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RESEARCH ARTICLE

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Robust adaptive control of nonlinearly parameterized systems with unmodeled dynamics

Abstract Many physical systems such as biochemical processes and machines with friction are of nonlinearly parameterized systems with uncertainties. How to control such systems effectively is one of the most challenging problems. This paper presents a robust adaptive controller for a significant class of nonlinearly parameterized systems. The controller can be used in cases where there exist parameter and nonlinear uncertainties, unmodeled dynamics and unknown bounded disturbances. The design of the controller is based on the control Lyapunov function method. A dynamic signal is introduced and adaptive nonlinear damping terms are used to restrain the effects of unmodeled dynamics, nonlinear uncertainties and unknown bounded disturbances. The backstepping procedure is employed to overcome the complexity in the design. With the proposed method, the estimation of the unknown parameters of the system is not required and there is only one adaptive parameter no matter how high the order of the system is and how many unknown parameters there are. It is proved theoretically that the proposed robust adaptive control scheme guarantees the stability of nonlinearly parameterized system. Furthermore, all the states approach the equilibrium in arbitrary precision by choosing some design constants appropriately. Simulation results illustrate the effectiveness of the proposed robust adaptive controller.

Keywords nonlinearly parameterized systems, adaptive control, unmodeled dynamics, robustness, stability

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1 Introduction

Much progress has been made recently in the research on adaptive control of nonlinear systems with uncertainties. Many approaches, such as adaptive feedback linearization, adaptive backstepping and adaptive switch control etc., have been presented. With some important results obtained in the research on the adaptive control of nonlinear systems with linear parameterization that are linearizable by feedback, the more difficult and practical problem, the adaptive control of nonlinearly parameterized systems, has been receiving great attention.

Based on the ideas of adding a power integrator and the separation principle, LIN and QIAN presented some important results in the adaptive control of a class of nonlinearly parameterized systems [1–3]. However, the adaptive control schemes presented in Refs. [1, 2] are not robust to disturbances and unmodeled dynamics. Although the control scheme given in Ref. [3] can be used in the case of systems with unmodeled dynamics, the adaptive law is of switch type, which may cause undesirable behavior such as chattering.

Since many physical systems are so complicated that it is impossible to describe them precisely by models, unmodeled dynamics are inevitably presented in almost all practical systems. It is pointed out in Refs. [4, 5] that unmodeled dynamics have serious effects on the stability and the performances of the systems. To enhance the robustness of the systems, unmodeled dynamics must be considered in the design of the controllers such as in Refs. [6–9]. However, the control schemes proposed in Refs. [6, 7, 9] only deal with the nonlinear systems with linear parameterization. The scheme presented in Ref. [8] can be used to control the nonlinearly parameterized systems, but it is for the nonlinear systems represented by input-output models.

This paper presents a robust adaptive state feedback control scheme for nonlinearly parameterized systems by combing control Lyapunov function method, backstepping, adaptive nonlinear damping [7, 8] and the nonlinear parameter separation technique. The scheme can be used in the cases where there exist parameter and nonlinear uncertainties, unmodeled dynamics and unknown bounded disturbances. Estimation of the unknown parameters of the system is not required with the proposed method. No matter how high the order of the system is and how many unknown parameters there are, there is only one adaptive parameter. It is proved theoretically that the proposed robust adaptive control scheme guarantees the stability of nonlinearly parameterized system. Furthermore, all the states approach the equilibrium in arbitrary precision by choosing some design constants appropriately. Since many physical systems such as biochemical processes, mechanical systems with frictions etc., are of nonlinearly parameterized systems [1, 2], the proposed scheme has widely potential uses in practice.

2 Problem statement

Consider nonlinearly parameterized system

$$\begin{aligned} \dot{x}_1 &= x_2 + \phi_1\left(x_1,\theta,\omega,d_1(t)\right) \\ &\cdots \\ \dot{x}_{n-1} &= x_n + \phi_{n-1}\left(x_1,\cdots,x_{n-1},\theta,\omega,d_{n-1}(t)\right) \\ \dot{x}_n &= u + \phi_n\left(x_1,\cdots,x_n,\theta,\omega,d_n(t)\right) \end{aligned} \tag{1}$$

where $u \in R$, $x \in R^n$ are the control and the state of the system, respectively; $\theta \in R^m$ is the unknown parameter, $d_i(t)$ is the unknown bounded disturbance; $\phi_i\left(x_1,\cdots,x_i,\theta,\omega,d_i(t)\right)$ represents the uncertain nonlinearity and the uncertainty related to the unmodeled dynamics ω and the disturbance $d_i(t)$, $\omega \in R^r$ is the state of unmodeled dynamics described by

$$\dot{\omega} = q(\omega, x_1) \tag{2}$$

where $q(\omega, x_1)$ is an unknown Lipschitz continuous function.

The objective of this paper is to design a robust adaptive controller for system Eq. (1) such that the closed-loop system is stable. We need the following assumptions:

Assumption 1 $\phi_i\left(x_1,\cdots,x_i,\theta,\omega,d_i(t)\right)$, $i = 1, 2, \cdots, n$, are unknown Lipschitz continuous functions satisfying

$$\begin{aligned} &\left|\phi_i\left(x_1,\cdots,x_i,\theta,\omega,d_i(t)\right)\right| \\ &\le a_i\left(x_1,\cdots,x_i,\theta\right) + c_{i,1}\|\omega\| + \hat{c}_{i,2}\left|d_i(t)\right| \end{aligned} \tag{3}$$

$$\phi_i\left(0,\cdots,0,\theta,\omega,d_i(t)\right) = 0 \tag{4}$$

where $a_i\left(x_1,\cdots,x_i,\theta\right)$ is a nonnegative function, $c_{i,1}, \hat{c}_{i,2} \ge 0$, $i = 1, 2, \cdots, n$ are unknown constants.

Assumption 2 The unmodeled dynamics $\dot{\omega} = q(\omega, x_1)$ is exponentially input-to-state practically stable (exp-ISpS) [6], i.e., there exists an exp-ISpS Lyapunov $V_\omega(\omega)$ satisfying

$$\alpha_1(\|\omega\|) \le V_\omega(\omega) \le \alpha_2(\|\omega\|) \tag{5}$$

$$\frac{\partial V_\omega(\omega)}{\partial \omega} q(\omega, x_1) \le -c_0 V_\omega(\omega) + \rho(|x_1|) + d_0 \tag{6}$$

where $\alpha_1(\cdot), \alpha_2(\cdot)$ are k_∞ functions, $\alpha_1(\cdot)$ is known; $c_0 > 0, d_0 \ge 0$ are known constants. Without loss of the generality, we assume that ρ has the form $\rho(h) = h^2\rho_0(h^2)$, where ρ_0 is a nonnegative smooth function. Otherwise, it suffices to replace $\rho(\cdot)$ by $x_1^2\rho_0(x_1^2) + \bar{\varepsilon}_0$ with $\bar{\varepsilon}_0 > 0$ being a sufficiently small real number [6].

Generally, unmodeled dynamics are immeasurable and unavailable for feedback control. To dominate the undesired effects of the ummodeled dynamics on the stability of the system needs to generate the following dynamic signal

$$\dot{s} = -\bar{c}_0 s + s_m(x_1),\ s(0) = s_0 > 0 \tag{7}$$

where $\bar{c}_0 \in (0, c_0)$, $s_m(x_1) = x_1^2\rho_0(x_1^2) + d_0$. It can be shown that the dynamic signal has the following property [6]. That is, for all $t \ge 0$

$$V_\omega(\omega(t)) \le s(t) + D(t) \tag{8}$$

and there is a finite T^0 such that $D(t) = 0$ for all $t \ge T^0 \ge 0$.

By Lemma 2.1 introduced in Ref. [1], there exist two smooth functions $b_i(\theta) \ge 1$ and $\gamma_i(x_1,\cdots,x_i) \ge 1$ such that

$$a_i(x_1,\cdots,x_i,\theta) \le \gamma_i(x_1,\cdots,x_i) b_i(\theta) \tag{9}$$

Let $\Theta = \sum_{i=1}^{n} b_i(\theta)$. Then, inequality Eq. (3) can be rewritten as

$$\begin{aligned} &\left|\phi_i\left(x_1,\cdots,x_i,\theta,\omega,d_i(t)\right)\right| \\ &\quad \le \gamma_i(x_1,\cdots,x_i)\Theta + c_{i,1}\|\omega\| + \hat{c}_{i,2}\left|d_i(t)\right| \end{aligned} \tag{10}$$

3 Design of the robust adaptive controller

In this section, the adaptive controller is constructed by the control Lyapunov function method and the backstepping procedure.

Step 1 Define $\tilde{\Theta} = \Theta^* - \hat{\Theta}(t)$, where $\hat{\Theta}(t)$ is the adaptive parameter of the controller; $\Theta^* > 0$ is an unknown constant representing the desired value of $\hat{\Theta}$, i.e., when $\hat{\Theta} = \Theta^*$, the control system has the desired performance. Choose the Lyapunov function candidate as

$$V_1 = \frac{1}{2}x_1^2 + \frac{1}{2}\Gamma^{-1}\tilde{\Theta}^2 \tag{11}$$

where $\Gamma > 0$ is a design constant. Then,

$$\begin{aligned} \dot{V}_1 &= x_1\dot{x}_1 - \Gamma^{-1}\tilde{\Theta}(t)\dot{\hat{\Theta}}(t) \\ &= x_1\left(x_2 + \phi_1\left(x_1,\theta,\omega,d_1(t)\right)\right) - \Gamma^{-1}\tilde{\Theta}(t)\dot{\hat{\Theta}}(t) \end{aligned}$$

Using Eq. (1) obtains

$$\dot{V}_1 \le x_1x_2 + |x_1|\gamma_1(x_1)\Theta + |x_1|\left(c_{1,1}\|\omega\| + \hat{c}_{1,2}|d_1(t)|\right) - \Gamma^{-1}\tilde{\Theta}(t)\dot{\hat{\Theta}}(t) \tag{12}$$

Based on Eq. (5), Eq. (8) and the properties of k_∞ functions, we get

$$c_{1,1}\|\omega\| \le c_{1,1}\alpha_1^{-1}(s + D(t)) \le c_{1,1}\alpha_1^{-1}(2s) + c_{1,1}\alpha_1^{-1}(2D(t)) \tag{13}$$

where $\alpha_1^{-1}(\cdot)$ is the inverse function of $\alpha_1(\cdot)$ and is again a function of class K_∞; Since $D(t)=0,\ \forall t \ge T^0$, we have $\alpha_1^{-1}(2D(t)) = 0,\ \forall t \ge T^0$. Let

$$c_{1,2} = \sup\left\{c_{1,1}\alpha_1^{-1}(2D(t)) + \hat{c}_{1,2}|d_1(t)|\right\}$$

Then, Eq. (12) becomes

$$\begin{aligned}\dot{V}_1 \le{}& x_1x_2 - \Theta^*\left(|x_1|\gamma_1(x_1) - \frac{\Theta}{2\Theta^*}\right)^2 - \Theta^*\left(|x_1|\alpha_1^{-1}(2s) - \frac{c_{1,1}}{2\Theta^*}\right)^2 - \Theta^*\left(|x_1| - \frac{c_{1,2}}{2\Theta^*}\right)^2 \\ &+ \frac{c_{1,1}^2}{4\Theta^*} + \frac{c_{1,2}^2}{4\Theta^*} + \frac{\Theta^2}{4\Theta^*} + \left(x_1^2\gamma_1^2(x_1) + x_1^2\left(\alpha_1^{-1}(2s)\right)^2 + x_1^2 - \sigma\hat{\Theta} - \Gamma^{-1}\dot{\hat{\Theta}} \right)\left(\Theta^* - \hat{\Theta}\right) + \hat{\Theta}\left(x_1^2\gamma_1^2(x_1) \right. \\ &\left. + x_1^2\left(\alpha_1^{-1}(2s)\right)^2 + x_1^2\right) + \sigma\hat{\Theta}\left(\Theta^* - \hat{\Theta}\right)\end{aligned} \tag{14}$$

Choose virtual control

$$x_2^*\left(x_1, \hat{\Theta}, s\right) = -\frac{3}{2}x_1 - \hat{\Theta}x_1\left(\gamma_1^2(x_1) + \left(\alpha_1^{-1}(2s)\right)^2 + 1\right) \tag{15}$$

Then

$$\dot{V}_1 \le -x_1^2 - \frac{1}{2}x_1^2 + x_1\left(x_2 - x_2^*\right) + \left(\Psi_1\left(x_1, \hat{\Theta}, s\right) - \Gamma^{-1}\dot{\hat{\Theta}}(t)\right)\left(\tilde{\Theta}(t) + \varphi_1\right) + N_1 + \sigma\hat{\Theta}\tilde{\Theta} \tag{16}$$

where $N_1 = \dfrac{c_{1,1}^2}{4\Theta^*} + \dfrac{c_{1,2}^2}{4\Theta^*} + \dfrac{\Theta^2}{4\Theta^*}$, $\varphi_1 = 0$, $\sigma > 0$ is a design constant. In Eq. (16)

$$\Psi_1\left(x_1, \hat{\Theta}, s\right) = x_1^2\gamma_1^2(x_1) + x_1^2\left(\alpha_1^{-1}(2s)\right)^2 + x_1^2 - \sigma\hat{\Theta} \tag{17}$$

Step $k\ (2 \le k \le n-1)$ Assuming that a series of virtual controllers had been developed before step k, $x_2^* = x_2^*\left(x_1, \hat{\Theta}, s\right), \cdots,\ x_{k+1}^* = x_{k+1}^*\left(x_1, \cdots, x_k, \hat{\Theta}, s\right)$

Define $\xi_1 = x_1$, $\xi_2 = x_2 - x_2^*, \cdots,\ \xi_{k+1} = x_{k+1} - x_{k+1}^*$. Thus $V_1 = 1/2\xi_1^2 + 1/2\Gamma^{-1}\tilde{\Theta}^2$, and

$$V_k\left(\xi_1, \cdots, \xi_k, \tilde{\Theta}\right) = \sum_{j=1}^{k}\frac{1}{2}\xi_j^2 + \frac{1}{2}\Gamma^{-1}\tilde{\Theta}^2 \tag{18}$$

It is also assumed that in Step k, similar to Step 1, by choosing an appropriate virtual controller x_{k+1}^*, we had obtained

$$\dot{V}_k \le -\left(\xi_1^2 + \cdots + \xi_k^2\right) - \frac{1}{2}\xi_k^2 + \xi_k\left(x_{k+1} - x_{k+1}^*\right) + \left(\Psi_k\left(\xi_1, \cdots, \xi_k, \hat{\Theta}, s\right) - \Gamma^{-1}\dot{\hat{\Theta}}\right)\left(\tilde{\Theta}(t) + \varphi_k\right) + N_k + \sigma\hat{\Theta}\tilde{\Theta} \tag{19}$$

where $N_k = N_{k-1} + \tilde{c}_{k,1}^2/4\Theta^* + \tilde{c}_{k,2}^2/4\Theta^* + k\Theta^2/4\Theta^*$, $k = 1, 2, \cdots$, $N_0 = 0$, $\tilde{c}_{k,1} \ge 0,\ \tilde{c}_{k,2} \ge 0$ are unknown constants, and $\tilde{c}_{11} = c_{1,1}, \tilde{c}_{1,2} = c_{1,2}$. In the following, we will show that inequality Eq. (19) also holds in Step $k+1$ by choosing virtual controller x_{k+2}^* appropriately, which also gives the design procedure and the general form of the controller. Define

$$V_{k+1}\left(\xi_1, \cdots, \xi_{k+1}, \tilde{\Theta}\right) = V_k\left(\xi_1, \cdots, \xi_k, \tilde{\Theta}\right) + \frac{1}{2}\xi_{k+1}^2 \tag{20}$$

Then

$$\begin{aligned}\dot{V}_{k+1} \le{}& -\left(\xi_1^2 + \cdots + \xi_k^2\right) - \frac{1}{2}\xi_k^2 + \xi_k\left(x_{k+1} - x_{k+1}^*\right) + \left(\psi_k\left(\xi_1, \cdots, \xi_k, \hat{\Theta}, s\right) - \Gamma^{-1}\dot{\hat{\Theta}}\right)\left(\tilde{\Theta}(t) + \varphi_k\right) \\ &+ \xi_{k+1}\left(x_{k+2} + \phi_{k+1}\left(x_1, \cdots, x_{k+1}, \theta, \omega, d_{k+1}(t)\right)\right) - \xi_{k+1}\left(\sum_{j=1}^{k}\frac{\partial x_{k+1}^*}{\partial x_j}\left(x_{j+1} + \phi_j(\cdot)\right)\right. \\ &\left. + \frac{\partial x_{k+1}^*}{\partial\hat{\Theta}}\dot{\hat{\Theta}} + \frac{\partial x_{k+1}^*}{\partial s}\dot{s}\right) + N_k + \sigma\hat{\Theta}\tilde{\Theta}\end{aligned} \tag{21}$$

We have

$$\begin{aligned}&|\xi_{k+1}|\left|\phi_{k+1}(\cdot) - \sum_{j=1}^{k}\frac{\partial x_{k+1}^*}{\partial x_j}\phi_j(\cdot)\right| \\ &\le |\xi_{k+1}|\left(\gamma_{k+1}(\cdot) + \sum_{j=1}^{k}\left|\frac{\partial x_{k+1}^*}{\partial x_j}\right|\gamma_j(\cdot)\right)\Theta \\ &+ |\xi_{k+1}|\left(1 + \sum_{j=1}^{k}\left|\frac{\partial x_{k+1}^*}{\partial x_j}\right|\right)\left(\tilde{c}_{k+1,1}\|\omega\| + c_{k+1,2}\left|\tilde{d}_{k+1}(t)\right|\right)\end{aligned} \tag{22}$$

where $\tilde{c}_{k+1,1} = \max\left\{c_{1,1}, \cdots, c_{k+1,1}\right\}$, $c_{k+1,2} = \max\left\{\hat{c}_{1,2}, \cdots, \hat{c}_{k+1,2}\right\}$, $\left|\tilde{d}_{k+1}(t)\right| = \sup\left\{|d_1(t)|, \cdots, |d_{k+1}(t)|\right\}$.

Treating $\|\omega\|$ as in Step 1 gives

$$\tilde{c}_{k+1,1}\|\omega\| \le \tilde{c}_{k+1,1}\alpha_1^{-1}(2s) + \tilde{c}_{k+1,1}\alpha_1^{-1}(2D(t)),$$

$$\begin{aligned}&|\xi_{k+1}|\left(1 + \sum_{j=1}^{k}\left|\frac{\partial x_{k+1}^*}{\partial x_j}\right|\right)\left(\tilde{c}_{k+1,1}\|\omega\| + c_{k+1,2}\left|\tilde{d}_{k+1}(t)\right|\right) \\ &\le |\xi_{k+1}|\left(1 + \sum_{j=1}^{k}\left|\frac{\partial x_{k+1}^*}{\partial x_j}\right|\right)\left(\tilde{c}_{k+1,1}\alpha_1^{-1}(2s) + \tilde{c}_{k+1,2}\right)\end{aligned} \tag{23}$$

where $\tilde{c}_{k+1,2} = \sup\left\{ \tilde{c}_{k+1,1}\alpha_1^{-1}(2D(t)) + c_{k+1,2}\left|\tilde{d}_{k+1}(t)\right|\right\}$.

In addition

$$\left|\xi_k\left(x_{k+1}-x_{k+1}^*\right)\right|\le\frac{1}{2}\xi_k^2+\frac{1}{2}\xi_{k+1}^2 \tag{24}$$

Substituting Eqs. (22)–(24) into Eq. (21) obtains

$$\begin{aligned}\dot V_{k+1}\le&-\left(\xi_1^2+\cdots+\xi_k^2\right)+\frac{1}{2}\xi_{k+1}^2+\xi_{k+1}x_{k+2}\\&+\left|\xi_{k+1}\right|\left(\gamma_{k+1}(\cdot)+\sum_{j=1}^{k}\left|\frac{\partial x_{k+1}^*}{\partial x_j}\right|\gamma_j(\cdot)\right)\Theta\\&-\xi_{k+1}\left(\sum_{j=1}^{k}\frac{\partial x_{k+1}^*}{\partial x_j}x_{j+1}+\frac{\partial x_{k+1}^*}{\partial\hat\Theta}\dot{\hat\Theta}+\frac{\partial x_{k+1}^*}{\partial s}\dot s\right)\\&+\left(\psi_k(\cdot)-\Gamma^{-1}\dot{\hat\Theta}\right)\left(\tilde\Theta(t)+\varphi_k\right)\\&+\left|\xi_{k+1}\right|\left(1+\sum_{j=1}^{k}\left|\frac{\partial x_{k+1}^*}{\partial x_j}\right|\right)\left(\tilde c_{k+1,1}\alpha_1^{-1}(2s)+\tilde c_{k+1,2}\right)\\&+N_k+\sigma\hat\Theta\tilde\Theta\end{aligned} \tag{25}$$

Take

$$\begin{aligned}\Psi_{k+1}\left(\xi_1,\cdots,\xi_{k+1},\hat\Theta,s\right)=&\Psi_k(\cdot)+\xi_{k+1}^2\left(1+\sum_{j=1}^{k}\left(\frac{\partial x_{k+1}^*}{\partial x_j}\right)^2\right)\\&\bullet\left(\left(\alpha_1^{-1}(2s)\right)^2+1\right)+\xi_{k+1}^2\left(\gamma_{k+1}^2(\cdot)+\sum_{j=1}^{k}\left(\frac{\partial x_{k+1}^*}{\partial x_j}\gamma_j(\cdot)\right)^2\right)\end{aligned} \tag{26}$$

$$\begin{aligned}\varphi_{k+1}=&\varphi_k+\Gamma\xi_{k+1}\frac{\partial x_{k+1}^*}{\partial\hat\Theta}\\x_{k+2}^*=&-2\xi_{k+1}+\sum_{j=1}^{k}\frac{\partial x_{k+1}^*}{\partial x_j}x_{j+1}+\Gamma\frac{\partial x_{k+1}^*}{\partial\hat\Theta}\Psi_{k+1}+\frac{\partial x_{k+1}^*}{\partial s}\dot s\\&-\hat\Theta\xi_{k+1}\left(1+\sum_{j=1}^{k}\left(\frac{\partial x_{k+1}^*}{\partial x_j}\right)^2\right)\left(\left(\alpha_1^{-1}(2s)\right)^2+1\right)\\&-\hat\Theta\xi_{k+1}\left(\gamma_{k+1}^2(\cdot)+\sum_{j=1}^{k}\left(\frac{\partial x_{k+1}^*}{\partial x_j}\gamma_j(\cdot)\right)^2\right)\end{aligned} \tag{27}$$

As in Step 1, from Eq. (25) we get

$$\begin{aligned}\dot V_{k+1}\le&-\left(\xi_1^2+\cdots+\xi_k^2+\xi_{k+1}^2\right)-\frac{1}{2}\xi_{k+1}^2\\&+\xi_{k+1}\left(x_{k+2}-x_{k+2}^*\right)+\left(\Psi_{k+1}(\cdot)-\Gamma^{-1}\dot{\hat\Theta}\right)\left(\tilde\Theta(t)+\varphi_{k+1}\right)\\&+N_{k+1}+\sigma\hat\Theta\tilde\Theta\end{aligned} \tag{28}$$

where $N_{k+1}=\frac{\tilde c_{k+1,1}^2}{4\Theta^*}+\frac{\tilde c_{k+1,2}^2}{4\Theta^*}+\frac{(k+1)\Theta^2}{4\Theta^*}+N_k$.

Hence, it has been proved that inequality Eq. (19) also holds in Step $k+1$ by choosing virtual controller x_{k+2}^* to be as Eq. (27) which gives the general form of the controller.

Step n It can be seen from Eq. (1) that equations Eqs. (19), (26) and (27) also hold in Step n provided that x_{n+1} is replaced by u. Let $k=n$

$$V_n\left(\xi_1,\cdots,\xi_n,\tilde\Theta\right)=\sum_{j=1}^{n}\frac{1}{2}\xi_j^2+\frac{1}{2}\Gamma^{-1}\tilde\Theta^2 \tag{29}$$

Choosing the adaptive controller

$$u=u^*=x_n^*\left(\xi_1,\cdots,\xi_n,\hat\Theta,s\right) \tag{30}$$

we obtain

$$\begin{aligned}\dot V_n\le&-\left(\xi_1^2+\cdots+\xi_n^2\right)-\frac{1}{2}\xi_n^2+\xi_n\left(u-u^*\right)\\&+\left(\psi_n(\cdot)-\Gamma^{-1}\dot{\hat\Theta}\right)\left(\tilde\Theta(t)+\varphi_n\right)+N_n+\sigma\hat\Theta\tilde\Theta\end{aligned} \tag{31}$$

Let the adaptive law for the parameter of the controller satisfy

$$\dot{\hat\Theta}=\Gamma\Psi_n\left(\xi_1,\cdots,\xi_n,\hat\Theta,s\right),\quad\hat\Theta(0)>0 \tag{32}$$

Then

$$\dot V_n\left(\xi_1,\cdots,\xi_n,\tilde\Theta\right)\le-\left(\xi_1^2+\cdots+\xi_n^2\right)+N_n+\sigma\hat\Theta\tilde\Theta \tag{33}$$

where $N_n=\sum_{k=1}^{n}\frac{\tilde c_{k,1}^2}{4\Theta^*}+\frac{\tilde c_{k,2}^2}{4\Theta^*}+\frac{k\Theta^2}{4\Theta^*}$.

4 Analysis of the stability

Since $\tilde\Theta=\Theta^*-\hat\Theta$, inequality Eq. (33) can be rewritten as

$$\begin{aligned}\dot V_n\le&-\left(\xi_1^2+\cdots+\xi_n^2\right)+N_n+\frac{1}{2}\sigma\Theta^{*2}-\frac{1}{2}\sigma\tilde\Theta^2\\\le&-\varepsilon V_n+N\end{aligned} \tag{34}$$

where $\varepsilon=\min\{2,\sigma\Gamma\}$

$$N=N_n+\frac{1}{2}\sigma\Theta^{*2}=\frac{1}{2}\sigma\Theta^{*2}+\sum_{k=1}^{n}\frac{\tilde c_{k,1}^2}{4\Theta^*}+\frac{\tilde c_{k,2}^2}{4\Theta^*}+\frac{k\Theta^2}{4\Theta^*} \tag{35}$$

Thus, the Lyapunov function $V_n\left(\xi_1,\cdots,\xi_n,\tilde\Theta\right)$ of the system decreases monotonically until $\left(\xi_1,\cdots,\xi_n,\tilde\Theta\right)$ reaches the compact set

$$C_s=\left\{\left(\xi_1,\cdots,\xi_n,\tilde\Theta\right)\in R^n\times R:V_n\left(\xi_1,\cdots,\xi_n,\tilde\Theta\right)\le\varepsilon^{-1}N\right\} \tag{36}$$

This shows that $\xi_1,\cdots,\xi_n,\tilde\Theta$ are uniformly bounded. Thus, $\hat\Theta$ is bounded due to that Θ^* being a constant.

Since ξ_1 is bounded and $x_1=\xi_1$, x_1 is bounded. It can be seen from Eq. (7) that s is bounded. Equation (15) shows that x_2^* is bounded due to the boundedness of $\xi_1,\hat\Theta,x_1,s$. Since ξ_2 is bounded, $x_2=\xi_2+x_2^*$ is bounded. Repeating the above procedure leads to the conclusion that $x_1,\cdots,x_n$ are bounded.

It can be seen from Eqs. (35)–(36) that the size $\varepsilon^{-1}N$ of the compact set can be made arbitrarily small by choosing Γ and Θ^* (Θ^* is a virtual and desired value) appropriately large and σ appropriately small, i.e. $\xi_1,\cdots,\xi_n,\tilde\Theta$ can approach zero in arbitrary precision. By a procedure similar to the above and using Eqs. (15)–(27), it can be shown that

$x_1, \cdots, x_n$ approach the equilibrium in arbitrary precision.

Thus, the control scheme proposed in this paper guarantees the stability of the closed-loop system consisting of Eqs. (1), (2), (7) and (32), and the boundedness of all the signals in the closed-loop system. Furthermore, the states of the system can approach the equilibrium in arbitrary precision by choosing some design constants appropriately.

5 Simulation study

Consider nonlinearly parameterized system

$$\dot{x}_1 = x_2 + \sin(\theta_1 x_1) x_1^2 + \theta_2 \sin(x_1)\omega + \theta_3 x_1 d_1(t)$$

$$\dot{x}_2 = u + \theta_4 x_1 + \frac{\theta_5 x_2^2}{1+\theta_6 x_1^2} + \theta_7 (1-\cos(x_2))\omega + \theta_8 x_1 d_2(t)$$

where $\theta_i, i = 0, 1, \cdots, 8$, are unknown parameters, ω is the unmodeled dynamics described by $\dot{\omega} = -\omega + x_1^2 + 0.5$

Obviously, $V_\omega(\omega) = \omega^2$, $\alpha_1(|\omega|) = |\omega|^2$, $\alpha_1^{-1}(s) = \sqrt{s}$

We design the dynamic signal [6, 8] to be $\dot{s} = -s + x_1^4 + 0.625$.

With the proposed control scheme, $\gamma_1(x_1) = x_1^2$, $\gamma_2(x_1, x_2) = x_1^2 + x_2^2$.

$$\Psi_1(x_1, \hat{\Theta}, s) = x_1^2 \gamma_1^2(x_1) + x_1^2 \left(\alpha_1^{-1}(2s)\right)^2 + x_1^2 - \sigma\hat{\Theta}$$

$$x_2^* = -1.5 x_1 - \hat{\Theta} x_1 \left(\gamma_1^2(x_1) + \left(\alpha_1^{-1}(2s)\right)^2 + 1\right)$$

$$\Psi_2(\xi_1, \xi_2, \hat{\Theta}, s) = \Psi_1(\cdot) + \xi_2^2 \left(1 + \left(\frac{\partial x_2^*}{\partial x_1}\right)^2\right) \bullet \left(\left(\alpha_1^{-1}(2s)\right)^2 + 1\right) + \xi_2^2 \left(\gamma_2^2(\cdot) + \left(\frac{\partial x_2^*}{\partial x_1}\gamma_1(\cdot)\right)^2\right)$$

$$\dot{\hat{\Theta}} = \Gamma \Psi_2(\cdot)$$

$$u = x_3^* = -2\xi_2 + \frac{\partial x_2^*}{\partial x_1} x_2 + \Gamma \frac{\partial x_2^*}{\partial \hat{\Theta}} \Psi_2(\cdot) + \frac{\partial x_2^*}{\partial s}\dot{s} - \hat{\Theta}\xi_2 \left(1 + \left(\frac{\partial x_2^*}{\partial x_1}\right)^2\right)\left(\left(\alpha_1^{-1}(2s)\right)^2 + 1\right) - \hat{\Theta}\xi_2 \left(\gamma_2^2(\cdot) + \left(\frac{\partial x_2^*}{\partial x_1}\gamma_1(\cdot)\right)^2\right)$$

The simulation was performed using MATLAB with the following disturbances $d_1(t) = \sin(t)$, $d_2(t) = \cos(t)$ and the parameters $\theta_1 = 2$, $\theta_2 = 2$, $\theta_3 = 0.5$, $\theta_4 = 5$, $\theta_5 = 6$, $\theta_6 = 2$, $\theta_7 = 1$, $\theta_8 = 0.5$. The initial conditions are $x_1(0) = 1$, $x_2(0) = 1$, $\omega(0) = 1$, $s(0) = 1$, $\hat{\Theta}(0) = 1$. The design constants are $\Gamma = 0.5$, $\sigma = 0.001$.

It can be seen from the simulation results shown in Fig. 1 that with the control scheme proposed in this pape, the states of the system approach the equilibrium in arbitrary precision regardless of the presence of unknown parameters and nonlinear uncertainties, unknown disturbances and unmodeled dynamics. The simulation results demonstrate that the proposed robust adaptive controller has a strong robustness against the parametric and nonlinear uncertainties, unknown disturbances and unmodeled dynamics. The results coincide with the conclusions obtained from the theoretical analysis.

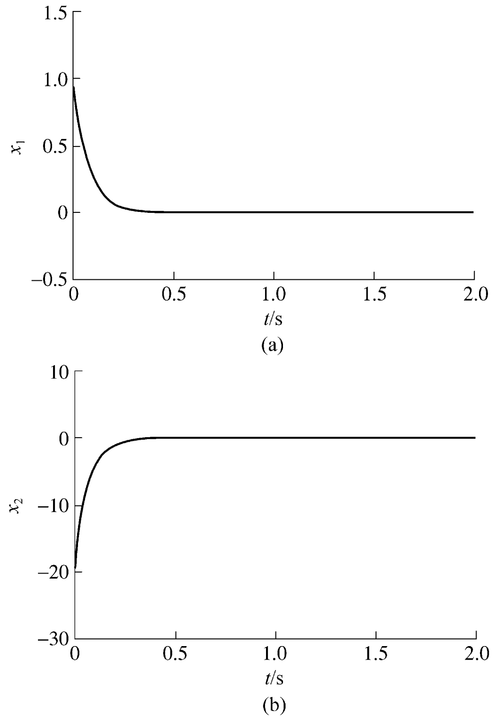

Fig. 1 States of the system approach the equilibrium in arbitrarily precise. (a) State x_1; (b) State x_2

6 Conclusions

A robust adaptive control scheme is proposed for an important class of nonlinearly parameterized systems. The control scheme can be used in the case of systems with unknown parameters, uncertain nonlinearities, disturbances and unmodeled dynamics. It is shown by the Lyapunov stability theory that the proposed robust adaptive control scheme guarantees the uniform boundedness of all the signals in the closed-loop system with unknown disturbances and unmodeled dynamics. Furthermore, the state of the system approaches the equilibrium in arbitrary precision by choosing some design constants appropriately. The scheme does not need to estimate the unknown parameters. No matter how high the order of the system is and how many unknown parameters the system has, there is only one adaptive parameter. Simulation results illustrate the conclusions obtained from the theoretical analysis. Since many physical systems such as biochemical processes, mechanical systems with frictions etc. are of nonlinearly parameterized systems, the proposed scheme will have widely potential uses in practice.

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