foundation of all of them [2].

Fast Fourier transformation (FFT) based on discrete-time sampling is the main method in harmonic detection and analysis. But this algorithm has spectral leakage when the sampling frequency is not synchronous with the actual signal frequency, and thus, the parameters calculated by FFT (amplitude, phase. etc.) involve errors. So it cannot meet the actual need. Although we have some ways (such as, by correcting sampling series, improving data window and increasing sampling frequency) to reduce frequency spectral leakage, they all have their own disadvantages. For example, the method of correcting sampling series to approximate actual value can only reduce up to 50 % frequency spectral leakage [3], and the calculation of add-window interpolation algorithm is very complicated, and so on.

The out-of-step between sampling frequency and actual signal frequency is the fundamental causation of frequency spectral leakage. So how to decrease synchronous error is the basic way to reduce frequency spectral leakage. Here, a software algorithm for harmonic analysis based on adaptive sampling frequency is presented. This algorithm can adjust sampling frequency according to the actual frequency, so it can decrease the influence of frequency spectral leakage, reduce sampling synchronous error and improve detecting-accuracy of power system signals whose frequency changes very slowly.

2 Frequency spectral leakage analysis

The general process of FFT harmonic analysis is represented in Fig. 1.

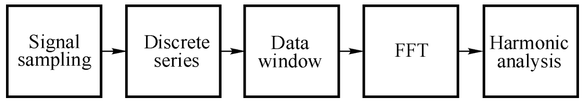

Fig. 1 Process of FFT harmonic analysis

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$x(t)$ is the actual signal, T_0 is the signal period, $f_0 = 1/T_0$ is the signal frequency, T_s is the sampling period, $f_s = 1/T_s$ is the sampling frequency, L is the number of intercepted periods, N is the number of sampling point, L and N are integers, $x(n)$ is the discrete series derived from $x(t)$ through a time-window whose width is LT_0, f_0 and f_s must meet the requirement of Eq. (1) [4]:

$$\frac{LT_0}{T_s} = \frac{Nf_s}{f_0} = N \tag{1}$$

During the progress of FFT signal analysis, if the sampling frequency is invariable, the time-window length of sampling will not be integer multiple periods because of the influence caused by the tiny change of actual signal frequency. This situation will make border point discontinuous in time-domain when period extends, and increase high-frequency component. In frequency-domain, a finite- length signal corresponds to the product of infinite-length signal with rectangular-window function, and its Fourier transformation corresponds to the convolution of the Fourier Transformation of actual signal with that of rectangular-window.

For an infinite-length harmonic signal, amplitude is A_m, angular frequency is ω_m, phase is θ, its Fourier transformation can be expressed as Eq. (2):

$$\begin{aligned} x_m(t) &= A_m e^{j(\omega_m t+\theta)} \\ X_m(\omega) &= 2\pi A_m e^{j\theta}\delta(\omega-\omega_m) \end{aligned} \tag{2}$$

The rectangular-window function and its Fourier transformation are interpreted as Eqs. (3) and (4):

$$W_T(t) = \begin{cases} 1, & 0 < t \leqslant T \\ 0, & \text{other} \end{cases} \tag{3}$$

$$W_T(\omega) = \frac{\sin\left(\frac{\omega T}{2}\right)}{\frac{\omega}{2}} e^{-j\frac{\omega T}{2}} \tag{4}$$

The finite-length harmonic signal is equal to the product of infinite-length harmonic signal with rectangular-window function, and the product in time-domain corresponds to the convolution in frequency-domain, so the finite-length harmonic signal and its Fourier transformation can be represented as Eqs. (5) and (6):

$$\bar{x}_m(t) = x_m(t)W_T(t) \tag{5}$$

$$\bar{x}_m(\omega) = A_m \frac{\sin\left(\frac{\omega-\omega_m}{2}T\right)}{\frac{\omega-\omega_m}{2}} e^{j\theta} e^{-j\frac{\omega-\omega_m}{2}T} \tag{6}$$

Equation (6) is a complex function, and the characteristic of frequency spectral amplitude is represented in Fig. 2:

The discrete spectrum is obtained from FFT algorithm: $\omega = 2\pi n/T$, $n=1, 2, \cdots, N$, N is the number of sampling point, T is sampling time, angular frequency is ω_m, so the spectral distribution can be represented as Eq. (7):

$$\bar{x}_m(n) = A_m N \frac{\sin(n-k)\pi}{(n-k)\pi} e^{j\theta} e^{-j(n-k)\pi} \tag{7}$$

From Eq. (7), we get Eq. (8):

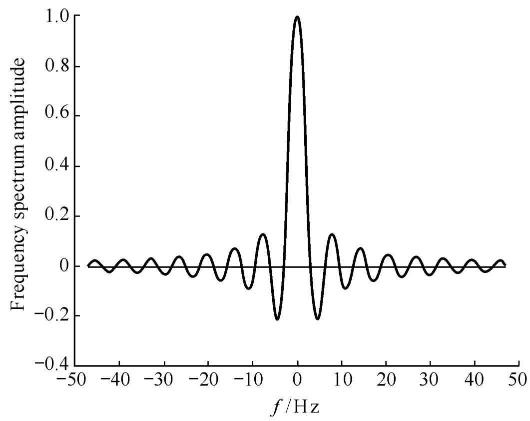

Fig. 2 Continuous spectrum of finite length harmonic signal

$$\bar{x}_m(n) = \begin{cases} \dfrac{A_m N e^{j\theta}}{2} & n = k \\ 0 & n \neq k \end{cases} \tag{8}$$

For synchronous sampling: $n = k$; asynchronous sampling: $n \neq k$, let $k = k_1 + r$ (k_1 is integer, $0 < r < 1$), so the spectral distribution:

$$\bar{x}_m(n) = A_m N \frac{\sin(n-k_1-r)\pi}{(n-k_1-r)\pi} e^{j\theta} e^{-j(n-k_1-r)\pi} \tag{9}$$

Let $t = n-k_1-r$, $M = \dfrac{-A_m N \sin(r\pi) e^{j(\theta+r\pi)}}{\pi}$

Then

$$\bar{x}_m(n) = \frac{M}{t} \tag{10}$$

We can learn from Eq. (9) that the width of sampling window is equal to integral signal period when synchronous sampling and the zero-crossing point of rectangular window parallels with discrete frequency point. When the width of sampling-window is not equal to integral signal period for asynchronous sampling, the harmonic spectrum does not appear to be a line but distributes mutually in the whole frequency-domain, and frequency spectral leakage happens.

3 Methods to improve analysis accuracy

Generally speaking, there are two main elements that affect the accuracy of frequency spectrum analysis: fence effect of FFT [6] and frequency spectrum caused by nonsynchronous sampling.

The current solutions to solve fence effect of FFT are interpolation such as rectangular-window interpolation, Hamming-window interpolation, Blackman-Harris interpolation [4, 5, 7, 9] and so on. Raising L (width of time-window) can reduce frequency spectral leakage while the synchronous error is invariable, and selecting suitable sampling-window function can reduce frequency spectral leakage while L is equal to the sampling data [6, 11].

The nonsynchronous sampling is the basic causation that leads to frequency spectral leakage, and there are several solutions to reduce this leakage:

1) Correct sampling series

If Eq. (1) cannot be met, that is to say, the sampling frequency is out of step, we can extend Taylor series for discrete series; after ignoring higher-order coefficient, we have the following [9, 10]:

$$\overline{x}_{m0}(n) = \overline{x}_{m}(n) + \frac{n}{N}\left|\overline{x}_{m}(n) - \overline{x}_{m}(n+N)\right| \tag{11}$$

And n = 0, 1, 2, ⋯, N–1.

Through Eq. (11) we can correct the discrete series, but its calculation is rather tedious, and it has little effect on reducing frequency spectral leakage.

2) Hardware synchronous circuit

This solution needs a hardware circuit (digital phase-locked loop) to get the synchronous frequency signal; its working principle is described in Fig. 3. The reliability and stability of wave-shape element and frequency division element in hardware circuit are not satisfying when intense harmonic and EMC (electromagnetic) interference appear.

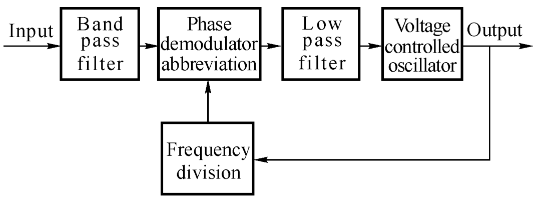

Fig. 3 Diagram of digital phase-locked loop

3) Sampling frequency adaptive adjust

This solution can realize adaptive sampling frequency through estimating the actual signal frequency accurately to adjust sampling interval based on software algorithm. Also, it can reduce frequency spectral leakage and improve detecting precision.

4 Adaptive sampling frequency algorithm

The frequency deviation and detecting precision are demanded in national standard GB/T-14549-93, and specific provisions are made for running frequency deviation of power system. Usually, the frequency fluctuation of power signal is not very fast, and frequency deviation of contiguous several cycles is also fairly small. So we can apply adaptive sampling frequency algorithm to harmonic detecting and frequency spectrum analysis of power signal. First of all, set principal wave frequency 50 Hz as the foundational sampling frequency, then obtain the actual signal frequency with software algorithm, and adjust sampling frequency in real-time based on this actual frequency to improve accuracy and to reduce synchronous error.

The process of software adaptive sampling frequency algorithm of actual power signal is illuminated as follows:

1) Set sampling interval: let T_s=78.125 μs (sampling frequency f_s = 12.8 kHz), sampling points: 2×512+256.

2) Obtain the actual signal frequency: calculate f_0 based on sampling series and sampling interval by using digital filtering zero-crossing modified algorithm.

3) Adjust the sampling frequency in real time based on actual frequency: let sampling-time integral multiple signal period $T_0 = 1/f_0$, sampling points: 2×512+256.

4) Obtain the result（N=512, only calculate 64 points）by using over-sampling FFT, output principal wave and every order harmonics data.

5) Set parameters that are equal to the new sampling interval and the sampling series of (2×512+256) points, then return to step 2).

The characteristics of this algorithm are interpreted as follows:

In step 2), the digital filtering zero-crossing modify method is adopted: the error of f_0 from the calculation of normal zero-crossing linearization algorithm is obvious; moreover, sometimes fault may even appear because of disturbance or other elements. To solve this problem, digital filtering is utilized for sampling series. Generally speaking, basic frequency signal is most intense, and it cannot be easily affected by other harmonic leakage. By digital filtering the elements beyond basic frequency can be filtered, and by utilizing 16 bits A/D chip, the frequency calculation error reduces obviously.

In step 3): use DSP chip TMS320VC5402 in the actual application, its accuracy of sampling interval adjust can reach 0.01 μs.

In step 4): Over-sampling FFT can reduce aliasing error. If the FFT series is not sufficiently band-limited, the frequency element higher than $f_s/2$ will fold over into 0–f_C. The highest analysis harmonic is 64-order to power signal, f_C = 3.2 kHz, Nyquist frequency $f_s = 2f_C$ = 6.4 kHz, so the frequency spectrum of f_s–f_C, $2f_s$–$3f_C$, $3f_s$–$5f_C$ and $4f_S$–$7f_C$ will fold- over into range 0–f_C and cause fold-over error[12]. The fold-over error is up to $f_s/2$; increasing f_s will reduce error when f_C is invariable. It means that over-sampling can reduce aliasing error in signal analysis.

It is necessary to note the effect of over-sampling FFT on calculation. Every order harmonic can be obtained by calculating ahead 64-point of the sampling series after digital filtering. The over-sampling reduces fold-over error, but it increases sampling points N, so it increases calculation work of N-point FFT. To reduce the calculation work, we can sample after filtering and calculating the ahead quarter data of the 256-point FFT.

5 Algorithm simulation

The harmonic signal is shown in Eq. (12), and its waveform is shown in Fig. 4. Simulation experiment is done with adaptive sampling and fixed sampling algorithms when the signal frequency fluctuates slowly, and we have:

$$x(t) = \sin(\omega_0 t) + 0.10\sin(2\omega_0 t) + 0.22\sin(3\omega_0 t) + 0.02\sin(4\omega_0 t) + 0.15\sin(5\omega_0 t) + 0.19\sin(7\omega_0 t) + 0.05\sin(9\omega_0 t) + 0.02\sin(11\omega_0 t) \quad (12)$$

When the signal frequency fluctuates slowly near 50 Hz, we get the signal spectrum by adaptive sampling algorithm and the fixed sampling algorithm is shown in Fig. 5. They are of relative value of each spectrum element, and the adaptive sampling algorithm can follow the actual frequency and reduce the spectral leakage when the frequency of the signal fluctuates slowly.

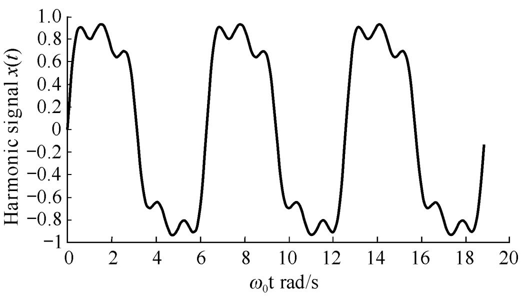

Fig. 4 Waveform of harmonic signal

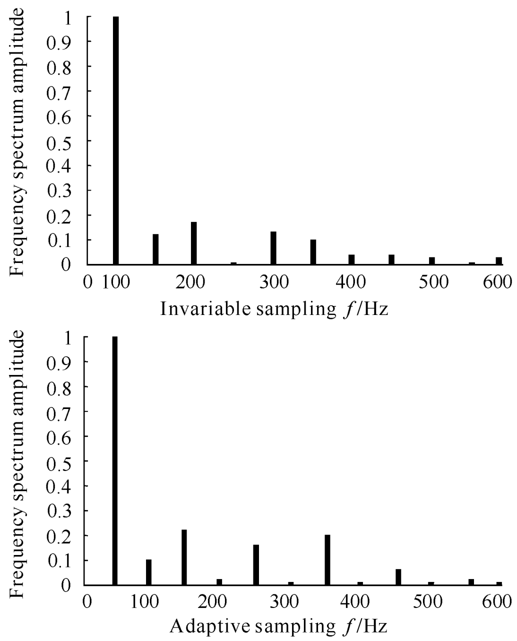

Fig. 5 Comparison of spectrum analysis

In terms of frequency spectral leakage calculation equation in Ref. [8], as Eq. (13), we can obtain the result of frequency spectral leakage in Table 1:

$$V = \sum_{k=0}^{\frac{N}{2}-1} \frac{|X(k)| - |X_{\max}|}{X_{\max}} \quad (13)$$

Table 1 Comparison of frequency spectral leakage while frequency changing

F/Hz	Fixed sampling/%	Adaptive sampling/%
49.6	0.052 0.905 0.883	0.006 0.422 0.628
49.8	0.048 0.699 0.743	0.003 0.035 0.487
50.0	0	0
50.3	0.042 0.705 0.787	0.004 0.367 0.584
50.5	0.075 0.966 0.883	0.008 0.543 0.791

$X(k)$ is the amplitude of NO. k order harmonic and $X_{\max}$ is the maximum amplitude of harmonic.

We can learn the following from Fig. 5: compared to fixed sampling, adaptive sampling algorithm can improve detecting accuracy and reduce frequency spectral leakage evidently while frequency fluctuation is not evident. The data in Table 1 also demonstrates this.

6 Conclusions

Generally, the power signal fluctuates very slowly and its fluctuating amplitude is not very obvious, so the software adaptive sampling frequency algorithm can reduce frequency leakage and synchronous error. Also, it can suppress the impact of frequency leakage and improve the accuracy of harmonic detection. And some content of this algorithm have been already applied to actual device [13, 14]. To some extent, the number of sampling points may meet the real-time requirement, and digital filtering and signal sampling parameter should be optimized and adjusted according to actual conditions. The adaptive sampling frequency software algorithm may have more errors when the signal frequency hugely fluctuates.

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