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RESEARCH ARTICLE

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Model and algorithm for optimization of rescue center location of emergent catastrophe

Abstract The location of rescue centers is a key problem in optimal resource allocation and logistics in emergency response. We propose a mathematical model for rescue center location with the considerations of emergency occurrence probability, catastrophe diffusion function and rescue function. Because the catastrophe diffusion and rescue functions are both nonlinear and time-variable, it cannot be solved by common mathematical programming methods. We develop a heuristic embedded genetic algorithm for the special model solution. The computation based on a large number of examples with practical data has shown us satisfactory results.

Keywords emergent event, emergency response, location optimization, genetic algorithm, heuristic algorithm.

1 Introduction

In recent years, emergent catastrophes such as terrorist strikes, earthquakes, fire and explosion accidents, SARS, and bird-flu, happen very frequently. They bring great loss of life and property to people. With the rising concern on people's lives from the whole society, the research on emergency rescue has attracted more attention from governments and public societies [1]. Supporting emergency rescue has become an active area of interest by many international and domestic scholars [2–4].

The location of rescue centers is very important to guarantee Logistics and resource optimization for catastrophe rescue [5, 6]. It is different from the classic location-allocation problems in general supply chain managernent [7–9]. The main difference is that the rescue arrival time becomes the most critical factor than others in our problem. This is because the loss caused by a catastrophe spread is usually much larger than the rescue cost.

We propose an optimization model for rescue center location based on the catastrophe occurrence probabilities, catastrophe spread functions and rescue functions. Since the catastrophe spread function and rescue function are both nonlinear functions of time, the model with time-varying functions cannot be solved by general mathematical programming approaches. Therefore, we propose a heuristic embedded genetic algorithm [10, 11]. By computing a number of examples from practice, satisfactory results have been achieved.

2 Problem description and model

The problem of rescue center location can be described as follows: there are m possible catastrophe Locations. For Location j, the occurrence probability in the plan horizon is p_i; the loss function caused by catastrophe spreading before rescue arrival is $d_j(t)$, the maximum number of acceptable rescue teams is K_j, $j = 1, 2, \cdots, m$. There are n candidate locations of rescue centers. For rescue center i, the setup cost is S_i; the rescue function is $R_i(t)$; the rescue cost is C_i, $i = 1, 2, \cdots, n$. The time from rescue center location i to catastrophe location j is t_{ij}, $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m$. The objective is to find out the optimal location of rescue centers and rescue scheme to minimize total catastrophe loss and rescue cost.

When a catastrophe happens, more than one team can respond. Assuming the arrival sequence for a location j is [1], [2], $\cdots$, [K_j], the united rescue function is noted by $U_j(R_{[1]}(t), R_{[2]}(t), \cdots, R_{[n]}(t))$. Here, we suppose that catastrophes never happen simultaneously. Because catastrophe is an event with very small probability, the hypothesis is acceptable.

The united rescue function is shown in Fig. 1. Once a

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catastrophe happens, the loss rises up very fast. When the first rescue team [1] arrives at $t_{j[1]}$, the loss begins to decrease. After teams [2] and [3] arrive, the loss declines very fast. Finally, the catastrophe is stopped. The area between axes and curves of the united rescue functions is the total loss caused by the catastrophe. It can be calculated by the integrals of loss function and united rescue functions.

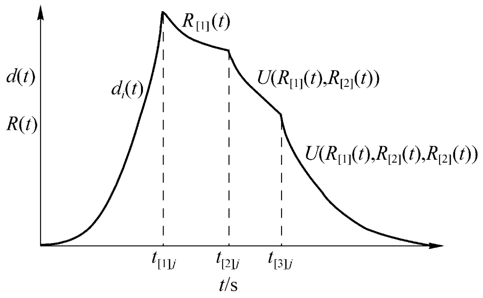

Fig. 1 The loss function and united rescue functions

The variable is defined as

$$x_{ij} = \begin{cases} 1 & \text{center } i \text{ attends the rescue of place } j \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$i = 1, 2, \cdots, n \quad j = 1, 2, \cdots, m$$

and

$$y_j = \begin{cases} 1 & \text{rescue center } j \text{ is set} \\ 0 & \text{otherwise} \end{cases} \quad j = 1, 2, \cdots, m. \tag{2}$$

Then, the model is describes in the following formulas:

$$\min z(x,y) = \sum_{i=1}^{n} S_i y_i + \sum_{j=1}^{m} p_j \left\{ \sum_{i=1}^{n} C_i x_{ij} + \int_0^{t_{[1]j}} d_j(t)\mathrm{d}t \right.$$

$$\left. + d_j(t_{[1]j}) \int_{t_{[1]j}}^{\infty} U(x_{[1]j} R_{[1]}(t - t_{[1]j}), \cdots, x_{[n]j} R_{[n]}(t - t_{[n]j}))\mathrm{d}t \right\} \tag{3}$$

$$\text{s.t.} \quad m\, y_i - \sum_{j=1}^{m} x_{ij} \ge 0, \; i = 1, 2, \cdots, n \tag{4}$$

$$\sum_{i=1}^{m} x_{ij} \le K_j, \; j = 1, 2, \cdots, m, \tag{5}$$

$$x_{ij}, \; y_i = 0 \text{ or } 1, i = 1, 2, \cdots, n, \; j = 1, 2, \cdots, m \tag{6}$$

Note: the rescue function $R_{[k]}(t)$ is not included in the united rescue function $U(\cdot)$ if $x_{[k]j}=0$.

The above model Eqs. (3)–(6) is a complex 0, 1 integer programming. In the objective Eq. (3), the first term is the cost to set the rescue center. The contents in the bracket $\{\cdot\}$ means the total loss by catastrophe j with probability p_j. Its first integral is the loss before any rescue team arrives; second integral is the loss in rescue process. The constraint Eq. (4) means the teams in the rescue have to be set. Equation (5) is the limitation of the number of acceptable rescue teams Eq. (6) is the restriction of variables.

Since the united rescue functions change with the variables x_{ij} and y_j, it cannot be solved by the common mathematical programming method.

3 A heuristic embedded genetic algorithm

If we take x_{ij} and y_i as the encoding for genetic algorithm directly, the length of a chromosome will be $(n+1)\, m$. Thus, the problem would be difficult even for very small problems. For example, in the 8×10 problem in the next section, the length of the chromosome will be 90 bites of 0,1.

As we know, once the selection of rescue centers is determined, the rescue scheme can be fixed by a heuristic of the shortest arrival time. Thus, the heuristic embedded genetic algorithm takes only $Y=[\, y_1, y_2, \cdots, y_n\,]$, y_i is 0, 1 variable defined by Eq. (2).

Since the loss of catastrophe spread is usually much larger than the rescue cost, the nearest rescue center will always be the first choice for the united rescue scheme. The other joined teams can be determined by a greedy heuristic. The procedure of heuristic is described as follows:

Step 1 Find the set of available rescue centers in chromosome Y, $Q=\{\, i \mid y_i=1, i=1, 2, \cdots, n\}$.

Step 2 For the possible catastrophe location j, $j=1, 2, \cdots, m$, do the following substeps 2.1–2.3.

Substep 2.1 If $i^*=\arg\min\{\, t_{ij} \mid i\in Q\}$, select i^* as the first arrival team [1], calculate

$$z_j([1]) = \int_0^{t_{[1]j}} d_j(t)\mathrm{d}t + d_j(t_{[1]j}) \int_{t_{[1]j}}^{\infty} U(x_{[1]j} R_{[1]}(t - t_{[1]j}))\mathrm{d}t \tag{7}$$

Substep 2.2 For $k=2, \cdots, K_j$, if $i^*=\arg\min\{\, z_j([k]) \mid i\in Q\backslash[k]\}$, where

$$z_j([k]) = \int_0^{t_{[1]j}} d_j(t)\mathrm{d}t + d_j(t_{[1]j})$$

$$\cdot \int_{t_{[1]j}}^{\infty} U(x_{[1]j} R_{[1]}(t - t_{[1]j}), \cdots, x_{[k]j} R_{[k]}(t - t_{[k]j}))\mathrm{d}t \tag{8}$$

select i^* as the kth arrival team $[k]$. If $z_j([k]) > z_j([k-1])$, go to Substep 2.3.

Substep 2.3 Calculate

$$z_j^* = p_j\{ \sum_{k=1,\cdots,K_j} C_{[k]} + z_j([K_j]) \text{ or } z_j([k-1])\}.$$

Step 3 Calculate $z(Y) = \sum_{i\in Q} S_i + \sum_{j=1}^{m} z_j^*$, return $z(Y)$ as the fitness value of chromosome Y.

4 Numerical example

The heuristic embedded genetic algorithm was coded by Fortran, and run at a PC P4/760. The satisfactory results have been achieved. We present a small example here. Let's say there are $m=10$ possible catastrophe locations. The catastrophe spread function is described as a quadratic function.

$$d_j(t) = a_j t^2 \quad j=1, 2, \cdots, 10 \tag{9}$$

The probability of catastrophe occurrence in location j is p_j; the parameter of the loss function is a_j; the limited number of rescue teams is K_j, j=1, 2, ⋯, 10. The data are shown in Table 1.

There are n = 8 candidates of rescue centers. Their single rescue function is presented as

$$R_i(t) = \mathrm{e}^{-\beta_i t}, \quad i=1, 2, \cdots, 8 \tag{10}$$

where the parameter β_i means the rescue speed of team i. Thus, the united rescue function is:

$$U(R_{[1]}(t), R_{[2]}(t), \cdots, R_{[k]}(t)) = \mathrm{e}^{-(\beta_{[1]}+\beta_{[2]}+\cdots+\beta_{[k]})t}$$

Table 1 Data of catastrophe possible points

j	p_j	a_j	K_j
1	0.035	800	2
2	0.020	1 000	3
3	0.030	1 200	3
4	0.037	700	4
5	0.034	880	2
6	0.035	900	3
7	0.020	980	3
8	0.047	1 100	3
9	0.035	1 200	4
10	0.020	650	4

The data of rescue centers are shown in Table 2.

Table 2 Data of candidate rescue centers

i	β_i	S_i	C_i
1	0.500	1.200	0.003 0
2	0.400	1.400	0.002 0
3	0.650	2.600	0.002 5
4	0.550	3.100	0.003 5
5	0.650	3.500	0.003 0
6	0.450	3.300	0.003 5
7	0.520	1.260	0.003 5
8	0.600	3.320	0.004 0

The arrival times from rescue centers to catastrophe locations are shown in Table 3.

Let the population size be 30. The maximum number of generation is 20. The probabilities of crossover and mutation are 0.90 and 0.05. After computation of the recommended algorithm, the rescue centers 2, 3, 5, 6 and 7 are selected for construction. The centers 1, 4 and 8 are gaveup. Once a catastrophe happens, the rescue scheme is shown in Table 3 and Fig. 2. The optimal objective value is 44.615 2.

From Table 4 we see that all catastrophe locations are rescued by the near centers. The rescue centers 2, 3 and 5 in the central area of the city have more rescue tasks. But the candidate centers 1, 4 and 8 in the suburban area have few tasks. Therefore, they cannot be selected in the planning. To check the optimality of this solution, we solve the example problem by a Branch and Bound algorithm. It proved that the solution is optimal.

Table 3 The times from rescue center to catastrophe points

M	1	2	3	4	5	6	7	8	9	10
1	0.30	0.80	0.70	1.20	1.50	1.70	2.20	3.00	2.80	3.20
2	0.25	0.70	1.20	0.30	0.35	1.45	2.50	1.90	2.20	2.80
3	1.20	0.30	0.30	1.55	0.45	0.30	0.90	2.00	1.95	1.98
4	2.50	1.90	0.40	3.00	2.50	1.50	0.40	3.00	2.50	1.90
5	1.90	1.80	2.20	1.00	0.50	0.95	2.00	0.40	0.30	1.50
6	3.10	2.30	1.70	3.00	2.00	0.50	0.30	2.50	1.20	0.30
7	3.00	3.10	3.50	2.20	1.80	2.40	2.80	0.50	0.40	1.50
8	3.20	3.00	3.10	3.00	2.50	1.90	2.10	1.80	0.50	0.30

Table 4 Rescue scheme once catastrophe happens

m	1	2	3	4	5	6	7	8	9	10
K_j	2	3	3	4	2	3	3	3	4	4
	[1]	[2]	[2]	[1]	[1]	—	—	[3]	—	—
2	[2]	[1]	[1]	[3]	[2]	[1]	[2]	—	[4]	[4]
3	—	[3]	—	[2]	—	[3]	[3]	[1]	[1]	[3]
5	—	—	[3]	—	—	[2]	[1]	—	[3]	[1]
6	—	—	—	[4]	—	—	—	[2]	[2]	[2]
7										

Note: the number in [ ] means the team arrival sequence.

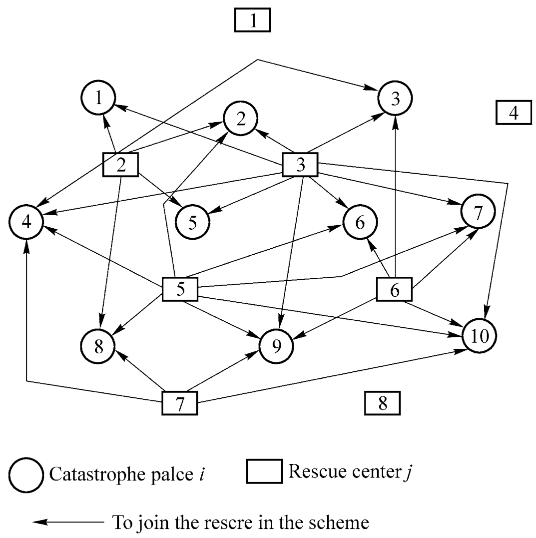

Fig. 2 The selection result of rescue centers and rescue scheme

5 Conclusions

The optimal location of rescue centers is a key research topic in the logistics of catastrophe rescue. Considering the urgent feature of catastrophe rescue, we apply the concepts of catastrophe spread function, the rescue function the distances between catastrophe places and rescue centers, the cost of rescue center construction, and the rescue costs into the location optimization model. Since the model involves some nonlinear time functions that form change with model variables, it cannot be solved by common mathematical programming methods. To solve the problem, we develop a heuristic embedded genetic algorithm. The computation on number examples from practice has proven that the recommended model and algorithm can achieve satisfactory results in actual applications.

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