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RESEARCH ARTICLE

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Separating cyclostationary signals from spectrally overlapping interference

Abstract This paper studies an algorithm about separating spectrally overlapping signals using the cyclostationary properties of signals. On the basis of direct sequence spread system (DSSS), frequency shift filter is added into the receiver of the communication system. Although the structure of frequency shift filter is more complicated than the time-domain filter, it uses both time correlations and frequency spectrum correlations so it can achieve better performances on separating the overlapping signals. After the analysis of cyclostationary characteristic and frequency spectrum correlation, the structure of the frequency shift filter can be gained. Then, a self-adaptive algorithm is utilized for the purpose of achieving optimum multidimensional tap weights of frequency shift components. The simulation results indicate that this method can efficiently separate overlapping signals, and its error rate is lower than the time-domain filter or DSSS system by two orders of magnitude on the condition that high-power interference is added into the system.

Keywords cyclostationary, frequency shift filter, overlap signal, spectral correlation

1 Introduction

Cyclostationary signals can be interpreted as the nonstationary signals whose statistical properties vary periodically, and exhibit spectral coherence in the frequency domain. Studies on theories of the nonstationary process began to develop rapidly in the mid-1980s, though it can be traced back to the 1950s. In the 21st century, with the further expansion of the mobile communication market, studies and applications on cyclostationarity have been dramatically improved and developed domestically in China and in other countries. For example, in code division multiple access (CDMA) cellular mobile communication systems, some algorithms based on cyclostationary characteristic of signals can be utilized to expand the system capacity. That is, the spectrally overlapping DSSS system is designed by inserting an additional channel in the middle of two neighboring channels, and then frequency shift filter (FSF) is introduced for channel separation and signals extraction. As a result, this system's capacity can increase several-times [1–7]. Studies on large narrowband interference suppression can be seen in other literatures [8–15], which apply the cyclostationary property of signals in wireless communi- cation systems with narrowband interference. Researches and investigations in related literatures show the fact that most modulated signals encountered in communication systems exhibit desirable spectral correlation and present obvious cyclostationary characteristic. Therefore, in order to separate spectrally overlapping signals effectively, these signals transmitted in actual channels should be modeled as a cyclostationary process and not a stationary process. Besides, spectral correlation of these cyclostationary signals should be utilized to extract the useful signals from the overlapping signals by FSF. This is because FSF applies both temporal correlation and spectral correlation properties of signals, while time-domain filter only utilizes temporal correlation properties. For these reasons, it is possible and realizable to separate overlapping spectra when the received signals are filtered by FSF.

Firstly, cyclostationary signals and its spectral correlation are briefly introduced and analyzed in this paper. Then the paper explains the material working principles of frequency shift filter and the designing of its key parameters. After this, a basic structure of FSF is introduced and realized, which has applied self-adaptive algorithms for determining the optimum tap weights of multiple frequency-shift components. In the following stage, an adaptive algorithm is presented for the purpose of separating spectral overlapping signals, which has utilized the cyclostationary properties of signals. In the simulation and discussion section, DSSS system is taken as the instance for theory analysis and further study. Finally, more discussions and conclusions are

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made in the end.

2 Cyclostationarity and spectral correlation

Cyclostationary signal is defined as the signal whose mean value and autocorrelation function have the characteristic of periodic variation, and for all time t

$$E[x(t)] = E[x(t + mT_0)] \tag{1}$$

$$\begin{aligned} R_x(t,\tau) &= E[x(t)x^*(t-\tau)] \\ &= E[x(t+mT_0)x^*(t+mT_0-\tau)] \end{aligned} \tag{2}$$

where m is an integer and T_0 is the basic cycle.

For cyclic Wiener filtering [1], the frequency domain theory usually reaches the optimum parameters under the criterion of time average-mean square error (TA-MSE). Adopting time average criterion is convenient for implementation of frequency shift filter using self-adaptive methods. This paper also takes time average criterion as the foundation for the following research.

In cyclostationary theory, the cyclic autocorrelation function is defined as [2]

$$R_x^\alpha(\tau) = \left\langle E\left[x\left(t+\frac{\tau}{2}\right)x^*\left(t-\frac{\tau}{2}\right)\right]\mathrm{e}^{-\mathrm{j}2\pi\alpha t}\right\rangle \tag{3}$$

where $\langle\rangle$ and $E[\cdot]$ denote infinite-time and ensemble average respectively, τ is the lag parameter, α denotes cyclic frequency.

For a complex signal, if the cyclic conjugate correlation function exists and is not zero, the signal is said to exhibit conjugate wide-sense cyclostationary (WSCS). Similarly, the cyclic conjugate correlation function is defined as

$$R_{xx^*}^\beta(\tau) = \left\langle E\left[x\left(t+\frac{\tau}{2}\right)x\left(t-\frac{\tau}{2}\right)\right]\mathrm{e}^{-\mathrm{j}2\pi\beta t}\right\rangle \tag{4}$$

where β denotes cyclic conjugate frequency. That is, when a complex signal *x(t)* is said to exhibit conjugate WSCS, it demonstrates that the cyclic conjugate correlation function $R_{xx^*}^\beta(\tau)$ ($\beta \neq 0$) exists and is not zero for some values of the lag parameter τ.

The complex envelope of the spectral component of $x(t)$ at frequency v with approximate bandwidth $1/T$ is given by

$$X_T(t,v) = \frac{1}{T}\int_{t-T/2}^{t+T/2} x(u)\exp(-\mathrm{j}2\pi vu)\,\mathrm{d}u \tag{5}$$

The frequency density for correlation of spectral components at frequency $v = f + \alpha/2$ and $v = f - \alpha/2$ is, therefore, given by

$$S_x^\alpha(f) = \lim_{T\to\infty}\frac{1}{T}\left\langle X_T\left(t, f+\frac{\alpha}{2}\right)X_T^*\left(\,\right)t, f-\frac{\alpha}{2}\right\rangle \tag{6}$$

And this spectral correlation density function and the cyclic autocorrelation function construct a Fourier transform pair:

$$S_x^\alpha(f) = \int_{-\infty}^{\infty} R_x^\alpha(\tau)\exp(-\mathrm{j}2\pi f\tau)\mathrm{d}\tau$$

If the two spectral components of a cyclic signal at $f+\alpha/2$ and $f-\alpha/2$ are correlated, the magnitude of correlation is measured by spectral correlation coefficients. For a cyclostationary signal, the spectral correlation coefficient is defined by

$$\rho = \frac{S_x^\alpha(f)}{(S_x(f+\alpha/2)S_x(f-\alpha/2))^{1/2}} \tag{7}$$

when ρ equals one, it can be known that the two frequency components at $f+\alpha/2$ and $f-\alpha/2$ have complete spectral redundancy. When the two spectral components are linearly dependent, the measure of using one of them to cancel or recover the other can be introduced, since the information carried by the two spectral components is identical. In fact, because of this spectral correlation, certain uncorrupted spectral components in a received signal can be used to eliminate or recover other severely corrupted spectral components in that signal. Therefore, when a signal exhibits cyclostationary characteristic, it must have certain spectral correlation properties in frequency domain, which can be used to separate the two cochannel spectrally overlapping signals.

3 Frequency shift filter

The principle and basic structure of frequency shift filter are on the basis of the cyclic Wiener filtering theory. It is well known that optimum filters for stationary signals are time-invariant. Similarly, optimum filters for signals that exhibit cyclostationarity with a single period or multiple periods are singly or multiply poly-periodically time-varying (PTV) filters [1].

A PTV linear filter has input-output relation [1]

$$y(t) = \int_{-\infty}^{\infty} h(t,u)x(u)\mathrm{d}u \tag{8}$$

where the impulse-response function $h(t,u)$ is a poly- periodic function of the time variable u, and can be expanded in a Fourier series

$$h(t,u) = \sum_\eta h_\eta(t-u)\exp(\mathrm{j}2\pi\eta u) \tag{9}$$

Therefore,

$$\begin{aligned} y(t) &= \sum_\eta \int_{-\infty}^{\infty} h_\eta(t-u)[x(u)\,\mathrm{e}^{\mathrm{j}2\pi\eta u}]\mathrm{d}u \\ &= \sum_\eta h_\eta(t)\otimes x_\eta(t) \end{aligned} \tag{10}$$

where $x_\eta(t) = x(t)\exp(\mathrm{j}2\pi\eta t)$ is a frequency shifted version of $x(t)$, the symbol $\otimes$ denotes convolution. Considering for the moment finite-energy signals, the Fourier transforms of both sides of Eq. (10) can be equated to obtain

$$Y(f) = \sum_\eta H_\eta(f)X(f-\eta) \tag{11}$$

Generally, a linear time-variant filter processes both the signal's complex envelope and its complex conjugate envelope. If a complex signal is used, then the issues of optimum and adaptive time-variant poly-periodic filtering must be treated as a bivariate filtering problem, where the signal and its conjugate are jointly filtered and then added together.

That is the linear-conjugate-linear (LCL) filter. The frequency shift filter discussed in this paper is just a certain kind of the LCL filters.

4 Separating algorithm for spectrally overlapping signals and analysis

This chapter discusses an algorithm for separating spectrally overlapping signals, which takes the model of DSSS communication as an example. Supposedly, both the useful signal and the interference signal take the type of BPSK modulation. Consequently, both the signal and the interference have the cyclostationary characteristic, namely, they both have spectral correlation.

The input signal $r(n)$ of the receiver has the discrete-time form. It is firstly filtered by self-adaptive frequency shift filter for signal separation, and then its output signal is demodulated by the BPSK demodulator. As shown in Fig. 1(b), the last two blocks represent accumulation during a period and sample decision, respectively. Figure 1 shows the whole system model.

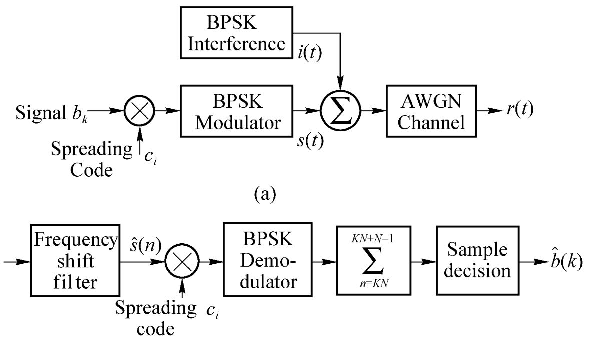

Fig. 1 Model of a communication system for separating spectrally overlapping signals.(a) Transmitter; (b) Receiver

Assuming that the discrete-time form of the input signal in the receiver part is

$$r(n)=s(n)+i(n)+w(n) \tag{12}$$

where $s(n)$ and $i(n)$ respectively represent the useful signal and the interference signal, and $w(n)$ is Gaussian white noise signal. Those three signals are assumed to be mutually independent.

In the statement [2], the general output form of frequency shift filter, on the condition that the input signal $r(t)$ is complex form, has been presented as

$$s(t)=\sum_{p} h_{\alpha_p}(t)\otimes[r(t)\mathrm{e}^{\mathrm{j}2\pi\alpha_p t}]+\sum_{q} h_{\beta_q}(t)\otimes[r^*(t)\mathrm{e}^{\mathrm{j}2\pi\beta_q t}] \tag{13}$$

where symbol $\otimes$ denotes convolution, α_p and β_q are the frequency shift parameters for input $r(t)$ and $r^*(t)$, respecti- vely.

From Eq. (13), the frequency shift filter can be regarded as two parallel banks of systems, the first driven by $r(t)$ and the second by $r^*(t)$. Each system performs frequency shifting of α_p or β_q to the input $r(t)$ or $r^*(t)$, followed by a finite impulse response (FIR) filter with impulse response $h_{\alpha_p}(t)$ or $h_{\beta_q}(t)$, respectively. The outputs of the two systems are then added together. Therefore, the basic structure of frequency shift filter is shown in Fig. 2, where the actual input signal is of complex number form. The frequency shift filter consists of the frequency shift part and conjugate frequency shift part [1].

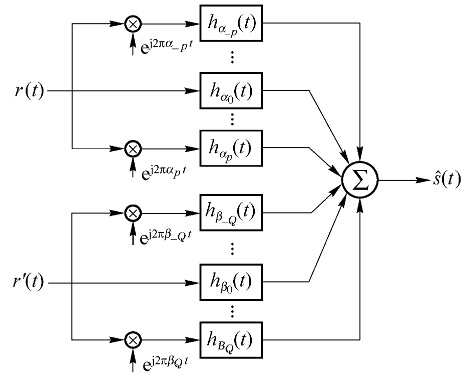

Fig. 2 Basic structure of frequency shift filter

One of the discrete-time forms for Eq. (13), which is the output of frequency shift filter, is presented as

$$\hat{s}(n)=\sum_{p=-P}^{P}\sum_{m=0}^{L-1} h_{\alpha_p}(m)r(n-m)\mathrm{e}^{\mathrm{j}2\pi\alpha_p(n-m)} +\sum_{q=-Q}^{Q}\sum_{m=0}^{M-1} h_{\beta_q}(m)r^*(n-m)\mathrm{e}^{\mathrm{j}2\pi\beta_q(n-m)} \tag{14}$$

where L and M respectively denote the lengths of $h_{\alpha_p}(m)$ and $h_{\beta_q}(m)$ of the FIR filters.

More, a matrix representation of Eq. (14) is

$$\hat{s}(n)=\boldsymbol{h}_{\alpha\beta}^{\mathrm{T}}(n)\boldsymbol{r}_{\alpha\beta}(n) \tag{15}$$

where

$$\boldsymbol{r}_{\alpha\beta}(n)=[\boldsymbol{r}_{\alpha_{-P}}(n)\ \cdots\boldsymbol{r}_{\alpha_0}(n)\ \cdots\ \boldsymbol{r}_{\alpha_P}(n)\ \boldsymbol{r}^*_{\beta_{-Q}}(n)\ \cdots\boldsymbol{r}^*_{\beta_0}(n)\cdots\ \boldsymbol{r}^*_{\beta_Q}(n)]^{\mathrm{T}} \tag{16}$$

$$\boldsymbol{h}_{\alpha\beta}(n)=[\boldsymbol{h}_{\alpha_{-P}}(n)\cdots\boldsymbol{h}_{\alpha_0}(n)\cdots\boldsymbol{h}_{\alpha_P}(n)\ \boldsymbol{h}_{\beta_{-Q}}(n)\cdots\boldsymbol{h}_{\beta_0}(n)\cdots\ \boldsymbol{h}_{\beta_Q}(n)]^{\mathrm{T}} \tag{17}$$

They are both K-dimensional column vectors, with

$$K=(2P+1)L+(2Q+1)M \tag{18}$$

where the frequency shift part of the filter is

$$\boldsymbol{r}_{\alpha_p}(n)=\begin{bmatrix} r(n)\mathrm{e}^{\mathrm{j}2\pi\alpha_p n} \\ r(n-1)\mathrm{e}^{\mathrm{j}2\pi\alpha_p(n-1)} \\ \vdots \\ r(n-L+1)\mathrm{e}^{\mathrm{j}2\pi\alpha_p(n-L+1)} \end{bmatrix},\quad \boldsymbol{h}_{\alpha_p}(n)=\begin{bmatrix} h_{\alpha_p}(0) \\ \vdots \\ h_{\alpha_p}(L-1) \end{bmatrix}$$

and the conjugate frequency shift part of the filter is

$$\boldsymbol{r}^*_{\beta_q}(n)=\begin{bmatrix} r^*(n)\mathrm{e}^{\mathrm{j}2\pi\beta_q n} \\ r^*(n-1)\mathrm{e}^{\mathrm{j}2\pi\beta_q(n-1)} \\ \vdots \\ r^*(n-M+1)\mathrm{e}^{\mathrm{j}2\pi\beta_q(n-M+1)} \end{bmatrix},\quad \boldsymbol{h}_{\beta_q}(n)=\begin{bmatrix} h_{\beta_q}(0) \\ \vdots \\ h_{\beta_q}(M-1) \end{bmatrix}$$

In Eq. (15), the vector $\boldsymbol{h}_{\alpha\beta}$ can be solved by classical Wiener filtering methods [15], where the parameters are optimized under the criterion of minimizing the TA-MSE,

$$\boldsymbol{h}_{\alpha\beta} = \boldsymbol{R}_{rr}^{-1}\boldsymbol{R}_{rs} \tag{19}$$

where $\boldsymbol{R}_{rr}$ denotes the autocorrelation matrix of received signal, $\boldsymbol{R}_{rs}$ denotes the cross-correlation matrix between the received signal and the filtered signal, $\boldsymbol{h}_{\alpha\beta}$ is the optimum multi-dimensional tap weights of frequency shift filter.

Equation (15) is the estimating expression of the useful signal. In order to reach the minimum mean square error (MMSE), the LMS self-adaptive filtering algorithms is adopted to adaptively reach the optimum K-dimensional weight parameters.

$$e(n) = d(n) - \boldsymbol{h}^{\mathrm{H}}(n-1)\,\boldsymbol{r}(n) \tag{20}$$

$$\boldsymbol{h}(n) = \boldsymbol{h}(n-1) + \mu(n)\,\boldsymbol{r}(n)e^{*}(n) \tag{21}$$

where $e(n) = s(n) - \hat{s}(n)$ denotes the error, $d(n)$ denotes the training signal, $\mu(n)$ is the convergence step size.

5 Simulation and results

To evaluate the particular separation effect of the adaptive algorithm for the spectral overlapping signals, computer simulations were adopted to evaluate the error probability on the transmitted information sequence signals by setting several representative parameters.

The simulation condition was set such that the useful signal rate was R_c = 400 kbps, and the interference signal rate was R_i = 200 kbps. The frequency shift parameter of frequency shift filter was set as: $\alpha = [0, R_c, -R_c, R_i, -R_i]$, where the frequency shift part had five frequency-shift components. Similarly, the conjugate frequency shift parameter was set as:

$$\beta = [0,\ 2F_i,\ -2F_i,\ R_c + 2F_i,\ R_c - 2F_i, -(R_c + 2F_i)\ ,\ -(R_c - 2F_i)]$$

where F_i denotes the carrier frequency deviation of the interference signal, which could control the spectrum overlapping rate, and the conjugate frequency shift part had seven frequency-shift components. In this simulation environment, each FIR-filter order of the frequency shift filter was set as $L = 6$ and $M = 6$. Therefore, from Eq. (18), the size of the K-dimensional vector was 72. The vector parameter then was determined by using LMS self-adaptive algorithm.

The Simulink toolboxes in Matlab were used for simulation, and the simulation results were presented from Fig. 3 to Fig. 6. In the following four figures, interference signal ratio (ISR) represents the power ratio of the interference signal and the useful signal, and signal-to-noise ratio (SNR) represents the power ratio of the overlapped signals and the Gaussian white noise in system channel.

After several simulations and the results analysis, the performance and capability in this adaptive frequency shift filtering algorithm under different ISR, SNR and spectrally overlapping rate can be seen. In this paper, the overlapping rate is defined as the ratio of the overlapping part of the interference signal and the useful signal and the effective spectrum of the useful signal.

From the simulation results, it can be seen that the effectiveness achieved by frequency shift filter only has a little advantage over time-domain filter or simple DSSS system, under the environment that the interference power is not very high (e.g., when the ISR is less than 5 dB), as illustrated in Fig. 3(a). When the ISR is quite high (e.g., when the ISR is bigger than 7 dB), either the time-domain filter or the simple DSSS system has unfavorable performance under the expectation, whereas the frequency shift filter is evidently much better in performance and capability, as shown in Fig. 3(b). The two simulation environments are under the condition that the spectral overlapping rate is 67 %.

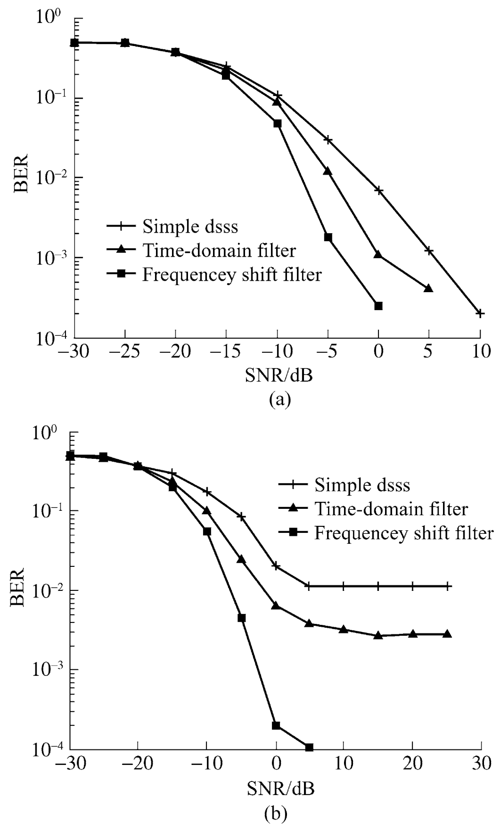

Fig. 3 Performance of frequency shift filter varying by SNR. (a) ISR is 5 dB; (b) ISR is 7 dB

Figure 4 demonstrates the varying trend of overlapping signals separation capability of frequency shift filter, when SNR is 0 dB. It carries out a comparison among the simple DSSS system, time-domain filter and frequency shift filter. As shown in Fig. 4, when the frequency shift filter is under the condition of high ISR (10 dB–20 dB), it can reach the favorable performance all the same, where the error rate is lower than that of time-domain filter and DSSS system by two orders of magnitude. However, it is only on the assumption that the cyclic frequency of interference signal is exactly known, so the capability of the frequency shift filter can thus reach its perfect extent. If the cyclic frequency error comes forth, the performance of the filter will severely drop.

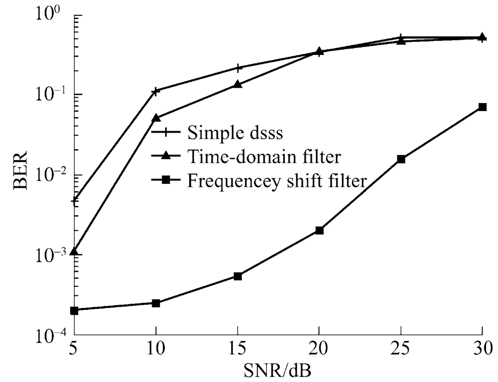

Fig. 4 Performance of frequency shift filter varying by interference signal ratio (SNR is 0 dB).

For the purpose of indicating more effectiveness for separating overlapping signals by the filter, the results can also been analyzed and compared by means of changing the overlapping rate or changing the interference signal rate, as shown in Figs. 5 and 6 under the two instances, when the ISR

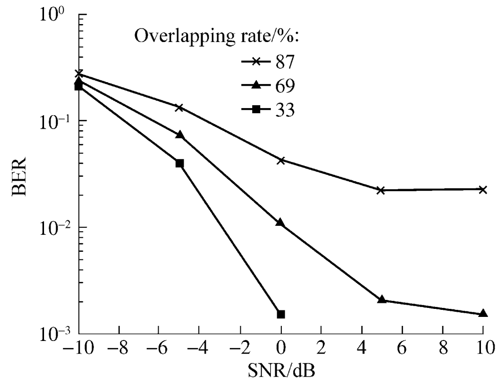

Fig. 5 Influence of frequency shift filter from different overlapping rate (ISR is 25 dB).

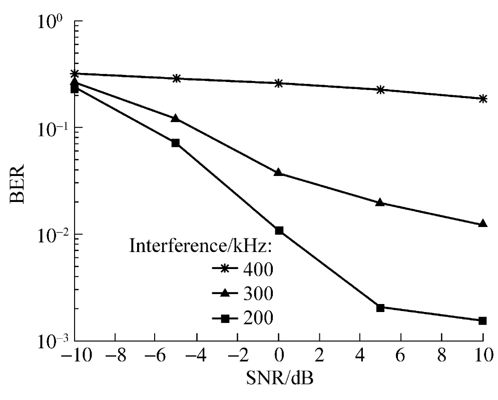

Fig. 6 Influence of frequency shift filter from different ISRs (useful signal rate is 400 kbps, ISR is 25 dB).

is set at 25 dB. It is obvious that the frequency shift filter can more effectively separate spectrally overlapping signals with the reduction of overlapping rate. It also can be seen that if the useful signal rate is the same as the interference signal rate, the separating effect is evidently worse than that on the condition of having different transmitting rates. Besides, if the ISR goes lower, the performance of this communication system will have further improvement. the performance of this communication system will have further improvement.

6 Conclusions

This paper has studied the kind of methods for separating spectrally overlapping signals on the basis of cyclostationary theory. It takes the DSSS system as an example and adds a frequency shift filter in the receiver, in order to realize the separation between the useful signal and the interference signal. The simulation results indicate that this method is effective and applicable. The communication system presented by this research has better performance and more effective capability of separation than simple DSSS system and time-domain filter system. Especially on the environment of high-power interference signals, the signal separation effectiveness is more obvious and evident.

The application of cyclostationary theory has extremely high potential in communications problems for solving the issue of an increasing lack of frequency resources. The method and simulation results presented in this paper have proved this point and have introduced an implementation in simulation. However, there are still other many topics in this area where much more research and energy should be done. For example, if the system has some cyclic frequency error, the performance of this research will have a sharp decline. Besides, how to realize the effective separation of signals under the unfavorable circumstances of high spectral overlapping rate or high-power interference is a question for discussion demanding an in-depth study.

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