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RESEARCH ARTICLE

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Multiuser detection for DS-CDMA systems in non-Gaussian channels

Abstract An adaptive multi-user detector was developed for direct-sequence code division multiple access (DS-CDMA) systems corrupted by non-Gaussian channel noise, which can be quite detrimental to the performance of the multi-user detectors based on classical white Gaussian assumption. This receiver simultaneously combats multiple-access interference (MAI) and non-Gaussian impulsive noise. The channel parameters are estimated and transmitted signals are jointly detected by a simple recursive algorithm derived from the EM/SAGE algorithm. Analytical and simulation results show that the proposed technique is robust with wider applicability than conventional multi-user detectors in terms of near-far resistance and bit-error ratio (BER) when either MAI or non-Gaussian impulsive noise is dominant.

Keywords DS-CDMA, Multi-user detection (MUD), Non–Gaussian impulsive noise, EM/SAGE algorithm, Multiple access interference (MAI)

1 Introduction

MUD in code division multiple access (CDMA) systems has been intensively investigated over the last decade. By and large, the study of this problem has focused on the situation in which the ambient noise is additive white Gaussian noise (AWGN), for the main aim is to mitigate the most severe interference—MAI. However, with the rapid development of MUD, the situation in which practical channels will be ambient-noise limited can be realistically envisioned.

In many practical wireless channels, the ambient noise is known as non-Gaussian by experimental measurements, such as radio channels and underwater acoustic channels [1]. Here we consider DS-CDMA systems corrupted by natural impulsive noise sources, which can be quite detrimental to the performance of the detectors based on the classical white Gaussian assumption. Although performance indices such as mean square-error (MSE) and signal-to-interference-plus-noise ratio (SINR) for linear multi-user detectors are not affected by the distribution of the noise (only the spectrum matters), the more crucial bit-error rate can depend heavily on the shape of the noise distribution [2]. There have been some approaches to MUD in non-Gaussian noise. Distinct features of the EM/SAGE- MUD algorithm in this paper include adaptive processing to the observed noise parameters and robust detection to user data. The proposed receiver combats both MAI and impulsive noise effectively with a simple structure and wide applicability.

2 System model and basic algorithm

2.1 DS-CDMA model in non-Gaussian channels

Consider a baseband DS-CDMA system with K active users operating with a BPSK modulation. Defining a rectangular pulse by $\prod_c(t)=1$ for $0<t\leqslant T_c$ and $\prod_c(t)=0$, otherwise, the normalized signature sequence waveform of the kth user can be expressed as

$$PN_k(t)=\sum_{j=1}^{\infty}S_{kj}\prod_c(t-jT_c) \quad (1)$$

where $\{S_{kj}\}$ takes on values of ± 1 with equal probability and T_c is the chip duration. The signature sequences here can be either short codes discussed in many literatures or long pseudo random sequences used in IS-95 or W-CDMA systems.

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The waveform received in the first data frame of interest with M bits is given by

$$r(t) = s(t) + w(t) = \sum_{i=1}^{M}\sum_{k=1}^{K} A_k b_{ki} PN_k(t-\tau_k) + w(t) \tag{2}$$

where A_k、τ_k、$b_{ki} \in \{1,-1\}$ denote, respectively, the received amplitude, delay, data stream when $t \in [(i-1)T_b, iT_b]$, $NT_c = T_b, N$ is the processing gain. w is the additive non-Gaussian channel noise.

For the sake of simplicity of discussion, our attention is first restricted to the synchronous case, i.e., $\tau_k = 0, 1 \le k \le K$.

The received signal is first filtered by a chip-matched filter and then sampled at the chip rate. The resulting discrete-time signal sample is given by

$$\boldsymbol{r}(m) = c\boldsymbol{e}(m) + \boldsymbol{w}(m) \tag{3}$$

where

$$c = \Big[\sum_{k=1}^{K} A_k b_{k1} S_{k1}, \sum_{k=1}^{K} A_k b_{k1} S_{k2}, \cdots, \sum_{k=1}^{K} A_k b_{k1} S_{kn},$$
$$\sum_{k=1}^{K} A_k b_{k2} S_{k(N+1)}, \cdots, \sum_{k=1}^{K} A_k b_{k2} S_{k(2N)}, \cdots,$$
$$\sum_{k=1}^{K} A_k b_{kM} S_{k(MN-N+1)}, \cdots, \sum_{k=1}^{K} A_k b_{kM} S_{MN}\Big]$$

$\boldsymbol{e}(m)$ is a $MN \times 1$ vector with all 0 elements except the mth element equals to 1. The sequence of noise samples $\{w_j(m)\}$ is a sequence of independent and identically distributed (i.i.d.) random variables with two-term Gaussian mixture probability density function (PDF), which serves as an approximation to the more fundamental Middleton Class A noise model [1] used to model physical noise. Its PDF has the form

$$f = \lambda_1 N(0, \sigma_1^2) + \lambda_2 N(0, \sigma_2^2) \tag{4}$$

where $\lambda_1 + \lambda_2 = 1, \sigma_1^2 \le \sigma_2^2, \lambda_1 \ge \lambda_2$.

The first term can be seen as the main background noise, and the second as an impulsive component. Here, it is assumed that the sets $\sigma^2 = \{\sigma_1^2, \sigma_2^2\}$, $\lambda = \{\lambda_1, \lambda_2\}$, $A = \{A_k\}$ do not vary for the duration of one frame.

2.2 EM & SAGE algorithm

It is known that the maximum likelihood estimation problem is usually with high computing complexity, such as

$$\varPhi(Y) = \arg_{\varPhi \in \varLambda} \{\max \log f(Y; \varPhi)\} \tag{5}$$

where $\varPhi \in \varLambda$ are the parameters to be estimated, $f(\cdot)$ is the parameterized PDF of the observable Y.

The idea of the expectation-maximization (EM) algorithm [3] is to choose a set of "missing data" $\boldsymbol{Z}$ to form a complete data $X = \{Y, Z\}$ and then to iteratively maximize the new objective function $\boldsymbol{Q}(\boldsymbol{\varPhi}; \boldsymbol{\varPhi}')$, which is relatively easier to be solved.

$$\boldsymbol{Q}(\varPhi; \varPhi') = \mathrm{E}\{\log f(Y, Z; \varPhi) | Y = y; \varPhi'\} \tag{6}$$

where $\varPhi'$ represent a priori estimate of the parameters from the previous iteration. EM estimates have been proved monotonically increasing the likelihood.

In order to accelerate the convergence for multidimensional parameter estimations, the SAGE algorithm [4] has been proposed, which divides the parameters into several groups, with only one group being updated in each iteration. Thus, SAGE algorithm improves the global convergence rate and the converging speed while maintaining the overall tractability of EM algorithm.

3 EM/SAGE-MUD algorithm

This paper focuses on two problems. One is channel estimation algorithm aimed at non-Gaussian channels. The other is MUD algorithm to mitigate MAI. Both are achieved by exploiting EM and SAGE algorithms in turns.

The complete data are formed by

$$\{y_{k(m)} = (r_{k(m)}, q(m)\}, m = 1, 2, \cdots, M, k = 1, 2, \cdots, K$$

where the label $q(m)$ is either 1 or 2, which indicates the two terms respectively in the mixture Gaussian model.

$$r_{k(m)} = c_k \boldsymbol{e}(m) + w_k(m)$$

$$c_k = [A_k b_{k1} S_{k1}, A_k b_{k2} S_{k2}, \cdots, A_k b_{k1} S_{kN},$$
$$A_k b_{k2} S_{k(N+1)}, \cdots, A_k b_{k2} S_{k(2N)}, \cdots,$$
$$A_k b_{kM} S_{k(MN-N+1)}, \cdots, A_k b_{kM} S_{k(MN)}]$$

$r_{k(m)}$ divides the received signal space according to the information of each user [5];

$$w_k(m) = w_{k1}(m) + w_{k2}(m) \quad \sum_{k=1}^{K} \beta_k = 1$$

$$w_{k1}(m) \sim N(0, \beta_k \sigma_1^2), w_{k2}(m) \sim N(0, \beta_k \sigma_2^2)$$

$w_{k1}(m), w_{k2}(m)$ differentiate the noise space according to the two terms in the mixture model [6]. It can be seen that the definition of the complete data is a comprehensive consideration between MAI and additive non-Gaussian noise.

Within this model, the estimated parameter data $\varPhi$ includes the amplitude A of received signals of each user, the user data bit b, and the Gaussian components mixture ratio λ and variances σ^2.

$\varPhi = \{A, b, \lambda, \sigma^2\}$ is the parameter set to be estimated, i.e., channel parameters will be estimated and user data will be detected simultaneously. Training sequences in this model are of no need but spreading sequences of all users should be known.

E-step: the objective function in EM algorithm is given by

$$Q(\Phi,\Phi') = E\left[\log f\left(y|\Phi\right)\ \middle| \boldsymbol{r},\Phi'\right]$$
$$= E\left[\sum_{m=1}^{MN}\sum_{k=1}^{K}\left\{\log\ \frac{\lambda_{q(m)}}{\sqrt{2\pi\beta_k}\,\sigma_{q(m)}} - \frac{[r_k(m)]^2\text{-}r_k(m)*c_k\boldsymbol{e}(m)+[c_k\boldsymbol{e}(m)]^2}{2\beta_k\sigma_{q(m)}^2}\right\}\boldsymbol{r},\Phi'\right]$$

Ignoring the terms which have no effect on the maximization process, we have

$$Q(\Phi\mid\Phi') = \sum_{q=1}^{2}\sum_{m=1}^{MN} g_q'\left(m\right)\Big\{K\log\lambda_q - K\log\sigma_q - \sum_{k=1}^{K}\frac{1}{2\beta k\sigma_q^2}\Big[\left(\beta_k-\beta_k^2\right)\sigma_q'^2+\big\{\boldsymbol{c}_k'\boldsymbol{e}\left(m\right) + \beta_k\left[r\left(m\right)-\boldsymbol{c}'\boldsymbol{e}\left(m\right)\right]\big\}^2 - 2\big\{\boldsymbol{c}_k'\boldsymbol{e}\left(m\right) + \beta_k\left[r\left(m\right)-\boldsymbol{c}'\boldsymbol{e}\left(m\right)\right]\big\}\ \boldsymbol{c}_k'\boldsymbol{e}\left(m\right)+A_k^2\Big]\Big\} \tag{7}$$

where $g_i'\left(m\right)=f\left[\ q\left(m\right)=i\middle|r\left(m\right),\Phi'\right]\ \ i=1\ \ or\ \ 2$.

M-step: the maximization problem separates into two items: the first involving the set λ alone and the second involving σ^2, *b, A*. For the parameters in each set, the maximization problem can be solved separately. However, it is difficult to obtain σ^2, *b, A* that jointly maximize $Q\left(\Phi|\Phi'\right)$. Therefore, the SAGE algorithm is adopted here and then we can assume other parameters are fixed in the derivation of one parameter set.

The final update equations are obtained:

($q=1,2$)

$$\lambda:\lambda_q=\frac{1}{MN}\sum_{m=1}^{MN}g_q'\left(m\right) \tag{8}$$

$$\sigma^2:\sigma_q^2=\frac{\sum_{m=1}^{MN}g_q'\left(m\right)\ \left[r\left(m\right)-c'\boldsymbol{e}\left(m\right)\right]^2}{\sum_{m=1}^{MN}g_q'\left(m\right)} \tag{9}$$

$$\mathrm{b}:b_{k_i}=\mathrm{sgn}\left(\sum_{m=(i-1)N+1}^{iN}\left\{\left[\frac{g_1'\left(m\right)}{\sigma_1'^2}+\frac{g_2'\left(m\right)}{\sigma_2'^2}\right]\cdot\left(b_{k_i}'+\frac{\beta_k}{A_k'}\left[r\left(m\right)-c'\boldsymbol{e}\left(m\right)\right]s_{km}\right)\right\}\right] \tag{10}$$

$$A\colon A_k=A_k'+\beta_k b_k'\bullet\frac{\sum_{m=1}^{MN}\left[\frac{g_1'\left(m\right)}{\sigma_1^2}+\frac{g_2'\left(m\right)}{\sigma_2^2}\right]\left[r\left(m\right)-c'\,\boldsymbol{e}\left(m\right)\right]s_{km}}{\sum_{m=1}^{MN}\left[\frac{g_1'\left(m\right)}{\sigma_1^2}+\frac{g_2'\left(m\right)}{\sigma_2^2}\right]} \tag{11}$$

EM/SAGE-MUD algorithm is summarized below:

1) Initialize;

2) Compute λ according to Eq. (8), *b* according to Eq. (10), and set $\lambda'=\lambda, b'=b$;

3) Compute λ according to Eq. (8), *A* according to Eq. (11), and set $\lambda'=\lambda,\ A'=A$;

4) Compute λ according to Eq. (8), σ^2 according to Eq. (9), and set $\lambda'=\lambda,\sigma'=\sigma$;

5) Repeat Steps 2)– 4) until the estimates converge.

4 Simulation results and analysis

4.1 Computer simulations

The basic simulation parameters are set as follows:

Three users employ Gold sequences as spreading codes with *N*=31.

The SNR of user 1 is $E_{b1}/\sigma_1^2=0\ \mathrm{dB}$, $E_{b1}/\sigma_2^2=-20\ \mathrm{dB}$; and λ_1 = 0.9, λ_2 = 0.1,

$E_{b2}/E_{b1}=10\ \mathrm{dB}$, $E_{b3}/E_{b1}=20\ \mathrm{dB}$. The hard limited output of the traditional single user detector, which is the output of the matching filters, is the initial value of *b*.

Figure1 plots the BER performance of user 1 by conventional maximum mean square error (MMSE) MUD, multistage parallel interference cancellation (PIC) MUD (the received amplitudes are known here) and EM/SAGE-MUD all in non-Gaussian additive noise channel. The figure indicates that even for user 1 with considerable low SINR, when the conventional receivers perform badly, the BER of the proposed receiver can reach below 10^{-3} after approximately 5 iterations. In fact, the BERs of user 2 and 3 are below 10^{-4}, 10^{-6} respectively under the same situations.

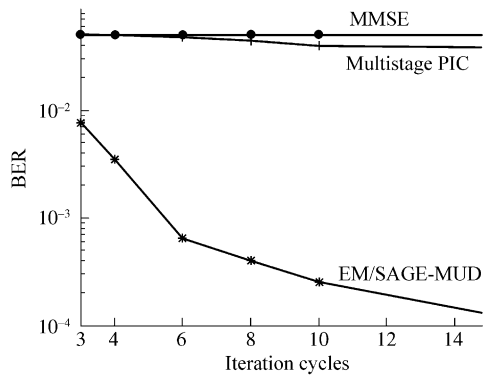

Fig. 1 BER versus iteration cycles

Figure 2 captures the effects of the observation period, i.e., the number of data frame on the precision of parameter estimations. Curve *A*, *B*, *C* each denotes the normalized sample standard deviation of estimated λ_2、σ_1^2、σ_2^2.

It is also shown by simulation that the number of frames has only a slight effect on estimation of amplitudes and detection of user data, although there is some certain effect on estimation of noise parameters. Due to our target on user data detection, the batch process on the whole frame data using the above algorithms should be changed into process bit by bit. This could enhance the calculation efficiency, reduce storage space, and be adaptive to fast time varying channel. The initial values of parameter data σ^2, λ, A in the next iteration could use their priori estimates to make full use of previous information.

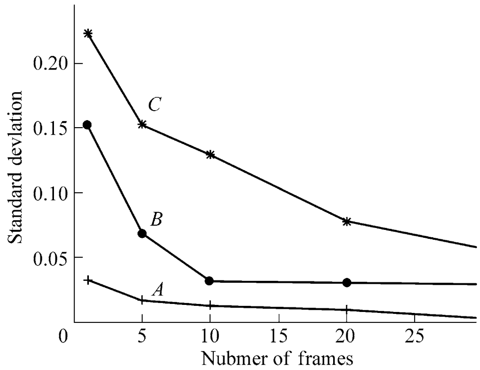

Fig. 2 Normalized standard deviation of the estimates versus the number of frames

4.2 Further algorithm analysis

1) If the noise source is additive white Gaussian, the difference between EM-MUD and multistage PIC lies in the factor β_k . However, just with this simple modification, better convergence is guaranteed by the EM algorithm. Besides, we can derive a result similar to the EM sequential algorithm [7] by adding a forgetting factor $1/(p+1)$ in the last term of Eq. (10), where p is the present iteration step. It has also been proved that the Fisher information contained in the complete data space, which is inversely proportional to the convergence rate of the algorithm, can be expressed as

$$I_c(b) = \mathrm{diag}\left(\left[\frac{A_1^2}{\beta_1\sigma^2}, \frac{A_2^2}{\beta_2\sigma^2}, \cdots, \frac{A_K^2}{\beta_K\sigma^2}\right]\right)$$

So β is inversely proportional to $I_c(b)$.

Generally speaking, the convergence of users with low SNR will be worse. Therefore, to obtain a good performance in limited stages, β should be chosen small for high SNR and large for low SNR.

2) If we use SAGE algorithm in the overall estimation procedure rather than use EM and SAGE interchangeably, the convergence rate can be improved further. Thus its computing complexity is equal to $O(K\Delta p)$, the same as the complexity of conventional PIC.

3) If the relative delays of the transmitting signals and relative propagation time of all the paths have been obtained by a synchronization device, we can have an approach over multiple paths using a mathematics model in Ref. [8]. And if we set $q(m) \in \{1, 2, \cdots, L\}$ and estimate the covariance matrix of the noise sources instead of σ^2, the algorithm proposed in this paper can be used in any L-term correlated Gaussian mixture model.

5 Conclusions

An approach to MUD for DS-CDMA systems in non-Gaussian channel noise is presented in this paper, which is based on modeling the PDF of additive noise with a finite mixture of Gaussian PDFs. An iterative algorithm for estimating signal and noise parameters is then derived using the EM/SAGE algorithm with a simple recursive structure. Analytical and simulation results show that the proposed technique is more robust and uniformly better than the conventional linear multi-user detectors when either MAI or impulsive noise is dominant.

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