

# Electronic Supplementary Material

## Optimal design of extractive dividing-wall column using an efficient equation-oriented approach

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### Appendix A: Steady-state distillation model with bypass efficiency

The MESH equations of steady-state distillation model is as eqs. (A1–A4).

Material balance equation is:

$$0 = V^{j+1}y_i^{j+1} + L^{j-1}x_i^{j-1} - V^{j,eq}y_i^{j,eq} - L^{j,eq}x_i^{j,eq} \quad i \in I, j \in J, \quad (\text{A1})$$

where  $I$  and  $J$  are the sets of all components and stages respectively. The stages are numbered from top to bottom.  $V^{j+1}$  and  $y_i^{j+1}$  are the flow rate and molar composition of the vapor stream entering a stage  $j$ , while  $L^{j-1}$  and  $x_i^{j-1}$  are the flow rate and molar composition of the liquid stream entering the stage.  $V^{j,eq}$ ,  $y_i^{j,eq}$  and  $L^{j,eq}$ ,  $x_i^{j,eq}$  are flow rate and molar composition of vapor stream and liquid stream in phase equilibrium respectively.

Enthalpy balance equation is:

$$0 = V^{j+1}h_V^{j+1} + L^{j-1}h_L^{j-1} - V^{j,eq}h_V^{j,eq} - L^{j,eq}h_L^{j,eq} \quad j \in J, \quad (\text{A2})$$

Here,  $h_V$  and  $h_L$  are vapor specific molar enthalpy and liquid specific molar enthalpy respectively.

Phase equilibrium equation is:

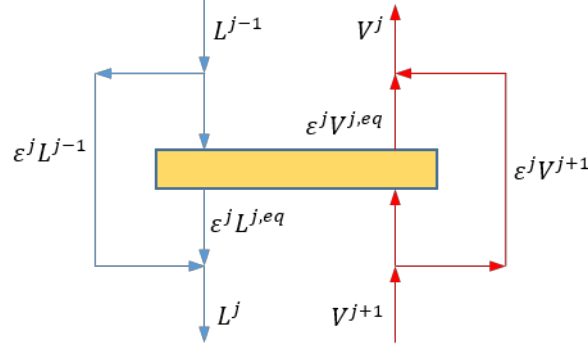
$$y_i^{j,eq} = k_i^j x_i^{j,eq} \quad i \in I, j \in J, \quad (\text{A3})$$

where  $k_i^j$  is phase equilibrium constant, which is a function of temperature, pressure and vapor and liquid composition on stage  $j$ .

The summations of vapor and liquid molar fractions both need to be 1, leading to the summation equation:

$$\sum_{i \in I} x_i^j = 1, \quad \sum_{i \in I} y_i^j = 1 \quad j \in J. \quad (\text{A4})$$

The vapor and liquid streams leaving a stage are got according to the bypass efficiency method [26], as shown in Fig. A1. In the model, a part of the inlet vapor (liquid) stream  $V^{j+1}$  ( $L^{j-1}$ ) feeds to the stage, while the other part of the stream bypasses the stage. The streams entering into the stage  $j$  reach to phase equilibrium, getting  $V^{j,eq}$  and  $L^{j,eq}$ . The vapor and liquid streams leaving the stage,  $V^j$  and  $L^j$  are the mixing of the bypass streams and equilibrium streams. The equations for bypass efficiency method are as Eqs. A5–A10.



**Fig. A1** Equilibrium stage applied with bypass efficiency method

$$V^j = \varepsilon^j V^{j,eq} + (1 - \varepsilon^j) V^{j+1} \quad j \in J, \quad (\text{A5})$$

$$L^j = \varepsilon^j L^{j,eq} + (1 - \varepsilon^j) L^{j-1} \quad j \in J, \quad (\text{A6})$$

$$y_i^{j,eq} = \varepsilon^j y_i^{j,eq} + (1 - \varepsilon^j) y_i^{j+1} \quad i \in I, j \in J, \quad (\text{A7})$$

$$x_i^{j,eq} = \varepsilon^j x_i^{j,eq} + (1 - \varepsilon^j) x_i^{j-1} \quad i \in I, j \in J, \quad (\text{A8})$$

$$h_V^j = \varepsilon^j h_V^{j,eq} + (1 - \varepsilon^j) h_V^{j+1} \quad j \in J, \quad (\text{A9})$$

$$h_L^j = \varepsilon^j h_L^{j,eq} + (1 - \varepsilon^j) h_L^{j-1} \quad j \in J. \quad (\text{A10})$$

where  $\varepsilon^j$  is the bypass efficiency on stage  $j$ , which is a continuous variable between 0 and 1.

### Appendix B: Economic evaluation model and parameters

The  $TAC$  include capital cost ( $CAP_{cost}$ ), energy cost ( $OPE_{cost}$ ), as shown in Eq. (B1). All the costs are in the unit of \$ or  $\text{\$}\cdot\text{year}^{-1}$ . The payback period is 3 years.

$$TAC = \frac{CAP_{cost}}{3} + OPE_{cost}, \quad (\text{B1})$$

where  $CAP_{cost}$  is composed of column cost ( $COL_{cost}$ ), heat exchanger cost ( $HX_{cost}$ ) and compressor cost ( $COMP_{cost}$ ) where  $COL_{cost}$  includes both column shell cost ( $SH_{cost}$ ) and column tray cost ( $TRAY_{cost}$ ).

$$CAP_{cost} = COL_{cost} + HX_{cost} + COMP_{cost}, \quad (\text{B2})$$

$$COL_{cost} = SH_{cost} + TRAY_{cost}, \quad (\text{B3})$$

The calculation of these costs are according to formulas in the book of Luyben [36], where  $SH_{cost}$  (\$) is:

$$SH_{cost} = 17640D^{1.066}H^{0.802}, \quad (\text{B4})$$

Here,  $D$  (m) and  $H$  (m) are column diameter and column height respectively.  $H$  is calculated according to the total number of stages  $NT$ ,

$$H = 1.2NT \cdot h_s, \quad (\text{B5})$$

Here,  $h_s$  is the tray space, which is 0.61 m in this work. The column diameter is the maximum diameter among all the diameters of trays. A diameter of a tray  $d_j$  (m) is:

$$d_j = \sqrt{\frac{4 \cdot V_{Sj}}{\pi u_j}}, \quad (\text{B6})$$

where  $V_{Sj}$  ( $\text{m}^3$ ) and  $u_j$  ( $\text{m}\cdot\text{s}^{-1}$ ) are the volume flow rate and velocity of vapor stream leaving stage  $j$  respectively. Note that, the column diameter is usually determined by the diameter of one specific column section, while the diameter of a column section is usually determined by the diameter of the bottom tray. Thus, for the simplification, in example 1, the maximum tray diameter in column section 5 is taken as column diameter, whilst in examples 2 and 3, the column diameter is got from the quadratic mean diameter of the diameters of the last trays in column

section 3 and 4.  $u_j$  is from

$$F = 0.8197u_j\rho_j^{0.5}, \quad (B7)$$

where  $\rho_j$  ( $\text{kg}\cdot\text{m}^{-3}$ ) is the mass density of the vapor stream.  $F$  is the F factor, which is set to be 1 according to heuristic [35].

The column tray cost is:

$$TRAY_{cost} = 229D^{1.55}NT, \quad (B8)$$

The heat exchanger cost is:

$$HX_{cost} = 7296A^{0.65}, \quad (B9)$$

where  $A$  ( $\text{m}^2$ ) is heat exchanger area got from

$$A = \frac{Q}{U\Delta T_m}, \quad (B10)$$

Here,  $Q$  (kW) is heat duty of heat exchanger,  $U$  ( $\text{kW}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ ) is heat transfer coefficient and  $\Delta T_m$  ( $^\circ\text{C}$ ) is logarithm mean temperature. The heat transfer coefficients for cooler and condenser are both  $0.852 \text{ kW}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ , while the  $U$  of reboiler is  $0.568 \text{ kW}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ . The heat transfer coefficient of process-process heat exchangers is  $0.43 \text{ kW}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ .

The centrifugal compressor is used, which is calculated from Eq. (B11) according to the work of Luo et al. [23].

$$COMP_{cost} = (1463.2/280)(664.1W^{0.65}). \quad (B11)$$

where  $W$  is the compressor power (kW). The prices of middle pressure steam and high pressure steam are 5.4 and 6.1  $\text{\$}\cdot\text{GJ}^{-1}$ , respectively and the price of cooling water is 0.54  $\text{\$}\cdot\text{GJ}^{-1}$ . The electricity price is assumed to be three times of the high pressure steam, so it is 18.3  $\text{\$}\cdot\text{GJ}^{-1}$ . The prices of entrainers DMSO and EG are 1557 and 1300  $\text{\$}\cdot\text{ton}^{-1}$  respectively.

### Appendix C: PTC distillation model

The PTC distillation model is from the work of Ma et al. [17]. It is briefly described as followings. Material balance equation is:

$$\frac{dM_i^j}{dt} = V^{j+1}y_i^{j+1} + L^{j-1}x_i^{j-1} - V^{j,eq}y_i^{j,eq} - L^{j,eq}x_i^{j,eq} \quad i \in I, j \in J, \quad (C1)$$

where  $M_i^j$  is component hold-up on stage  $j$ . The other notations in the equation have the same meaning as those in Appendix A.

Enthalpy balance equation is:

$$\frac{dH^j}{dt} = V^{j+1}h_V^{j+1} + L^{j-1}h_L^{j-1} - V^{j,eq}h_V^{j,eq} - L^{j,eq}h_L^{j,eq} \quad j \in J, \quad (C2)$$

Here,  $H$  is the total hold-up enthalpy. The phase equilibrium equation and summation equations are the same as Eqs. (A3–A4).  $M_i^j$  and  $H^j$  are calculated from the following equations,

$$M_i^j = M_L^j x_i^{j,eq} + M_V^j y_i^{j,eq} \quad i \in I, j \in J, \quad (C3)$$

$$H^j = M_L^j h_L^{j,eq} + M_V^j h_V^{j,eq} \quad j \in J, \quad (C4)$$

Here,  $M_L^j, M_V^j$  are liquid molar hold-up and vapour molar hold-up on stage  $j$  respectively, which are calculated from the pseudo hydraulic correlation Eqs. B5–B6.

$$L^{j,eq} = C_L M_L^j \quad j \in J, \quad (C5)$$

$$V^{j,eq} = C_V M_V^j \quad j \in J. \quad (C6)$$

Here,  $C_L, C_V$  are two constant coefficients, which are both set to be  $1800 \text{ h}^{-1}$  according to our computational experience. The vapour and liquid streams leaving a stage are also got according to the bypass efficiency method [26] as with Eqs. (A5–A10).