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RESEARCH ARTICLE

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Hybrid Method of combined difference and spectrum for time-domain maxwell's equations

Abstract For the finite-difference time domain (FDTD) method, the electromagnetic scattering problem, which requires the characteristic structure size to be much smaller than the wavelength of the exciting source, is still a challenge. To circumvent this difficulty, this paper presents a novel hybrid numerical technique of combined difference and spectrum for time-domain Maxwell's equations. With periodical continuation of each time-dependent quantity in Maxwell's equations, the solutions before and after the continuation remain consistent in the first period, which results in the conversion of the continuous spectrum problem to a discrete one. The discrete spectrum of the field after continuation is obtained from difference methods for Maxwell's curl equations in frequency-domain, and the time domain solution of the original problem is derived from their inverse Fourier transform. Due to its unconditional stability, the proposed scheme excels FDTD in resolving the aforementioned problems. In addition, this method can simulate dispersive media whose electric susceptibility cannot be expressed with Debye or Lorentz types of models. In dealing with boundary conditions, it can utilize the perfectly matched layer (PML) without extra codes. Numerical experiments demonstrate its effectiveness, easy implementation and high precision.

Keywords Difference-spectrum hybrid method, Periodical continuation, Unconditional stability, Dispersive media

1 Introduction

Yee's FDTD method has a wide application in the field of computational electromagnetics [1]. It is easy to understand, simple to implement and is highly efficient. Particularly, the calculation is performed in the time domain, so that one simulation can obtain wideband response. However, with the restriction of conditional stability, the scattering problem where the characteristic structure size is much smaller than the wavelength of the exciting source, such as those with slot and ventage, is still a challenge since the early days of FDTD. For the simulation of dispersive media, as far as we know, FDTD is limited to taking care of Debye, Lorentz and Derude types of dielectrics [2-4]. The more general dispersive media such as human body tissue is beyond the capability of FDTD. This is because of the high cost of calculating the convolution of electric fields and the permittivity in time domain with the classical FDTD method. Also, there are many dielectrics whose time domain forms about permittivity are yet unknown. To circumvent the two difficulties, this paper combines difference and spectrum techniques to construct a novel hybrid method for time domain Maxwell's equations. The time variable is dealt with using the spectrum scheme while the spatial variables use the difference technique. The proposed method is unconditional stable and capable of simulating any dispersive dielectric.

This paper is arranged as follows. Sect. 2 describes the difference-spectrum hybrid method in detail. In Sect. 3, we numerically validate the method. Sect. 4 concludes this paper.

2 Difference-spectrum hybrid method

Consider the following frequency-domain Maxwell's equation:

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$$\begin{cases} \mathrm{i}\omega\overset{=}{\varepsilon}(\boldsymbol{r},\omega)\boldsymbol{E}(\boldsymbol{r},\omega)=\nabla\times\boldsymbol{H}(\boldsymbol{r},\omega)-\boldsymbol{J}(\boldsymbol{r},\omega) \\ \mathrm{i}\omega\overset{=}{\mu}(\boldsymbol{r},\omega)\boldsymbol{H}(\boldsymbol{r},\omega)=-\nabla\times\boldsymbol{E}(\boldsymbol{r},\omega) \end{cases} \tag{1}$$

where $\overset{=}{\varepsilon}$ and $\overset{=}{\mu}$ are tensor permittivity and permeability respectively, and $\boldsymbol{J}$ is the exciting current source. Using Yee's leap-frog grids to cancel $\boldsymbol{H}$ and discretizing Eq. (1) with central finite difference leads to a linear system,

$$\boldsymbol{ME}=\boldsymbol{J} \tag{2}$$

where $\boldsymbol{E}=\{\boldsymbol{E}_x,\boldsymbol{E}_y,\boldsymbol{E}_z\}^{\mathrm{T}}$ and $\boldsymbol{J}=\{\boldsymbol{J}_x,\boldsymbol{J}_y,\boldsymbol{J}_z\}^{\mathrm{T}}$. The coefficient matrix $\boldsymbol{M}$ is a sparse one with only 13 elements nonzero each row. In addition, the corresponding element is identical at different points of the same electromagnetic properties, by which we can utilize special means to save computation cost.

After setting the exciting current source $\boldsymbol{J}$, one can solve Eq. (2) for the electric field response at frequency point ω. If the spectrum of the exciting source is discrete, then the frequency domain solution can be obtained by solving Eq. (2) at each frequency point. Its Fourier transform is the time domain solution. However, a lot of exciting sources last finite time, and their spectrum is band-limited. To get the time domain solution, one needs all the responses at every frequency point in the bandwidth of the exciting source. Apparently, this is impossible to realize at infinite frequency points. For now, let us consider the time-domain Maxwell's equation:

$$\begin{cases} \varepsilon_0\,\mathrm{E}(\boldsymbol{r},t)+\dfrac{\partial}{\partial t}\displaystyle\int_0^t\overset{=}{\chi}_e(\boldsymbol{r},\tau)\mathrm{E}(\boldsymbol{r},t-\tau)\,\mathrm{d}\tau=\nabla\times\mathrm{H}(\boldsymbol{r},t)-\boldsymbol{J}(\boldsymbol{r},t) \\ \mu_0\,\mathrm{H}(\boldsymbol{r},t)+\dfrac{\partial}{\partial t}\displaystyle\int_0^t\overset{=}{\chi}_m(\boldsymbol{r},\tau)\mathrm{H}(\boldsymbol{r},t-\tau)\,\mathrm{d}\tau=-\nabla\times\mathrm{E}(\boldsymbol{r},t) \end{cases} \tag{3}$$

where $\overset{=}{x}_e$ and $\overset{=}{x}_m$ are tensor electric and magnetic susceptibility respectively. Under the condition of finite time exciting, each field quantity approaches zero after a long period of time. If we select this time as the period T and periodically continue all the time-dependent quantity to $t\in(-\infty,+\infty)$, the solutions of Eq. (3) before and after this operation are identical in $t\in[0,T]$. Due to the convergence of the Fourier series, one can obtain the approximate solution by a few numbers of the spectrum. The number of discrete spectrum N depends on the selection of T, bandwidth of the exciting source, and precision requirement.

Suppose the discrete spectrum of a quantity at a point from Eq. (2) is

$$f\left(n\frac{2\pi}{\mathrm{T}}\right) n\in\{-N,-N+1,\cdots,N-1,N\} \tag{4}$$

Then the corresponding time domain solution in time $t\in[0,T]$ is

$$F(t)=\sum_{n=-N}^{N}f(2\pi n/T)\,\mathrm{e}^{\mathrm{i}n2\pi t/T} \tag{5}$$

The above scheme can be physically explained as one electromagnetic process that repeats infinitely. Also, Eq. (5) can be understood in another way. Before continuation, the spectrum of a field quantity is $f(\omega)$. With Fourier transform, its time domain form is

$$F(t)=\frac{\int_{-\infty}^{+\infty}f(\omega)\mathrm{e}^{\mathrm{i}\omega t}\mathrm{d}\omega}{2\pi} \tag{6}$$

Because the bandwidth of $f(t)$ is finite, the integration of Eq. (6) can be replaced with piecewise summation approximately. Choosing step length as $2\pi/T$, rewrite Eq. (6) as

$$F(t)=\sum_{n=-\Omega T/2\pi}^{\Omega T/2\pi}T^{-1}f(2\pi n/T)\,\mathrm{e}^{\mathrm{i}n2\pi t/T} \tag{7}$$

where

$$f(2\pi n/T)=\int_0^T F(t)\,\mathrm{e}^{-\mathrm{i}n2\pi t/T}\mathrm{d}t \tag{8}$$

When $\omega=0$, there exists a difficulty. As it has been assumed that any field quantities approach zero after a long period of time and the time domain form of zero-frequency spectrum is a constant *C*, one can conclude that:

$$F(T)\approx 0 \tag{9}$$

At last we obtain the time domain solution

$$\begin{aligned} F(t)=n=&\sum_{n=-N}^{-1}f(2\pi n/T)\,\mathrm{e}^{\mathrm{i}n2\pi t/T}+\sum_{n=1}^{N}f(2\pi n/T)\,\mathrm{e}^{\mathrm{i}n2\pi t/T} \\ &-\left(\sum_{n=-N}^{-1}f(2\pi n/T)+\sum_{n=1}^{N}f(2\pi n/T)\right) \end{aligned} \tag{10}$$

To find the solution, Eq. (2) is solved at discrete frequency points. There is no restriction on the form of $\overset{=}{\varepsilon}$ and $\overset{=}{\mu}$, so that the proposed technique is suitable for any linear dispersive medium, which benefits the setting up of perfectly matched layer (PML) boundary [5]. It needs no extra codes to implement UPML also [6]. Apparently, from the course of derivation, this scheme is unconditional stable.

3 Numerical validation

To validate the above technique, a few numerical experiments are performed. First an infinite dielectric plank irradiated vertically by a plane electromagnetic pulse is simulated for simplification without lost generalization. The thickness d of the plank is 40 mm. Profile of the exciting current source is $J(t)=(st)^2\mathrm{e}^{-st/2}(s=10^8s^{-1})$. Spatial step Δ equals 4 mm. The exciting source locates at 5 Δ apart from the left side of the plank while the check point is 10 Δ apart from the plank at the right side. Assume the plank is a Debye type dielectric with permittivity $\varepsilon=\varepsilon_s+(\varepsilon_s-\varepsilon_\infty)/(1+\mathrm{i}\omega\tau)$ $(\varepsilon_s=1.8,\varepsilon_\infty=81,\ \tau=9.4\times10^{-12}\mathrm{s})$. Because the upper limit frequency of the exciting source is about 100 MHz, the period of the continuation T is set to be 400 ns. The calculation results are shown in Fig. 1, which are compared to the simulation with FDTD. It can be seen that they have very good agreement. To illustrate the application of the proposed scheme to the generic dispersive medium, let $\varepsilon = \varepsilon_s + (\varepsilon_s - \varepsilon_\infty)/\left(1+(\mathrm{i}\omega\tau)^{1-a}\right)$ $(a = 0.8)$ and the other simulation parameters remain unchanged. Fig. 2 demonstrates the electric quantity of the check point as a function of time. For comparison, we also compute the responses at a number of frequency points with FDTD. Fig. 3 gives the frequency spectrum obtained with the two methods. Also, they are nearly identical. Here, Mur's absorption boundary condition is adopted rather than PML for simplification.

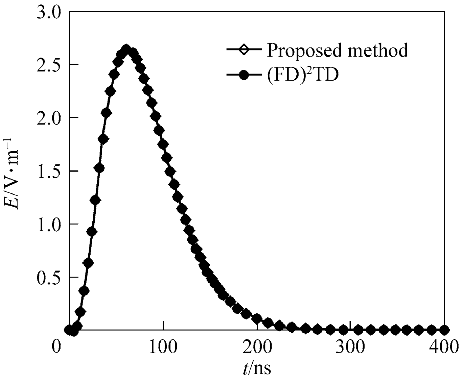

Fig. 1 The transmitted electric field changes with time

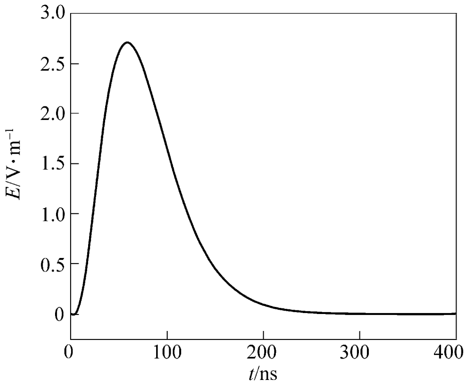

Fig. 2 Transmitted electric field as a function of time

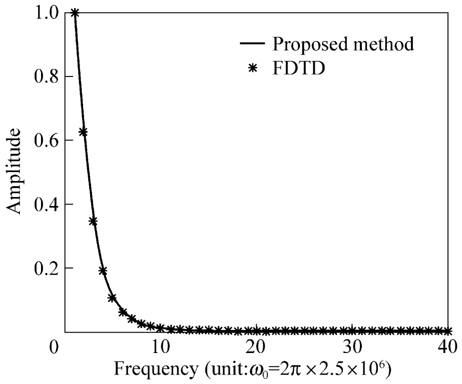

Fig. 3 Normalized spectra of the electric field in Fig. 2

4 Conclusions

In this paper we put forth a hybrid numerical technique for time-domain Maxwell's equations. The unconditional stable scheme can deal with generic dispersive medium. Its effectiveness is validated with numerical experiments.

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