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RESEARCH ARTICLE

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# Bifurcation and chaos in multi-parallel-connected current-mode controlled boost DC–DC converters



**Abstract** This paper studied the bifurcation and chaos phenomenon in a multi-parallel-connected current-mode controlled boost DC–DC converter system with the use of nonlinear mapping bifurcation theory of two dimensions, and the changing rules of the bifurcation charts with the increase of the control parallels and control parameters were concluded. The method of discrete mapping modeling was utilized to construct the difference equations of the system operating in continuous conduction mode (CCM). Analyses and computer emulations were made.

**Keywords** Bifurcation, Chaos, Boost converter

## 1 Introduction

Modern power technology is based on DC–DC converters. In some cases, sub-harmonics, bifurcation and chaos phenomenon may appear in the DC–DC converters because of their strong nonlinearity, which make the systems unpredictable and uncontrollable. We are used to using small-signal modeling method to analyze and evaluate the design performance of the feedback loop while designing the control scheme of a new type of DC–DC converters, but, at the same time, some nonlinear dynamics and latent operation modes were neglected. So, some researchers turned their steps to that area. Deane and Hamill discussed the theory of chaos phenomenon that happen in current-mode controlled boost DC–DC converters in Continual Conduction Mode (CCM) [1]. Banerjee studied the chaos phenomenon of the PWM Buck converter in CCM [2]. Some other articles analyzed the chaos phenomenon in the power systems [3–10]. The modeling methods they adopted include average or quasi-average modeling methods, sample data modeling and discrete time modeling based on nonlinear mapping. The discrete mapping modeling, which can be used to predict the dynamics in the time domain, is most suitable for the observation and analysis of the nonlinear dynamics of the switched converters [11].

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## 2 Mapping formulas for the construction of the multi-parallel-connected current-mode controlled boost DC–DC converter system based on the discrete time modeling principle

### 2.1 System circuit and operation current waveforms

During the design process of the DC–DC conversion, several DC–DC converters have to be parallel connected to decrease the current value across the semiconductor switch. Fig.1(a) shows the typical circuit diagram, from which we get to know that every converter module is a simple current feedback controlled converter and every switch is controlled by the output of a comparator. On the assumption that the system is operating in the CCM and the modules are controlled synchronously, the current across the inductor $i_{L_i}$ ($i=1,2,\cdots,m$) can be shown in Fig. 1(b).

### 2.2 Discrete time mapping formulas of the system in CCM

For a single DC–DC converter, the difference equation can be written as [12]:

$$\boldsymbol{x}_{n+1}=f(\boldsymbol{x}_n,I_{\text{ref}}) \tag{1}$$

where $\boldsymbol{x}_n=(v_n,i_n)'$, is the state vector of the converter, and the subscript $n$ stands for the initial state of the $n$th conversion cycle. The difference equation (Eq. (1)) connects

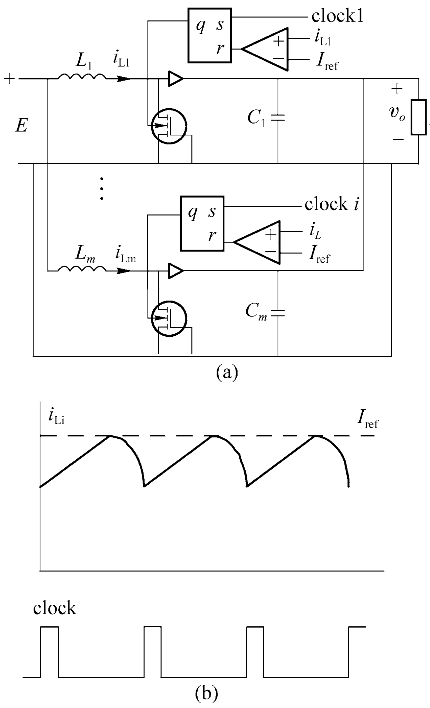


**Fig. 1** The block diagram and current waveforms of parallel connected DC-DC converters

the state vector of the ($n$+1)th cycle $(v_{n+1}, i_{n+1})'$ to that of the $n$th cycle $(v_{n+1}, i_{n+1})'$, and the following analyses are based on it. For the $m$-module parallel connected DC–DC converter system, as all the modules share the same input voltage, so the state vector can be set as:

$$\boldsymbol{x}_n = (v_n, i_{nL_1}, \cdots, v_n, i_{nL_m})'$$

Suppose that state 1 is the state of switch-on, and state 2, switch-off, and the duty cycle of switches is $d$, and the system is operated in the CCM, this means the current across the inductor is above zero. Then, the state equations of the system can be written as:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}_1\boldsymbol{x} + \boldsymbol{B}_1 E,\ nT \le t < (n+d)T$$

$$\dot{\boldsymbol{x}} = \boldsymbol{A}_2\boldsymbol{x} + \boldsymbol{B}_2 E,\ (n+d)T \le t < (n+1)T$$

where

$$\boldsymbol{A}_1 = [\boldsymbol{A}_{11} \ \cdots \ \boldsymbol{A}_{1j} \ \cdots \ \boldsymbol{A}_{1m}]',$$

$$\boldsymbol{B}_1 = [\boldsymbol{B}_{11} \ \cdots \ \boldsymbol{B}_{1j} \ \cdots \ \boldsymbol{B}_{1m}]'$$

$$\boldsymbol{A}_{1j} = \frac{1}{RC_j}\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{B}_{1j} = \begin{bmatrix} 0 \\ 1/L_j \end{bmatrix}$$

$$\boldsymbol{A}_{2j} = \frac{1}{RC_j}\begin{bmatrix} -1 & R \\ -RC_j/L_j & 0 \end{bmatrix}, \boldsymbol{B}_2 = \begin{bmatrix} 0 \\ 1/L_j \end{bmatrix}$$

By using iteration techniques, $x_{n+1}$ can be expressed as the function of $x_n$ and $d_n$:

$$\dot{\boldsymbol{x}}_{n+1} = \Phi_2(T - d_nT)\varPhi_1(d_nT) \cdot \left[\dot{x}_n + \int_{nT}^{nT+d_nT} \varPhi_1(nT - \tau)\boldsymbol{B}_1 E \mathrm{d}\tau\right]$$

$$+\Phi_2(T - d_nT) \cdot \int_{nT+d_nT}^{(n+1)T} \varPhi_2(nT + d_nT - \tau)\boldsymbol{B}_2 E \mathrm{d}\tau \qquad (2)$$

where

$$\varPhi_i(\xi) = 1 + \sum_{k=1}^{\infty} \frac{1}{k!} \boldsymbol{A}_i^{\,k} \xi^k \qquad (i = 1, 2)$$

Having derived the discrete time mapping formulas of the system, we can now use numerical simulation methods to view the chaos phenomenon in the parallel connected DC–DC converters. The exhibited dynamic characteristics of the system can also be studied as the control parameter $I_{\text{ref}}$ changes. Then, the method of the Jacobian matrix can be used to analyze the chaos accurately [13].

## 3 Bifurcation and chaos with designated control parameters

### 3.1 Bifurcation charts with the reference current as the control parameter

First of all, we will put forward the vivid bifurcation charts through calculation, from which some principles can be gained. Circuit parameters of the system will be chosen as follows to ensure the system is operating under CCM.

Clock period: $T = 100\ \mu\text{s}$; input voltage: $E = 10\ \text{V}$; load resister: $R = 20\ \Omega$; $L_i = 1\ \text{mH}$; $C_i = 10\ \mu\text{F}$ $(i = 1, 2, \cdots, m)$.

The math toolbox of the software MATLAB will be used to realize the calculation and draw the bifurcation charts. First of all, a control parameter $I_{\text{ref}}$ and the initial state will be designated according to Eq. (2), and then through iterating calculation thousands of times with front part results omitted, the bifurcation charts of single-module, two-module, three-module and four-module systems can be drawn as Fig. 2(a)–(d). The charts on the top left corner are their magnified parts.

From Fig. 2, we can see that the parallel connected DC–DC converter systems go through a stable state to chaos with the increase of the parameter $I_{\text{ref}}$, but the processes of the different number of modules under the same circuit parameters are different from each other, which pushes us to analyze the systems particularly with the use of bifurcation theory of two-dimensional discrete mapping system.

### 3.2 Bifurcation theory of two-dimensional discrete mapping system

For the $m$-module parallel connected DC–DC converter system, let

$$i_{nL_1} = \cdots = i_{nL_m} = i_n$$

Only the first three items of $\varPhi_j(\xi)$ in Eq. (2) are retained for simplicity, and the function

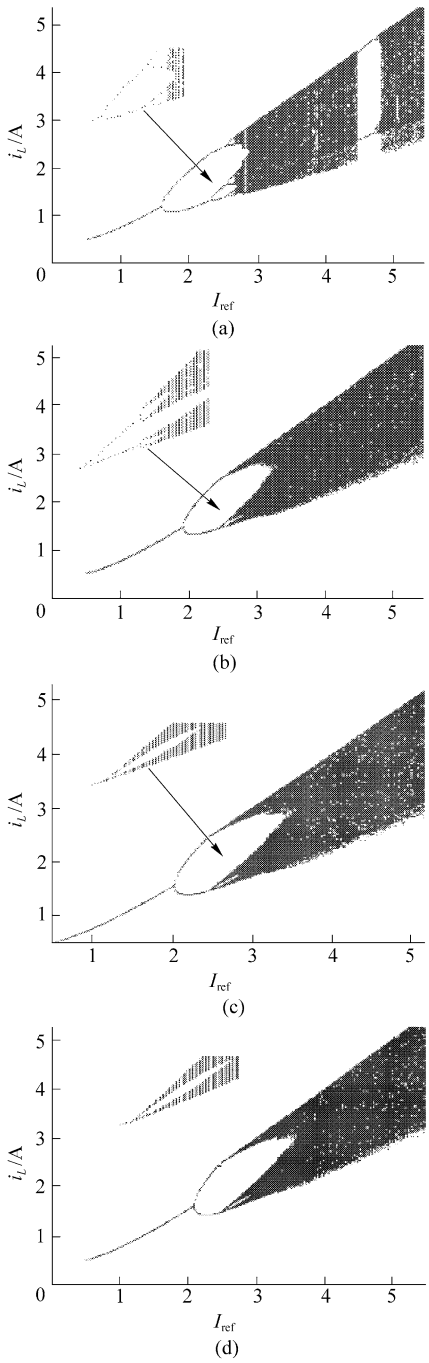


**Fig. 2** Bifurcation charts of the parallel connected DC–DC converters with reference current as the control parameter. (a) Single module system; (b) Two-module system; (c) Three-module system; (d) Four-module system

$$d_n = I_{\mathrm{ref}} - i_n / ET / L$$

can be used to get the expression as Eq. (1), which can be expanded to be a two-dimensional discrete map:

$$\begin{cases} v_{n+1} = f(v_n, i_n, I_{\mathrm{ref}}) \\ i_{n+1} = g(v_n, i_n, I_{\mathrm{ref}}) \end{cases} \quad (3)$$

We can also use the bifurcation theory of two-dimensional discrete mapping system to analyze the $m$-module parallel connected DC–DC converter system based on Eq. (3).

Bifurcation theory is a modern math theory that is used to investigate the characteristics of the solutions of nonlinear equations. It is initially used to indicate the qualitative change in features of the system, such as the number and the type of solutions, under the variation of one or more parameters on which the considered system depends. When bifurcation occurs, the different states of the system begin to transit to one another discontinuously, which brings about more bifurcations, and finally, chaos occurs. The state space is composed of state variables and control parameters, and the states of the system change in the state space. The point where the change happens is called bifurcation point. There exists a fixed point (balance point) in the difference equation as Eq. (1), whose multiplier

$$\lambda = f'(x, I_{\mathrm{ref}})$$

can be used to judge the stability of the fixed point [14].

According to the bifurcation theory of two-dimensional discrete mapping system [14], the multiplier $\lambda$ of

$$\boldsymbol{J} = \begin{bmatrix} \dfrac{\partial f}{\partial v_n} & \dfrac{\partial f}{\partial i_n} \\ \dfrac{\partial g}{\partial v_n} & \dfrac{\partial g}{\partial i_n} \end{bmatrix}_P$$

has to be calculated to judge the stability of the fixed point $P$, and the multiplier $\lambda$ can be determined by the solution of

$$\det(\lambda \boldsymbol{I} - \boldsymbol{J}) = 0$$

where $\boldsymbol{I}$ is an identity matrix.

Orbits near the fixed point take the form of :

$$P_n = c_1 \lambda_1{}^n + c_2 \lambda_2{}^n$$

where , $\lambda_1$ and $\lambda_2$ are two multipliers.

The fixed point is stable when $|\lambda_1| < 1$ and $|\lambda_2| < 1$, while is unstable when either of the absolute values is larger than 1. The value of $\lambda_1$ and $\lambda_2$ corresponds to extension or shorten times in their own direction, which keeps to the flowing rules:

- If $0 < \lambda_1 < 1$ and $0 < \lambda_1 < 1$, then, the fixed point is stable;
- If $\lambda_1 > 1$ and $\lambda_2 > 1$, then, the fixed point is unstable;
- If $0 < \lambda_1 < 1$ and $\lambda_2 > 1$, then, the fixed point is a saddle point;
- If $\lambda_1$ and $\lambda_2$ are conjugate, which it is to say, $\lambda_1 = a\mathrm{e}^{\mathrm{i}\alpha}$, $\lambda_2 = a\mathrm{e}^{-\mathrm{i}\alpha}$ ( $0 < \alpha < \pi$ ), the fixed point is a stable focus when $a < 1$, an unstable focus when $a > 1$, a centre when $a = 1$.
- If $\alpha / \pi$ is a rational number, the orbits are periodic, else, quasi-periodic.

When $I_{\mathrm{ref}} < 1.60$, the absolute values of $\lambda_1$ and $\lambda_2$ are both less than 1, and the single-module DC–DC converter system is stable under given parameters. When $I_{\mathrm{ref}} = 1.60$, $\lambda_1 = 1.000$ and $\lambda_2 = 0.437$ are real numbers, we can see that the multiplier transits the Floquet circle and transcritical bifurcation occurs. With the increase of $I_{ref}$, cyclic-fold bifurcations occur frequently and finally, the system turns into chaos state, at the same time, calculation of the multipliers becomes more difficult. Lyapunov indexes of the whole bifurcation process were figured out in Ref. [4], and it pointed out that there are quasi-periodic orbits before chaos. For the *m*-module parallel connected DC–DC converters with the same circuit parameters, we can draw the conclusions:

1) In the bifurcation chart with $I_{ref}$ as the control parameter, we can see that the double-period bifurcation point moves backward when *m* increases. For example, in the two-module parallel connected DC–DC converter system, double-period bifurcation occurs when $I_{ref} \approx 1.91$ ($\lambda_1$ and $\lambda_2$ are $-1.000$, $0.571$ respectively); in the three-module system, double-period bifurcation occurs when $I_{ref} \approx 2.02$; in the four-module system, double-period bifurcation occurs when $I_{ref} \approx 2.08$.

2) Double-period bifurcation occurs less with the increase of *m*. A single-module system goes through period-2, period-4, period-8 and period-16 bifurcation orbits to chaos, and two-module system goes through period-2, period-4 and quasi-period-8 bifurcation orbits to chaos, and when *m* is greater than 3, system goes through period-2 and quasi-period-4 bifurcation orbits to chaos.

3.3 Bifurcation chart with input voltage as the control parameter

Now we take the input voltage as the control parameter to observe the influence of the input voltage to the multi-parallel connected DC–DC converters.

Let $I_{ref} = 3.0$, with $E$ increasing from 10 V to 22 V, and other parameters are as the same as mentioned above, bifurcation charts with input voltage as the control parameter of single-module, two-module, three-module and four-module parallel connected DC–DC converter systems are shown in Fig. 3(a)–(d).

As can be seen from the charts above, the systems go through stable state to chaos with the decrease of the input voltage when the input voltage is taken as the control parameter, and the changing principles are identical as those with reference current as the control parameter.

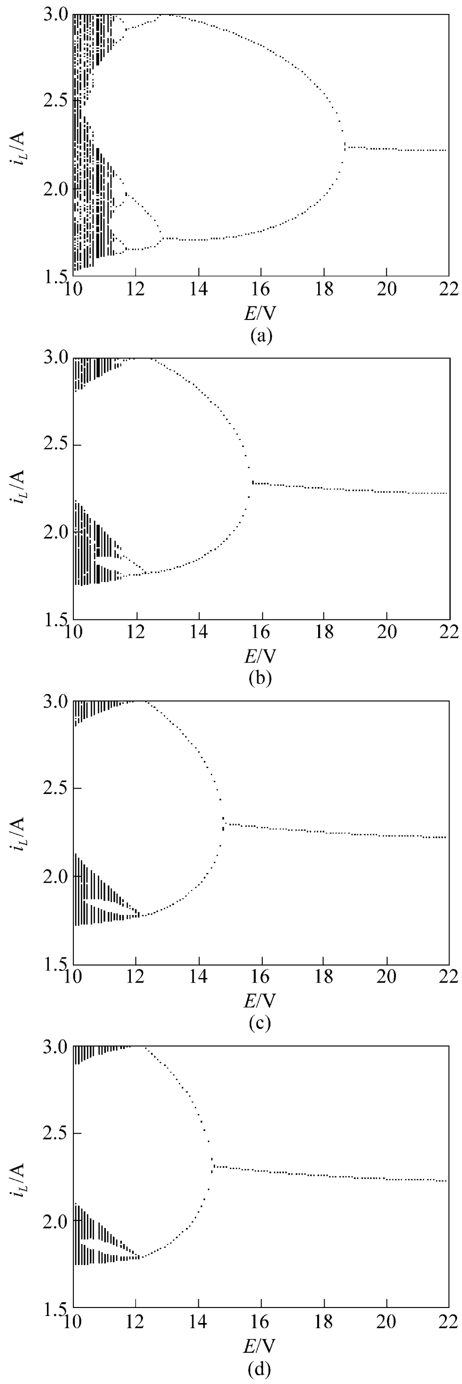


**Fig. 3** Bifurcation charts of the parallel connected DC–DC converters with input voltage as the control parameter. (a) Single-module system; (b) Two-module system; (c) Three-module system; (d) Four-module system

## 4 Conclusions

Firstly, the method of discrete-time mapping modeling was used to construct the difference equation of the multi-parallel connected DC–DC converter system operating in continual conduction mode. Secondly, bifurcation charts with reference current and input voltage as the control parameters were drawn by using the math toolbox of the MATLAB respectively. Thirdly, analyses were made on the systems with the use of bifurcation theory of two-dimensional discrete mapping system. Finally, changing conclusions were made with the increase of *m* under the same circuit parameters.

Though small-signal modeling is the main method of designing a DC–DC converter, nonlinearity dynamics are inevitable. Chaos happens frequently as the DC–DC converters were presented in the world [15]. Analyses and forecasting can be made conveniently on the multi-module parallel connected DC–DC converter system by using the methods presented in this paper.

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