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DR-model-based estimation algorithm for NCS

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Abstract A novel estimation scheme based on dead reckoning (DR) model for networked control system (NCS) is proposed in this paper. Both the detailed DR estimation algorithm and the stability analysis of the system are given. By using DR estimation of the state, the effect of communication delays is overcome. This makes a controller designed without considering delays still applicable in NCS. Moreover, the scheme can effectively solve the problem of data packet loss or timeout.

Keywords Networked control system (NCS), Time-delay, Time synchronization, Dead reckoning (DR)

1 Introduction

Networked control system (NCS) is applied more and more widely with the scale extension of the control system and the development of computers and networks. However, communication delay is inevitable in NCS and the delay is time-variant, because the communication band is limited and transferring data requires time—waiting time and retransfer time. The delay brings many problems to the control, which means the existing control method cannot be applied directly to the design and analysis of the real-time NCS.

The design and analysis of NCS with communication delay have been studied by many scholars in different ways. Branicky et al. [1] studied the stability of the NCS by analyzing the influence of the sample rate and delay on NCS. Nilsson and Bernhardsson [2] modeled the communication delay as independent random delay and Markov chain random delay, and analyzed the stability and LQR of the closed system with the max random delay

limited to one sample time. Halevi and Ray [3] introduced an expanded model with the current system state, regarding delayed input and delayed output as a new state. However, these studies focused on statistics characters of the delay involving lots of mean and variance calculation, or hypothesizing an upper bound of the delay. Moreover, the problem of data packet loss or timeout was not taken into account and these studies tended to redesign the controller despite the controller being designed without considering delays.

In this paper, a novel estimation scheme based on DR model for NCS is proposed and the stability analysis of the closed-loop system is given. By using DR estimation of the state, the effect of the communication delays is overcome. This makes the controller designed without considering delays still applicable in NCS.

2 DR-model-based estimation algorithm

2.1 DR algorithm

Dead reckoning (DR) is a technology widely used in distributed interaction simulation (DIS). For example, there are two nodes, Node *A* and Node *B*, interacting in simulation. Node *A* predicts the actual state of Node *B* with the DR model. If the difference between the actual state and the predicted one is within a certain threshold, the new state information is not sent out [4]. The DR algorithm not only greatly reduces the communication load, but also compensates the communication delay.

2.2 Interface delay

Definition 1 (Interface delay) At time kT , if the most fresh state information received by Node *A* from *B* was generated at the time $t(n)$, $\tau(k) = t(k) - t(n)$ is defined as the interface delay from *B* to *A* at kT .

The interface delay can be calculated from the actual communication delay series as follows:

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$$\begin{aligned} \tau(k) &= t(k) - t(k'), \\ k' &= \max \{n \mid t(n) + d(n) \leq t(k), n = 1, 2, \dots, k\} \end{aligned} \quad (1)$$

Therefore, the interface delay is not the actual communication delay. Communication delay is from the view of data communication, while interface delay is from the view of data usage, i.e., the period from the time the data is generated to the time that it is used.

An important property for interface delay is $\tau(k+1) \leq \tau(k) + 1, \forall k$. Therefore, the interface delay may be small even if the actual communication is large (e.g., timeout or data loss). That is, interface delay model can automatically shield the invalid communication delay.

2.3 Closed-loop NCS based on DR estimation

Suppose that the controller and the plant are time-driven, and the interface delay model and DR estimator are applied in NCS. Fig. 1 shows how to estimate the state X_k with interface delay $\tau(k)$. Fig. 2 illustrates the closed-loop NCS with estimation algorithm based on DR model. Where $\tau^{\text{cp}}(k)$ is the interface delay from controller to plant, and $\tau^{\text{pc}}(k)$ is the interface delay from plant to controller.

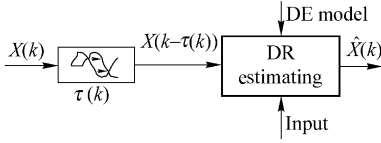


Fig. 1 Interface delay and DR estimator

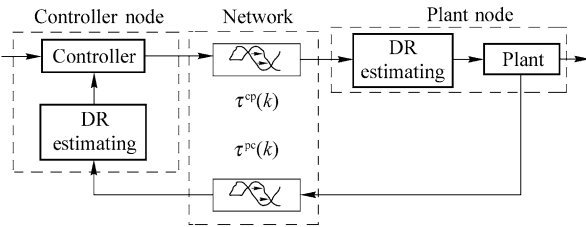


Fig. 2 DR-estimation-based NCS

From the definition of interface delay, we know that the latest information that the plant received from the controller is generated at time $(k - \tau^{\text{cp}}(k))T$, while the latest information for the controller is generated at time $(k - \tau^{\text{pc}}(k))T$.

2.4 Estimation algorithm based on DR model in NCS

Suppose that when there is no delay the plant and the controller can be respectively described as

$$\begin{cases} X_{k+1} = AX_k + BU_k \\ Y_k = CX_k + DU_k \end{cases} \quad \begin{cases} Z_{k+1} = FZ_k + GY_k \\ U_k = HX_k + KY_k \end{cases} \quad (2)$$

If time-delay exists, the state can be estimated by DR

algorithm, and the plant model and the controller model can be rewritten respectively as

$$\begin{cases} X_{k+1} = AX_k + B\hat{U}_{k|\tau^{\text{cp}}(k)} \\ Y_k = CX_k + D\hat{U}_{k|\tau^{\text{cp}}(k)} \end{cases} \quad \begin{cases} Z_{k+1} = FZ_k + G\hat{Y}_{k|\tau^{\text{pc}}(k)} \\ U_k = HX_k + K\hat{Y}_{k|\tau^{\text{pc}}(k)} \end{cases} \quad (3)$$

where $\hat{U}_{k|\tau^{\text{cp}}(k)}$ denotes the DR estimation of U_k by using $U_{k-\tau^{\text{cp}}(k)}$ and the controller DR model; and $\hat{Y}_{k|\tau^{\text{pc}}(k)}$ denotes the DR estimation of Y_k by using $Y_{k-\tau^{\text{pc}}(k)}$ and the plant DR model.

In real application, the estimation DR model is given on-line by the estimated plant to other nodes in the network. It could be time-varying, low-order or high-order, identified on-line or off-line. For example, for first-order DR model, the DR algorithm can be written as

$$\hat{X}_k = X_{k-\tau(k)} + \dot{X}_{k-\tau(k)} \tau(k)$$

and for second-order model, it is

$$\hat{X}_k = X_{k-\tau(k)} + \dot{X}_{k-\tau(k)} \tau(k) + \frac{1}{2} \ddot{X}_{k-\tau(k)} \tau(k)^2$$

2.5 Estimation algorithm when DR model is selected as the real model

Let the DR model of the plant and controller be selected as their real model. Then $\hat{U}_{k|\tau^{\text{cp}}(k)}$ and $\hat{Y}_{k|\tau^{\text{pc}}(k)}$ in Eq. (3) can be calculated by the following DR algorithm:

$$\begin{aligned} \hat{U}_{k|\tau^{\text{cp}}(k)} &= H(F^{\tau^{\text{cp}}(k)} \hat{Z}_{k-\tau^{\text{cp}}(k)} \\ &\quad + \sum_{i=1}^{\tau^{\text{cp}}(k)} F^{\tau^{\text{cp}}(k)-i} GY_{k-i}) + KY_k \\ \hat{Y}_{k|\tau^{\text{pc}}(k)} &= C(A^{\tau^{\text{pc}}(k)} X_{k-\tau^{\text{pc}}(k)} \\ &\quad + \sum_{i=1}^{\tau^{\text{pc}}(k)} A^{\tau^{\text{pc}}(k)-i} BU_{k-i}) + DU_k \end{aligned} \quad (4)$$

Combining Eqs. (3)-(4), we can get the closed-loop system.

3 Stability analysis based on DR estimation algorithm

3.1 Analysis of system model

Let $D = 0$ or $K = 0$, and define

$$\begin{cases} \bar{A} := A + BKC \\ \bar{B} := BH \end{cases} \quad \begin{cases} \bar{F} := F + GDH \\ \bar{G} := GC \end{cases} \quad (5)$$

By Eqs. (3)-(5), we can get

$$\bar{X}_{k+1} = \Phi(k) \bar{X}_k \quad (6)$$

where

$$\begin{aligned} \bar{X}_k &= [X_{k|0}^T, X_{k|1}^T, X_{k|2}^T, \dots, X_{k|\tau^{\text{pc}}(k)-1}^T, \\ &X_{k|\tau^{\text{pc}}(k)}^T, \dots, X_{k|\max \tau^{\text{pc}}(k)}^T, Z_{k|0}^T, Z_{k|1}^T, Z_{k|2}^T, \\ &\dots, Z_{k|\tau^{\text{cp}}(k)-1}^T, Z_{k|\tau^{\text{cp}}(k)}^T, \dots, Z_{k|\max \tau^{\text{cp}}(k)}^T]^T \end{aligned}$$

$$\Phi(k) = \mathcal{Q}(\tau^{\text{cp}}(k), \tau^{\text{pc}}(k)) = \begin{array}{c|ccc|ccc|c} \bar{A} & 0 & 0 & 0 & \bar{B} & 0 & 1 \cdot m \\ \hline \bar{A} & \bar{A} & 0 & \bar{B} & 0 & 0 & \tau^{\text{pc}}(k) \cdot m \\ & \ddots & & \vdots & & & \\ & 0 & \bar{A} & \bar{B} & 0 & 0 & \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & (\max \tau^{\text{pc}}(k) - \tau^{\text{pc}}(k)) \cdot m \\ \hline 0 & 0 & \bar{G} & \bar{F} & 0 & 0 & 1 \cdot n \\ \hline \bar{G} & & & \bar{F} & 0 & & \\ \bar{G} & & & \bar{F} & & & \\ \vdots & & & & \ddots & & \\ \bar{G} & 0 & 0 & & & & \tau^{\text{cp}}(k) \cdot n \\ \bar{G} & & & 0 & \bar{F} & & \\ \hline 0 & 0 & 0 & 0 & \bar{F} & 0 & (\max \tau^{\text{cp}}(k) - \tau^{\text{cp}}(k)) \cdot n \end{array} \quad (7)$$

$m = \dim(\bar{A}), n = \dim(\bar{F})$

The augmented closed-loop NCS Eq. (6) is a special discrete-time system, with the system matrix $\Phi(k)$ (defined in Eq. (7) varying with $\tau^{\text{pc}}(k), \tau^{\text{cp}}(k)$).

3.2 Stability analysis

Definition 2 (Bounded random delay) $\tau(k)$ is called bounded random delay in $\{\tau_{\min}, \dots, \tau_{\max}\}$, if it can be an arbitrary value in $\{\tau_{\min}, \dots, \tau_{\max}\}$ at any time, and its value cannot be influenced by others.

Equation (6) is an uncertain system, and its stability must be analyzed in the worst case, i.e., Eq. (6) should be stable for any delay $\tau^{\text{cp}}(k), \tau^{\text{pc}}(k)$ varies in $\{\tau_{\min}, \dots, \tau_{\max}\}$.

Suppose that the interface delay is $\tau^{\text{cp}}(k)$ and $\tau^{\text{pc}}(k) \in \{\tau_{\min}, \dots, \tau_{\max}\}$ is a bounded random delay. The system matrix Eq. (7) of system Eq. (6) varies with $\tau^{\text{cp}}(k), \tau^{\text{pc}}(k)$, which can be any value in $\{\tau_{\min}, \dots, \tau_{\max}\}$ at any time. Let \mathcal{Q}^I be the set composed by all the time-varying matrix $\Phi(k)$, i.e., $\{\Phi(k)\} = \mathcal{Q}^I$,

$$\mathcal{Q}^I = \mathcal{Q}(\tau^{\text{cp}}(k), \tau^{\text{pc}}(k)), \forall \tau^{\text{cp}}(k), \tau^{\text{pc}}(k) \in \{\tau_{\min}, \dots, \tau_{\max}\} \quad (8)$$

where $\mathcal{Q}(\tau^{\text{cp}}(k), \tau^{\text{pc}}(k))$ is defined in Eq.(8), and $\dim(\mathcal{Q}^I) = (\tau_{\max} - \tau_{\min} + 1)^2$.

Since the delay $\tau^{\text{cp}}(k), \tau^{\text{pc}}(k)$ can be any value in $\{\tau_{\min}, \dots, \tau_{\max}\}$ at time kT , the system matrix Eq. (7) $\Phi(k)$ of system Eq. (6) can be any element in \mathcal{Q}^I . The stability of system Eq. (6) can be decided by \mathcal{Q}^I . If the time-delay is constant, then the dimension of \mathcal{Q}^I is 1, and then system Eq. (6) is a linear time-invariant system.

Theorem 1 If $\exists n > 0$ and certain norm $\|\cdot\|$, such that $\max \|\prod_{k=1}^n \Phi(i_k)\| < 1$, then system Eq.(6) is asymptotically stable.

Proof Let $N = \lfloor \frac{k}{n} \rfloor$, i.e., the integer part of k/n , and

$$M = \max \|\Phi(i)\|, \Phi(i) \in \mathcal{Q}^I. \text{ Denote } \lambda = \max \|\prod_{k=1}^n \Phi(i_k)\|, \text{ by}$$

the assumption of the theorem we know that $\lambda < 1$.

From Eq.(6), it can be seen that

$$\begin{aligned} &\|x(k)\| \\ &= \|\Phi(1)\Phi(2)\cdots\Phi(n)\Phi(n+1)\Phi(n+2)\cdots\Phi(2n)\cdots \\ &\quad \Phi((N-1)n+1)\Phi((N-1)n+2)\cdots\Phi(Nn)\cdots \\ &\quad \Phi(N\cdot n+1)\cdots\Phi(k-1)\Phi(k)\cdot x(0)\| \\ &\leq \|\Phi(1)\Phi(2)\cdots\Phi(n)\| \cdot \|\Phi(n+1)\Phi(n+2)\cdots\Phi(2n)\| \cdots \\ &\quad \cdot \|\Phi((N-1)n+1)\Phi((N-1)n+2)\cdots\Phi(Nn)\| \cdot \|\Phi(Nn+1)\| \cdots \\ &\quad \cdot \|\Phi(k-1)\| \cdot \|\Phi(k)\| \cdot \|x(0)\| \\ &\leq \lambda^N \|\Phi(N\cdot n+1)\| \cdots \|\Phi(k-1)\| \cdot \|\Phi(k)\| \cdot \|x(0)\| \\ &\leq \lambda^N M^{k-N\cdot n} \|x(0)\| \end{aligned}$$

Since $0 \leq k - Nn < n$, $\|x(k)\| \leq \lambda^N M^n \cdot \|x(0)\|$ if $M \geq 1$, or else $\|x(k)\| \leq \lambda^N \cdot \|x(0)\|$. Therefore $\|x(k)\| \leq \lambda^N \cdot \max(M^n, 1) \cdot \|x(0)\|$. It is obvious that system Eq. (6) is asymptotically stable with $\lambda < 1$

Theorem 2 If \exists sequence i_1, i_2, \dots, i_n , such that $\rho(\prod_{k=1}^n \Phi(i_k)) > 1$, where $\rho(\Phi)$ is the spectrum radius of square matrix Φ , then system Eq. (6) is unstable.

Proof If \exists sequence i_1, i_2, \dots, i_n , such that $\rho(\prod_{k=1}^n \Phi(i_k)) > 1$, construct the following:

$$\begin{aligned} \Phi(1) &= \Phi(i_1), \\ \Phi(2) &= \Phi(i_2), \dots, \\ \Phi(n) &= \Phi(i_n), \\ \Phi(1+n) &= \Phi(i_1), \\ \Phi(2+n) &= \Phi(i_2), \dots, \\ \Phi(n+n) &= \Phi(i_n), \\ \Phi(1+2n) &= \Phi(i_1), \\ \Phi(2+2n) &= \Phi(i_2), \dots, \\ \Phi(n+2n) &= \Phi(i_n), \\ &\vdots \end{aligned}$$

Then $x(mn) = \prod_{k=1}^n \Phi(i_k) x((m-1)n) = (\prod_{k=1}^n \Phi(i_k))^m x(0)$, inside which $\rho(\prod_{k=1}^n \Phi(i_k)) > 1$, so system Eq. (6) is unstable in this case.

We can see, if $\exists \Phi(i_k) \in \mathcal{Q}^I$, such that $\rho(\Phi(i_k)) > 1$, then system Eq. (6) is unstable. Furthermore, if only $\rho(\Phi(i_k)) < 1$,

$\forall \Phi(i_k) \in \mathcal{Q}'$, system Eq. (6) may be unstable in the worst situation.

Theorem 3 For system Eq. (6), given $\tau^{\text{cp}}(k) = \tau^{\text{pc}}(k) = \tau^{\text{cp}}(k)$, system Eq. (6) is unstable if $\rho\left(\begin{bmatrix} \bar{A} & \bar{B} \\ \bar{G} & \bar{F} \end{bmatrix}\right) > 1$.

Proof Omitted. Remark. $\begin{bmatrix} \bar{A} & \bar{B} \\ \bar{G} & \bar{F} \end{bmatrix}$ is the system matrix of Eq.(6) with no delay. If $\tau^{\text{cp}}(k) = \tau^{\text{pc}}(k)$, all the eigenvalues of $\begin{bmatrix} \bar{A} & \bar{B} \\ \bar{G} & \bar{F} \end{bmatrix}$ are always those of $\Phi(k)$. If $\rho\left(\begin{bmatrix} \bar{A} & \bar{B} \\ \bar{G} & \bar{F} \end{bmatrix}\right) > 1$, system Eq. (6) is unstable according to Theorem 2. If \exists sequence i_1, i_2, \dots, i_n , such that $\rho\left(\prod_{k=1}^n \Phi(i_k)\right) > 1$, where $\rho(\Phi)$ is the spectrum radius of square matrix Φ , then system Eq. (6) is unstable.

Theorem 3 For system Eq. (6), suppose $\tau^{\text{cp}}(k) = \tau^{\text{pc}}(k) = \text{constant}$. If there exists certain norm $\|\cdot\|$, such that $\left\| \begin{bmatrix} \bar{A} & 0 \\ 0 & \bar{F} \end{bmatrix} + \begin{bmatrix} 0 & \bar{B} \\ \bar{G} & 0 \end{bmatrix} \right\| < 1$, then system Eq. (6) is asymptotically stable.

Proof Omitted. If $\tau^{\text{cp}}(k) = \tau^{\text{pc}}(k) = \text{constant}$, system Eq. (6) is LTI and hence is always asymptotically stable whatever the interface delay is.

4 Simulation

4.1 Description of controller and plant

Consider the longitudinal short period equation with elevator dynamic character: $d\xi/dt = a\xi + bu$, $y = c\xi$, where $\xi = [\delta_e \quad W \quad q]^T$, $u = \delta_a$, $y = [\alpha \quad a_n \quad q]^T$, where δ_e , W , q , δ_a , α , and a_n are elevator angle, linear velocity, pitch rate, elevator control, attack angle, and gravity acceleration, respectively.

The system could be parameterized and discretized with the sample time $T = 0.025$ s, then the coefficient matrixes of the plant in Eq. (2) are :

$$A = \begin{pmatrix} 0.6065 & 0 & 0 \\ -4.1997 & 0.9216 & 6.8975 \\ -0.7504 & -0.0008 & 0.9059 \end{pmatrix}$$

$$B = (0.3935 \quad -0.8547 \quad -0.2072)$$

$$C = \begin{pmatrix} 0 & 0.0033 & 0 \\ 21.6204 & 0.3328 & 1.9801 \\ 0 & 0 & 1.0000 \end{pmatrix}$$

$$D = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Let the coefficient matrixes of the controller in Eq. (2) as follows:

$$F = \begin{pmatrix} -0.5211 & 0.0007 & 0.4058 \\ -27.6428 & 0.4772 & -1.3282 \\ -0.5389 & -0.0088 & 0.4636 \end{pmatrix}$$

$$G = \begin{pmatrix} -0.1248 & 0.0156 & -0.0456 \\ 13.8962 & 1.1637 & 5.0717 \\ 0.5959 & 0.0095 & 0.2175 \end{pmatrix}$$

$$H = (-2.0081 \quad 0.0139 \quad 0.9942)$$

$$I = (0 \quad 0 \quad 0)$$

4.2 Simulation result

Suppose the pitch rate $q_0 = 0.1$ and the other states $X_0 = 0$. The simulation is made in three cases:

1) The communication delay is a constant, such as $1T$, $2T$ and $3T$. The simulation result is shown in Fig. 3.

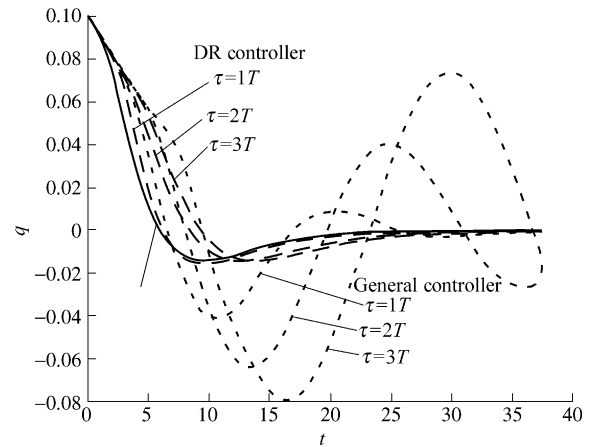


Fig. 3 Response with constant delay

2) The communication delay is an independent random value, and the minimum value is $0T$. The maximum is $1T$, $2T$ and $3T$. The simulation result is shown in Fig. 4.

3) The data loss possibility is 20%. The simulation result is shown in Fig. 5.

4) The DR model error is 10%. This is to test the robustness of the system. The simulation result is shown in Fig. 6.

It can be drawn from Figs. 3-5 and 6 that the system is sensitive to delay and data loss. When the general controller is designed without considering delay, even the system becomes unstable. However, with the estimating algorithm based on DR model, the control result is similar to the idea

effect with a constant delay, random delay or data loss. And the delay value and DR model error have little influence on the system control performance.

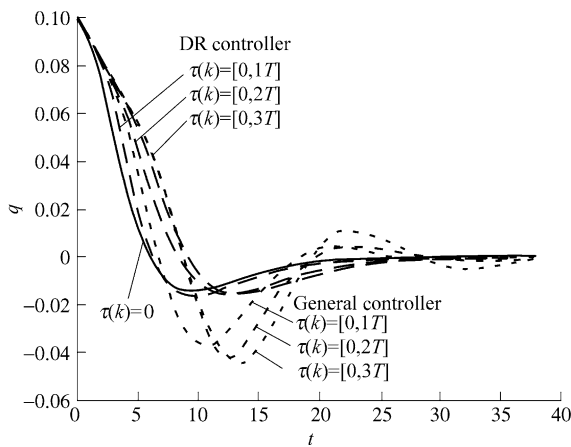


Fig. 4 Response with random delay

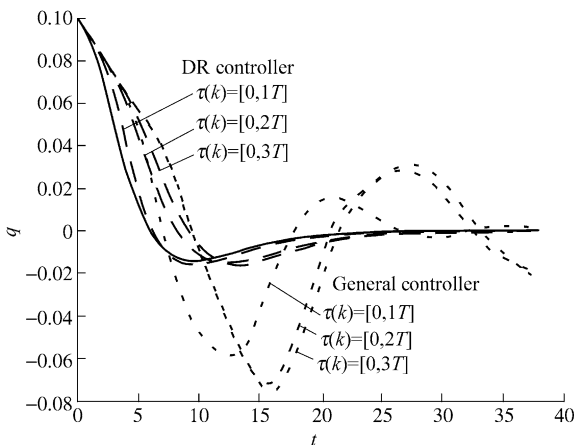


Fig. 5 Response with random delay and data packets lost

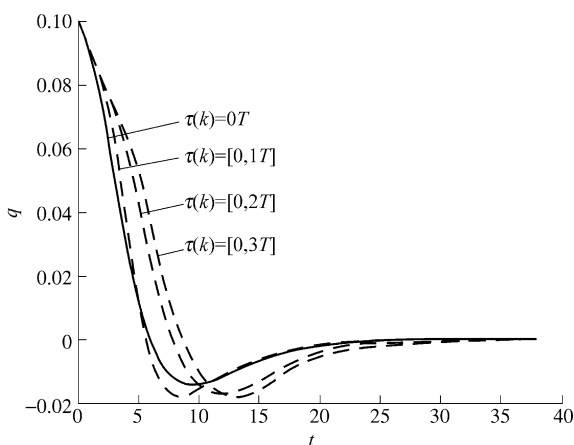


Fig. 6 Response with random delay and error in DR model

5 Some comment

1) The system must be synchronized and all the data packets must be stamped.

2) UDP is preferred rather than TCP/IP to send data to reduce delay.

3) All the states X_k of the plant must be sent rather than the general output information Y_k ; if there is no X_k , a state observer could be applied to get \hat{X}_k .

4) The DR model is dynamically determined by the estimated node and is communicated to other nodes by network.

6 Conclusions

A novel estimating scheme based on DR model for NCS is proposed. Both the detailed DR estimation algorithm and the stability analysis of the system are given. By using DR estimation of the state, the effect of the communication delays is overcome, that makes the controller designed without considering delays still applicable in NCS. Moreover, the scheme can effectively solve the problem of data packets loss or timeout.

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