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RESEARCH ARTICLE

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An integrated approach to modeling and adaptive control

Abstract In the book (Adaptive Identification, Prediction and Control—Multi Level Recursive Approach), the concept of dynamical linearization of nonlinear systems has been presented. This dynamical linearization is formal only, not a real linearization. From the linearization procedure, we can find a new approach of system identification, which is on-line real-time modeling and real-time feedback control correction. The modeling and real-time feedback control have been integrated in the identification approach, with the parameter adaptation model being abandoned. The structure adaptation of control systems has been achieved, which avoids the complex modeling steps. The objective of this paper is to introduce the approach of integrated modeling and control.

Keywords Real time identification, Feedback control, Structure adaptive

1 Introduction

In order to design a model-free controller, we discussed the dynamical linearization problem of a nonlinear system in Ref. [1]. In fact, this linearization is formal. In that article, we presented that on some conditions, the following nonlinear model

$$y(k) = f[Y_{k-1}^{k-n}, \boldsymbol{u}(k-1), U_{k-2}^{k-m}, k] \quad (1)$$

could be described in real-time by the following model combined with control law

$$y(k) - y(k-1) = \boldsymbol{\varphi}(k)^{\tau}[\boldsymbol{u}(k-1) - \boldsymbol{u}(k-2)] \quad (2)$$

where $y(k)$ is one-dimension system output, $u(k)$ is multi-dimension input, k is discrete time. And

$Y_{k-1}^{k-n} = \{y(k-1), \cdots, y(k-n)\}$, n is a positive integer.

$U_{k-2}^{k-m} = \{\boldsymbol{u}(k-2), \cdots, \boldsymbol{u}(k-m)\}$, m is a positive integer.

φ (k) is a function of $U_{k-2}^{k-m}, Y_{k-1}^{k-n}$. This is called character parameter or a pseudo gradient of Eq. (1)

This dynamical linearization is only formal rather than real. But why does this linearization play an important role in the model-free control law? We found through this study that the process called dynamical linearization contains a new idea of modeling.

Generally speaking, the mathematical model of a dynamical system needs to be constituted in the process of designing a control law. The mathematical model must first be constituted in the classical method, or at least its structure must be determined first. The more accurate the model is, the better it is. In the design of a model-free controller, this restriction was removed, in which a more accurate mathematical model must be built beforehand in designing a control law.

In this case the modeling process is followed with feedback control. The initial model may be inaccurate, while convergence of the controller used must be held. The model-free controller is such a kind of controller, which models while controlling, then obtains new data, and goes back to modeling while controlling again. This process is continued and the model that we obtain from this process tends to be more precise. The property of control law is hence improved. We call this procedure the integrated approach of modeling and adaptive control.

Here, the traditional way of adaptive control law was abandoned. The traditional way is to achieve an adaptive aim by designing a control law according to a model initially built for the controlled object, and identifying the model or controlling parameter on-line. This is traditional adaptation. The disadvantage of this kind of adaptation is that it is a parameter adaptation only, because the basic structure (e.g., the order) of the model has been determined off-line at first. It is difficult to realize the structure adaptation.

The traditional way of adaptive control law appears to have more drawbacks when applied in complex systems. Because the model structure (e.g., the order) has been determined off-line, it is difficult to satisfy the requirements of some differences in system structure.

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On the other hand, if the modeling is to design a controlling law for a controlled system that is complex, it is currently very difficult to model this system. This should be avoided to the utmost extent in designing a control law.

Real-time identification and real-time feedback correction was presented in the paper. It combines the identification and the design of a control law. It can avoid a series of difficulties mentioned above. The control law structure is adaptive.

2 Universal model and character parameter

The following universal model has been presented in Refs. [1-2],

$$y(k)-y(k-1)=\boldsymbol{\varphi}(k-1)^{\tau}[\boldsymbol{u}(k-1)-\boldsymbol{u}(k-2)] \quad (3)$$

Without losing generality, suppose that the time-delay of controlled system is 1, $y(k)$ is a one-dimension system output, $\boldsymbol{u}(k-1)$ is a p-dimension input vector, $\varphi(k-1)$ is character parameter, which can be on-line estimated using some identification method, and k is discrete time. It will show that: in the integrated approach of real-time identification, real feedback correction $\varphi(k)$, has clear mathematics and engineering significance.

Suppose a dynamical system can be denoted in the following form:

$$y(k)=f[Y_{k-1}^{k-n},\boldsymbol{u}(k-1),U_{k-2}^{k-m},k] \quad (4)$$

where $Y_{k-1}^{k-n}, U_{k=2}^{k-m}$ is the same with the above meaning. Assume that $f[Y_{k-1}^{k-n}\boldsymbol{u}(k-1),U_{k-2}^{k-m},k]$ has a continuous gradient with respect to $\boldsymbol{u}(k-1)$. When the system S is at a steady state, assume it satisfies the condition that if $\boldsymbol{u}(k-1)=\boldsymbol{u}(k-2)$, then $y(k)=y(k-1)$ (in random case, $E\{y(k)\}=E\{y(k-1)\}$).

Hence,

$$y(k)-y(k-1)=f[Y_{k-1}^{k-n},\boldsymbol{u}(k-1),U_{k-2}^{k-m},k]$$
$$-f[Y_{k-2}^{k-n-1},\boldsymbol{u}(k-2),U_{k-3}^{k-m-1},k-1]\;=f[Y_{k-1}^{k-n},\boldsymbol{u}(k-1),U_{k-2}^{k-m},k]$$
$$-f[[Y_{k-1}^{k-n},\boldsymbol{u}(k-2),U_{k-2}^{k-m},k]+f[[Y_{k-1}^{k-n},\boldsymbol{u}(k-2),U_{k-2}^{k-m},k]$$
$$-f[Y_{k-2}^{k-n-1},\boldsymbol{u}(k-2),U_{k-3}^{k-m-1},k-1]$$

Let

$$f[Y_{k-1}^{k-n},\boldsymbol{u}(k-2),U_{k-2}^{k-m},k]-f[Y_{k-2}^{k-n-1},\boldsymbol{u}(k-2),U_{k-3}^{k-m-1},k-1]=\xi(k)$$

Using the mean value theorem in the calculus, we obtain:

$$f[Y_{k-1}^{k-n},\boldsymbol{u}(k-1),U_{k-2}^{k-m},k]-f[Y_{k-1}^{k-m},\boldsymbol{u}(k-2),U_{k-2}^{k-m},k]$$
$$=\nabla_{u(k-2)}f[Y_{k-1}^{k-n},\overline{\boldsymbol{u}(k-2)},U_{k-2}^{k-m},k]^{\tau}[\boldsymbol{u}(k-1)-\boldsymbol{u}(k-2)]$$

where $\overline{\boldsymbol{u}(k-2)}=\boldsymbol{u}(k-2)+\theta(\boldsymbol{u}(k-1)-\boldsymbol{u}(k-2)),\theta$ satisfying $0\le\theta\le1$ Therefore we have:

$$y(k-y(k-1))=\nabla_{\boldsymbol{u}(k-2)}f[\overline{\boldsymbol{u}(k-2)},k]^{\tau}$$
$$\bullet[\boldsymbol{u}(k-1)-\boldsymbol{u}(k-2)]+\xi(k) \quad (5)$$

where $\nabla_{\boldsymbol{u}(k-2)}f[\overline{\boldsymbol{u}(k-2)},k]=\nabla_{\boldsymbol{u}(k-2)}f[Y_{k-1}^{k-n},\overline{\boldsymbol{u}(k-2)},U_{k-2}^{k-m},k]$

Definition 1 If from conditions $Y_{k-1}^{k-n}=Y_{k-2}^{k-n-1}, U_{k-2}^{k-m}=U_{k-3}^{k-m-1}$, $\boldsymbol{u}(k-1)=\boldsymbol{u}(k-2)$, it can be obtained that :

$$f[Y_{k-1}^{k-n},\boldsymbol{u}(k-1),U_{k-2}^{k-m},k]=f[Y_{k-2}^{k-n-1},\boldsymbol{u}(k-2),U_{k-2}^{k-m},k-1]$$

then system (4) is an auto time-invariant system..

Obviously, if system (4) is auto time-invariant and the function $f[y_{k-1}^{k-n},\boldsymbol{u}(k-1),U_{k-2}^{k-m},k]$ is continuous with respect to $Y_{k-1}^{k-n},\boldsymbol{u}(k-1),U_{k-2}^{k-m}$, we certainly have

$$\lim_{\substack{u(k-1)\to u(k-2)\\ Y_{k-1}^{k-n}\to Y_{k-2}^{k-n-1}\\ U_{k-1}^{k-m}\to U_{k-3}^{k-m-1}}} f[Y_{k-1}^{k-n},\boldsymbol{u}(k-1),U_{k-2}^{k-m},k]$$
$$=f[Y_{k-2}^{k-n-1},\boldsymbol{u}(k-2),U_{k-3}^{k-m-1},k-1]$$

Especially, we have

$$\lim_{\substack{Y_{k-1}^{k-n}\to Y_{k-2}^{k-n-1}\\ U_{k-1}^{k-m}\to U_{k-3}^{k-m-1}}}\xi(k)=0$$

Furthermore, if we let $\psi(k)=\nabla_{\boldsymbol{u}(k-2)}f[\overline{\boldsymbol{u}(k-2)},k]$, then Eq. (5) can be written as:

$$y(k)-y(k-1)=\psi(k)^{\tau}[\boldsymbol{u}(k-1)-\boldsymbol{u}(k-2)]+\xi(k) \quad 6)$$

If $\|\boldsymbol{u}(k-1)-\boldsymbol{u}(k-2)\|\neq0$, let

$$\boldsymbol{\varphi}(k-1)=\psi(k)+\frac{\boldsymbol{u}(k-1)-\boldsymbol{u}(k-2)}{\|\boldsymbol{u}(k-1)-\boldsymbol{u}(k-2)\|^{2}}\xi(k)$$

then Eq. (6) can be rewritten as :

$$y(k)-y(k-1)=\boldsymbol{\varphi}(k-1)^{\tau}[\boldsymbol{u}(k-1)-\boldsymbol{u}(k-2)] \quad (7)$$

Note: When the system is in a steady state, when $\|\boldsymbol{u}(k-1)-\boldsymbol{u}(k-2)\|=0$, we have $y(k)=y(k-1)$, in this case, Eq. (7) can be considered naturally valid.

Eq. (7) is the universal model, which has been presented. $\boldsymbol{\varphi}(k-1)$ is the character parameter vector.

3 Integration of real time modeling and feedback controlling

It is clear that the necessary condition for the universal model:

$$y(k)-y(k-1)=\boldsymbol{\varphi}(k-1)^{\tau}[\boldsymbol{u}(k-1)-\boldsymbol{u}(k-2)] \quad (8)$$

to be used in practice is that the estimate value $\hat{\boldsymbol{\varphi}}(k-1)$ of $\boldsymbol{\varphi}(k-1)$ can be obtained in real-time with enough accuracy. As for estimating $\boldsymbol{\varphi}(k-1)$, there are several ways. For example:

1) Recursive least square

Let

$$z(k)=y(k)-y(k-1)$$
$$\boldsymbol{\phi}(k)=\boldsymbol{u}(k-1)-\boldsymbol{u}(k-2)$$

the Eq. (8) becomes

$$z(k)=\boldsymbol{\phi}(k)^{\tau}\boldsymbol{\varphi}(k-1) \quad (9)$$

when $y(k),\boldsymbol{u}(k-1)$ are observed in real time, $z(k)$ and

$\phi(k)$ can be obtained too. The estimate value $\hat{\varphi}(k-1)$ of $\varphi(k-1)$ can thus be derived from the following recursive algorithm:

$$\hat{\varphi}(k-1)=\hat{\varphi}(k-2)+M(k)\{z(k)-\phi(k)^{\tau}\hat{\varphi}(k-2)\}$$

$$M(k)=\frac{P(k-1)\phi(k)}{\delta_k+\phi(k)^{\tau}P(k-1)\phi(k)}$$

$$P(k)=\frac{1}{\delta_k}\{I-M(k)\phi(k)^{\tau}]P(k-1) \qquad (10)$$

where δ_k is forget factor, I is suitable dimension unit matrix.

2) Recursive gradient algorithm [1]:

For Eq. (9), the estimate value $\hat{\varphi}(k-1)$ of $\varphi(k-1)$ can be derived as follows:

$$\hat{\varphi}(k-1)=\hat{\varphi}(k-2)+\frac{\delta}{\eta k+\|\phi(k)\|^2}\phi(k)\bullet$$

$$\{z(k)-\phi(k)^{\tau}\hat{\varphi}(k-2)\} \qquad (11)$$

where η_k is an applicable small positive number, δ is an applicable constant .

The design problem of control law could be described as follows:

The observed data $\{\boldsymbol{u}(k-1), y(k)\}$ of system S are known, and the expected output $y_0(k+1)$ of time $k+1$ is given. Find a controller $\boldsymbol{u}(k)$, by the action of which, the output of system S, $y(k+1)$, equals $y_0(k+1)$ .If the system S can be described as the following universal model:

$$y(k+1)-y(k)=\varphi(k)^{\tau}[\boldsymbol{u}(k)-\boldsymbol{u}(k-1)] \qquad (12a)$$

and suppose $\hat{\varphi}(k)$ is known, which is the best estimate of $\varphi(k)$, then Eq. (12a) can be substituted by the following approximate formula:

$$y(k+1)-y(k)=\hat{\varphi}(k)^{\tau}[\boldsymbol{u}(k)-\boldsymbol{u}((k-1)] \qquad (12b)$$

In control law designing, the left of Eq. (12) becomes $y_0(k+1)-y(k)$. Eq. (12b) thus becomes the following indefinite equation:

$$y_0(k+1)-y(k)=\hat{\varphi}(k)^{\tau}[\boldsymbol{u}(k)-\boldsymbol{u}(k-1)]$$

whose unknown variable is $\boldsymbol{u}(k)$. It can easily be known that the equation has a special solution presented as follows:

$$\boldsymbol{u}(k)=\boldsymbol{u}(k-1)+\frac{1}{\|\hat{\varphi}(k)\|^2}\hat{\varphi}(k)\{y_0(k+1)-y(k)\} \qquad (13)$$

Equation (13) can be regarded as the control law of system S. However, when $\|\hat{\varphi}(k)\|^2=0$, Eq. (13) becomes singular which is not allowed in practice. In order to avoid the denominator being zero, we substitute $\|\hat{\varphi}(k)\|^2$ by $a+\|\hat{\varphi}(k)\|^2$

$$\boldsymbol{u}(k)=\boldsymbol{u}(k-1)+\frac{\lambda_k}{a+\|\hat{\varphi}(k)\|^2}\hat{\varphi}(k)\{y_0(k+1)-y(k)\}$$

where a is an applicable small positive number, in case $y_0(k+1)=y_0$, y_0 is a constant, the above formula becomes

$$\boldsymbol{u}(k)=\boldsymbol{u}(k-1)+\frac{\lambda_k}{a+\|\hat{\varphi}(k)\|^2}\hat{\varphi}(k)\{y_0-y(k)\} \qquad (14)$$

λ_k is the control parameter, whose selection is closely related to the convergence of control law. Eq. (14) is the basic form of a model-free control law.

Thus far, we have obtained the framework for the integrated approach of modeling and feedback control as follows:

1) According to the observe data and universal model:

$$y(k)-y(k-1)=\varphi(k-1)^{\tau}\{\boldsymbol{u}(k-1)-\boldsymbol{u}(k-2)\}$$

Using suitable estimate method, i.e., 1 or 2, we obtain the estimated value $\hat{\varphi}(k-1)$ of $\varphi(k-1)$;

2) To find the one-step-forward estimation $\hat{\varphi}^*(k)$ of $\hat{\varphi}(k-1)$, a simple way is to let $\hat{\varphi}^*(k)=\hat{\varphi}(k-1)$. In control law design, we still denote $\hat{\varphi}^*(k)$ by $\hat{\varphi}(k)$

3) Let the control law

$$\boldsymbol{u}(k)=\boldsymbol{u}(k-1)+\frac{\lambda_k}{a+\|\hat{\varphi}(k)\|^2}\hat{\varphi}(k)\{y_0-y(k)\}$$

act on system S. A new output $y(k+1)$ can be obtained, thus forming a group of new data $\{y(k+1), \boldsymbol{u}(k)\}$ subsequently.

Steps 1, 2 and 3 are repeated based on these new data, introducing new data $\{y(k+2), \boldsymbol{u}(k+1)\}$. Repeat these steps continuously. The following discussion will show that, if only system S satisfies some conditions, the above scheme will lead the output of system S, $y(k)$, to converge to y_0 asymptotically.

4 Analysis on convergence of the control law

The necessary condition that the integrated approach of real-time modeling and feedback control can be used is that both the estimation algorithm of $\varphi(k)$ and control algorithm Eq. (14) can be converged. The convergence of estimation algorithm has been analyzed in a lot of studies. Here, we will focus on analyzing the convergence of control law Eq. (14).

The model-free control law is the result of the combination and interactively carrying out on-line of identification and control law. The starting point of analyzing the convergence of control law should be both the universal model

$$y(k+1)-y(k)=\varphi(k)^{\tau}[\boldsymbol{u}(k)-\boldsymbol{u}(k-1)] \qquad (15)$$

and control law

$$\boldsymbol{u}(k)=\boldsymbol{u}(k-1)+\frac{\lambda_k}{a+\|\hat{\varphi}(k)\|^2}\hat{\varphi}(k)\{y_0-y(k)\} \qquad (16)$$

Introducing Eq. (16) to Eq. (15) yields;

$$y(k+1)-y(k)=\lambda_k \frac{\varphi(k)^\tau \hat{\varphi}(k)}{a+\|\hat{\varphi}(k)\|^2}\{y_0-y(k)\} \quad (17)$$

where $y(k+1)$ is the actual output of system in respect to $\boldsymbol{u}(k)$. From Eq. (17), it can be determined that:

$$\begin{aligned} & y_0-y(k+1) \\ & = y_0-y(k)-\lambda k \frac{\varphi(k)^\tau \hat{\varphi}(k)}{a+\|\hat{\varphi}(k)\|^2}\{y_0-y(k)\} \\ & = \left(1-\lambda k \frac{\varphi(k)^\tau \hat{\varphi}(k)}{a+\|\hat{\varphi}(k)\|^2}\right)(y_0-y(k)) \end{aligned}$$

Let $\Delta_k = \frac{\varphi(k)^\tau \hat{\varphi}(k)}{a+\|\hat{\varphi}(k)\|^2}$

then the above formula becomes

$$y_0-y(k+1)=(1-\lambda_k \Delta_k)(y_0-y(k)\}$$

Based on the universal model, use identification again to get the estimation $\hat{\varphi}(k+1)$ of $\varphi(k+1)$. Let:

$$\begin{aligned} \boldsymbol{u}(k+1)=\boldsymbol{u}(k)+ & \frac{\lambda_{k+1}}{a+\|\hat{\varphi}(k+1)\|^2} \\ & \bullet \hat{\varphi}(k+1)\{y_0-y(k+1)\} \end{aligned}$$

then get the actual output of system $y(k+2)$. Thus far it can be obtained:

$$\begin{aligned} & y_0-y(k+2) \\ & =(1-\lambda_{k+1}\Delta_{k+1})(y_0-y(k+1)) \\ & =(1-\lambda_{k+1}\Delta_{k+1})(1-\lambda_k \Delta_k)(y_0-y(k)) \end{aligned}$$

Repeat these steps. Then get:

$$y_0-y(k+h)=\prod_{j=0}^{h-1}(1-\lambda_{k+j}\Delta_{k+j})(y_0-y(k)) \quad (18)$$

This formula is the basis of convergence analyzing. One direct result is:

Theorem 1 *Suppose that*

1) $0 \le \lambda_{k=h}\Delta_{k+h} \le 1 \qquad \forall\ h \ge 0$

2) $\sum_{h=0}^{\infty} \lambda_{k+h}\Delta_{k+h} = \infty$

then:

$$\lim_{h\to\infty} y(k+h)=y_0$$

The following theorem shows that if the difference between $\varphi(k)$ *and* $\hat{\varphi}(k)$ *satisfies some condition and* $\|\hat{\varphi}(k)\|$ *is bounded, the convergence of control law can be assured.*

Theorem 2 *Suppose that the relation between* $\varphi(k)$ *and its estimation* $\hat{\varphi}(k)$ *satisfies:*

$$\varphi(k)-\hat{\varphi}(k)=\varepsilon(k)$$

$$-\frac{a}{2} \le \varepsilon(k)^T \hat{\varphi}(k) \le \frac{a}{2}$$

and

$$a \le \|\hat{\varphi}(k)\|^2 \le b$$

then there exists suitable number $\lambda_k = \lambda$ *, that*:

$$\lim_{h\to\infty} y(k+h)=y_0$$

Proof Notice that

$$\Delta_k = \frac{\varphi(k)^\tau \hat{\varphi}(k)}{a+\|\hat{\varphi}(k)\|^2} = \frac{\|\hat{\varphi}(k)\|^2+\varepsilon(k)\hat{\varphi}(k)}{a+\|\hat{\varphi}(k)\|^2}$$

so

$$0 \le \frac{\|\hat{\varphi}(k)\|^2-\frac{a}{2}}{\|\hat{\varphi}(k)\|^2+a} \le \Delta_k \le \frac{\|\hat{\varphi}(k)\|^2+\frac{a}{2}}{\|\hat{\varphi}(k)\|^2+a} \le 1$$

from Eq. (18), it can be derived that:

$$|y_0-y(k+h)|=\prod_{j=0}^{h-1}|1-\lambda_{k+j}\Delta_{k+j}||y_0-y(k)|$$

provided $\lambda_{k+j} \ge 0$,we have

$$\begin{aligned} & |y_0-y(k+h)| \\ & \le |y_0-y(k)|\prod_{j=0}^{h-1}\left|1-\lambda_{k+j}\frac{\|\hat{\varphi}(k)\|^2-\frac{a}{2}}{\|\hat{\varphi}(k)\|^2+a}\right| \\ & \le |y_0-y(k)|\prod_{j=0}^{h-1}\left|1-\lambda_{k+j}\frac{a-\frac{a}{2}}{b+a}\right| \end{aligned}$$

Therefore, once we let $\lambda_{k+j} = b+a/a$, it comes as

$$\begin{aligned} |y_0-y(k+h)| & \le |y_0-y(k)|\left(1-\frac{1}{2}\right)^h \\ & =|y_0-y(k)|\left(\frac{1}{2}\right)^h \to 0 \quad (h\to\infty) \end{aligned}$$

i.e. $\lim_{h\to\infty} y(k+h)=y_0$

If the system is single-input and single-output, the convergence can be obtained if $\varphi(k)$ and $\hat{\varphi}(k)$ satisfy certain conditions of bound.

Theorem 3 *If there exist constants* $\delta > 0$ *and* $\beta > 0$ *that satisfy*:

$$\delta \le \varphi(k) \le \beta$$

$$\delta \le \hat{\varphi}(k) \le \beta$$

then there exists a suitable number $\lambda_k = \lambda$,which makes:

$$\lim_{h\to\infty} y(k+h) = y_0$$

Proof Notice

$$\Delta_k = \frac{\varphi(k)^\tau \hat{\varphi}(k)}{a + \|\hat{\varphi}(k)\|^2}$$

then

$$\frac{\delta^2}{a+\beta^2} \le \Delta_k \le \frac{\beta^2}{a+\delta^2}$$

Provided $\lambda_{k+j} > 0$, from Eq. (18), it can be derived that:

$$|y_0 - y(k+h)| \le |y_0 - y(k)| \prod_{j=0}^{h-1} \left| 1 - \lambda_{k+j} \frac{\delta^2}{a+\beta^2} \right|$$

therefore, when

$$\lambda_{k+j} = \frac{a+\beta^2}{2\delta^2}$$

we have

$$|y_0 - y(k+h)| \le |y_0 - y(k)| \left(\frac{1}{2}\right)^h \to 0 \ \ (h \to \infty)$$

i.e. $\lim_{h\to\infty} y(k+h) = y_0$

5 The convergence of control variable and analysis of character parameter

In the integrated approach of modeling and feedback control, when the system satisfies some conditions, the convergence of control law has been proved. Here, it shows that the control variable $\boldsymbol{u}(k)$ is convergent too, under the conditions of Theorem 2 and Theorem 3.

In fact, the following theorem exists:

Theorem 4 *If the conditions of Theorem 2 can be satisfied, then it can be determined that*:

1) $\|\boldsymbol{u}(k+h) - \boldsymbol{u}(k+h-1)\| \le C \frac{(a+b)\sqrt{b}}{a} \left(\frac{1}{2}\right)^h$, *where* C *is a constant independent of* h .

2) $\sum_{h=1}^{\infty} \|\boldsymbol{u}(k+h) - \boldsymbol{u}(k+h-1)\| < +\infty$

3) There exists $\boldsymbol{u}_0$ such that

$$\lim_{h\to\infty} \boldsymbol{u}(k+h) = \boldsymbol{u}_0$$

Proof For 1), since

$$\boldsymbol{u}(k+h) = \boldsymbol{u}(k+h-1) + \frac{\lambda_{k+h}}{a + \|\hat{\varphi}(k+h)\|^2} \bullet \hat{\varphi}(k+h)\left[y_0 - y(k+h)\right]$$

then

$$\|\boldsymbol{u}(k+h) - \boldsymbol{u}(k+h-1)\| = \frac{|\lambda_{k+h}|}{a + \|\hat{\varphi}(k+h)\|^2} \|\hat{\varphi}(k+h)\| \ |y_0 - y(k+h)|$$

It has been obtained in the proof of Theorem 2 (under the condition $\lambda_{k+j} \ge 0$) that:

$$|y_0 - y(k+h)| \le |y_0 - y(k)| \prod_{j=0}^{h-1} \left(1 - \lambda_{k+j} \frac{a}{2(a+b)}\right)$$

Let $\lambda_{k+j} = (a+b)/a$, then:

$$|y_0 - y(k+h)| \le |y_0 - y(k)| \left(\frac{1}{2}\right)^h$$

hence

$$\begin{aligned} &\|\boldsymbol{u}(k+h) - \boldsymbol{u}(k+h-1)\| \\ &\le \frac{(a+b)\|\hat{\varphi}(k+h)\|}{a\left(a + \|\hat{\varphi}(k+h)\|^2\right)} |y_0 - y(k)| \ \left(\frac{1}{2}\right)^h \\ &\le \frac{(a+b)\sqrt{b}}{2a^2} |y_0 - y(k)| \ \left(\frac{1}{2}\right)^h = C\frac{(a+b)\sqrt{b}}{2a^2} \left(\frac{1}{2}\right)^h \end{aligned}$$

where $C = |y_0 - y(k)|$

For 2), notice that

$$\|\boldsymbol{u}(k+h) - \boldsymbol{u}(k+h-1)\| \le C\frac{(a+b)\sqrt{b}}{2a^2} \left(\frac{1}{2}\right)^h$$

we have:

$$\begin{aligned} &\sum_{h=1}^{\infty} \|\boldsymbol{u}(k+h) - \boldsymbol{u}(k+h-1)\| \\ &= C\frac{(a+b)\sqrt{b}}{a} \sum_{h=0}^{\infty} \left(\frac{1}{2}\right)^h \le 2C\frac{(a+b)\sqrt{b}}{a} < +\infty \end{aligned}$$

For 3), for any *n*>*m*, it comes that:

$$\begin{aligned} &\|\boldsymbol{u}(k+n) - \boldsymbol{u}(k+m)\| \\ &= \|\boldsymbol{u}(k+n) - \boldsymbol{u}(k+n-1) + \boldsymbol{u}(k+n-1) + \cdots \\ &+ \boldsymbol{u}(k+m+1) - \boldsymbol{u}(k+m)\| \le \|\boldsymbol{u}(k+n) - \boldsymbol{u}(k+n-1)\| \\ &+ \|\boldsymbol{u}(k+n-1) - \boldsymbol{u}(k+m-2)\| + \cdots + \|\boldsymbol{u}(k+m+1) - \boldsymbol{u}(k+m)\| \\ &= \sum_{h=m}^{n-1} \|\boldsymbol{u}(k+h+1) - \boldsymbol{u}(k+h)\| \end{aligned}$$

since

$$\lim_{\substack{m\to\infty \\ n\to\infty}} \sum_{h=m}^{n-1} \ \|\boldsymbol{u}(k+h-1) - \boldsymbol{u}(k+h)\| = 0$$

$\{\boldsymbol{u}(k+h)\}$ should be a Cauchy sequence. There must exist an $\boldsymbol{u}_0$ satisfying:

$$\lim_{h\to\infty} \boldsymbol{u}(k+h) = \boldsymbol{u}_0$$

Theorem 5 *Under the condition of Theorem 3, we have* :

1) $\|\boldsymbol{u}(k+h)-\boldsymbol{u}(k+h-1)\| \le C\dfrac{(a+\beta^2)\beta}{2(a+\delta^2)\delta^2}\left(\dfrac{1}{2}\right)^h$

2) $\sum_{h=1}^{\infty} \|\boldsymbol{u}(k+h)-\boldsymbol{u}(k+h-1)\| < t\infty$

3) *There exists* $\boldsymbol{u}_0$ *that* $\lim_{h\to\infty}\boldsymbol{u}(k+h)=\boldsymbol{u}_0$

Proof It is completely similar with Theorem 4.

Thus far, it has shown that, in the process of the integrated approach of modeling and feedback control, when some requirements are met, it turns out that:

$$\lim_{h\to\infty} y(k+h) = y_0$$
$$\lim_{h\to\infty} \boldsymbol{u}(k+h) = u_0$$

then the following conclusion can be derived:

Conclusion The control law produced by the integrated approach of modeling and feedback control can be used, and the approach is reasonable.

The meaning of character parameter will be analyzed next, for which reason we notice that:

$$\lim_{\substack{\boldsymbol{u}(k-1)\to\boldsymbol{u}(k-2)\\ Y_{k-1}^{k-n}\to Y_{k-2}^{k-n-1}\\ U_{k-2}^{k-m}\to U_{k-3}^{k-m-1}}} \varphi(k-1)$$
$$= \lim_{\substack{\boldsymbol{u}(k-1)\to\boldsymbol{u}(k-2)\\ Y_{k-1}^{k-n-1}\to Y_{k-2}^{k-n}\\ U_{k-2}^{k-m}\to U_{k-3}^{k-m-1}}} (\psi(k)+\frac{\boldsymbol{u}(k-1)-\boldsymbol{u}(k-2)}{\|\boldsymbol{u}(k-1)-\boldsymbol{u}(k-2)\|^2}\xi(k))$$

In some special cases, it can be obtained that

$$\lim_{\substack{\boldsymbol{u}(k-1)\to\boldsymbol{u}(k-2)\\ Y_{k-1}^{k-n}\to Y_{k-2}^{k-m-1}\\ U_{k-2}^{k-m}\to U_{k-3}^{k-m-1}}} \frac{\boldsymbol{u}(k-1)-\boldsymbol{u}(k-2)}{\|\boldsymbol{u}(k-1)-\boldsymbol{u}(k-2)\|^2}\xi(k)=0$$

For example, consider the system:

$$y(k)=a\exp^{bu(k-1)}$$

we have

$$f[Y_{k-1}^{k-n}, \boldsymbol{u}(k-2), U_{k-2}^{k-m}, k] = a\exp^{bu(k-2)}$$

and

$$f[Y_{k-2}^{k-n-1}, \boldsymbol{u}(k-2), U_{k-3}^{k-m-1}, k] = a\exp^{bu(k-2)}$$

it can be deduced that

$$\xi(k) = f[Y_{k-1}^{k-n}, \boldsymbol{u}(k-2), U_{k-2}^{k-m}, k] - f[Y_{k-2}^{k-n-1}, \boldsymbol{u}(k-2), U_{k-3}^{k-m-1}, k-1]=0$$

In this special case, it certainly turns to that

$$\lim_{\substack{\boldsymbol{u}(k-1)\to\boldsymbol{u}(k-2)\\ Y_{k-1}^{k-n}\to Y_{k-2}^{k-m-1}\\ U_{k-2}^{k-m}\to U_{k-3}^{k-m-1}}} \frac{\boldsymbol{u}(k-1)-\boldsymbol{u}(k-2)}{\|\boldsymbol{u}(k-1)-\boldsymbol{u}(k-2)\|^2}\xi(k)=0$$

Let

$$\nabla_{\boldsymbol{u}(k-2)} f[\boldsymbol{u}(k-2),k] = \nabla_{\boldsymbol{u}(k-2)} f[Y_{k-2}^{k-n-1}, \boldsymbol{u}(k-2), U_{k-3}^{k-m-1}, k]$$

Obviously

$$\nabla_{\boldsymbol{u}(k-2)} f[\boldsymbol{u}(k-2),k] = ab\exp^{bu(k-2)}$$

From $\boldsymbol{u}(k-1)\to\boldsymbol{u}(k-2)$, it can be obtained that

$$\overline{\boldsymbol{u}(k-2)} \to \boldsymbol{u}(k-2)$$

So we have

$$\lim_{\substack{\boldsymbol{u}(k-1)\to\boldsymbol{u}(k-2)\\ Y_{k-1}^{k-n}\to Y_{k-2}^{k-n-1}\\ U_{k-2}^{k-m}\to U_{k-3}^{k-m-1}}} \varphi(k-1) = \lim_{\substack{\boldsymbol{u}(k-1)\to\boldsymbol{u}(k-2)\\ Y_{k-1}^{k-n}\to Y_{k-2}^{k-n-1}\\ U_{k-2}^{k-m}\to U_{k-3}^{k-m-1}}} \psi(k) = \nabla_{\boldsymbol{u}(k-2)} f[\boldsymbol{u}(k-2),k]$$

The above discussion presented that character parameter $\varphi(k-1)$ of the universal model has close relationship with the gradient $\nabla_{\boldsymbol{u}(k-2)} f\ [\boldsymbol{u}(k-2),k]$. Accordingly, the character parameter $\varphi(k-1)$ can be called the pseudo- gradient.

6 Conclusions

From the above analyses, it shows that the integrated approach of modeling and feedback control is practicable. Therefore, when considering the modeling problem of control system design, the classical "modeling then designing" method can be deserted. Provided that we start from the universal model, the convergence of the controller can be assured under some conditions. Reversely, this can assure the universal model of gradually becoming accurate, on which designing a controller depends. The model-free controller is designed in this way, with many successful applications in practice [7] showing the rationality of this approach.

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