

Discontinuous mechanical behaviors of existing shield tunnel with stiffness reduction at longitudinal joints

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ABSTRACT An analytical model is proposed to estimate the discontinuous mechanical behavior of an existing shield tunnel above a new tunnel. The existing shield tunnel is regarded as a Timoshenko beam with longitudinal joints. The opening and relative dislocation of the longitudinal joints can be calculated using Dirac delta functions. Compared with other approaches, our method yields results that are consistent with centrifugation test data. The effects of the stiffness reduction at the longitudinal joints (α and β), the shearing stiffness of the Timoshenko beam GA , and different additional pressure profiles on the responses of the shield tunnel are investigated. The results indicate that our proposed method is suitable for simulating the discontinuous mechanical behaviors of existing shield tunnels with longitudinal joints. The deformation and internal forces decrease as α , β , and GA increase. The bending moment and shear force are discontinuous despite slight discontinuities in the deflection, opening, and dislocation. The deflection curve is consistent with the additional pressure profile. Extensive opening, dislocation, and internal forces are induced at the location of mutation pressures. In addition, the joints allow rigid structures to behave flexibly in general, as well as allow flexible structures to exhibit locally rigid characteristics. Owing to the discontinuous characteristics, the internal forces and their abrupt changes at vulnerable sections must be monitored to ensure the structural safety of existing shield tunnels.

KEYWORDS tunnel–soil interaction, discontinuous analysis, longitudinal joints, existing shield tunnel, Timoshenko beam, Dirac delta function

1 Introduction

Over the last three decades, underground spaces have developed rapidly worldwide. A significant number of tunnels have been constructed, which inevitably cross existing underground structures such as tunnels, pipelines, and subway stations. Thus, tunnel–soil interaction (TSI) is vital for guaranteeing the safety and serviceability of existing structures. The mechanical behaviors of existing tunnels have been investigated extensively owing to the construction of new tunnels via experiments and numerical simulations [1–8], empirical methods [9–12], and analytical methods [13–17]. Centrifuge tests can

capture the correct stress–strain behavior of a tunnel and soil in TSI problems. Empirical analyses, accompanied by numerical simulations, are typically performed to analyze specific engineering applications. The analytical method is a general and convenient method for analyzing TSI problems. Additionally, many influencing factors have been investigated, such as ground condition [18–20], structure configuration [12], crossing angle [21], crossing type [22,23], potential soil gap formation [24], and weak joints [25,26].

Weak joints include movement joints, construction joints of the composite-lining tunnel, and longitudinal and circumferential joints of the shield tunnel [27–29]. Liu et al. [30] investigated the bending deflection, rotation, and shearing dislocations of movement joints in a

composite-lining tunnel. Six deformation modes were included in the composite-lining tunnel based on the location of the movement joints. Wu et al. [31,32] analyzed the longitudinal behaviors of an existing shield tunnel by considering shear dislocation at the longitudinal joints. Two deformation modes of segmental rings are presented: an opening between the segmental rings (Fig. 1(a)) and dislocation between the segmental rings (Fig. 1(b)). The third deformation mode combines both the opening and dislocation modes, as shown in Fig. 1(c). The joint opening and dislocation result in problems such as groundwater leakage [33–35], concrete cracks [36,37], track distortion, and separation between a ballast-less bed and the lining, which may jeopardize the safety of a tunnel during its operation [38]. Artificial intelligence technology, such as image processing and machine learning [39,40], is used extensively in geotechnical engineering for recognizing, analyzing, and processing hazards. The differential deformation at the joints of shield tunnels during their construction and operation has garnered significant attention.

In terms of analytical methods, the beam–spring model [41] and longitudinal continuous model [42] are typically used to investigate TSI problems involving longitudinal joints. In the beam–spring model, the standard structural segment is regarded as a beam and the weak joints as elastic springs to model the moment, axial, and shear forces. The longitudinal continuous model considers the existing shield tunnel as a homogenous continuous beam with stiffness reduction at weak joints, in which the stiffness of the tunnel is obtained by an equivalent

method. Previous studies have primarily considered the existing shield tunnel as an Euler–Bernoulli beam or a Timoshenko beam. The TSI problem is solved using conventional elastic foundation models.

Various combinations of beam and foundation models were applied to simulate the mechanical behaviors of an existing shield tunnel above a new tunnel. Some analytical models fail to capture either the joint opening or joint dislocation. Moreover, the longitudinal continuous model disregards weak joints and uses the equivalent stiffness, unlike the beam–spring model. The beam–spring model cannot be easily used to solve TSI problems as it requires complicated calculations. In addition, the aforementioned models assume that the existing shield tunnel behaves continuously during deformation; thus, the discontinuous deformation and internal forces of the existing shield tunnel cannot be predicted accurately.

Previously, a composite-lining tunnel modeled using Euler–Bernoulli beams with movement joints were investigated [30]. Six deformation modes were summarized based on the relative location and quantity of movement joints. Using the findings obtained, we estimate the discontinuous mechanical behaviors of an existing shield tunnel with a series of longitudinal joints in this study. The segment rings are modeled using Timoshenko beams instead of Euler–Bernoulli beams. The usage conditions of these two beam models are discussed and confirm that a rigid (flexible) tunnel generates flexible (rigid) deformation owing to the longitudinal joints (movement joints).

Herein, we propose an analytical method that uses a Timoshenko beam with Dirac delta functions on the Winkler foundation, abbreviated as the TDW method. Using the Dirac delta functions, the mathematical model can predict the discontinuous deformation and internal forces at the longitudinal joints, at which bending and shearing stiffness reductions are considered. Our proposed method is validated based on comparison with measured data and previous approaches. The effects of the stiffness reduction coefficients α and β at the longitudinal joints, the shear stiffness GA , and the additional pressure profiles on the responses of an existing shield tunnel are investigated. Finally, the deformation characteristics of a Timoshenko beam in modeling a shield tunnel with longitudinal joints vs. those of an Euler–Bernoulli beam in modeling a composite-lining tunnel with movement joints are compared and discussed.

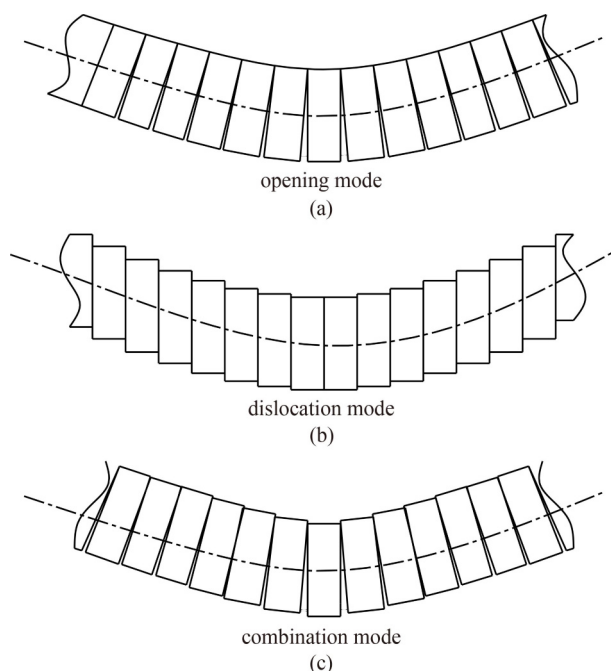


Fig. 1 Deformation modes of segmental rings: (a) opening mode; (b) dislocation mode; (c) combination mode.

2 Mathematical model of longitudinal joints

A novel mathematical model is proposed to analyze the deformation modes at the longitudinal joints of an

existing shield tunnel, as shown in Fig. 1. We applied two virtual pressures, $e_1(x)$ and $e_2(x)$, at the longitudinal joints to obtain the joint opening and dislocation. The virtual pressures are represented by the first-order derivative $\delta'(x - x_j)$ and second-order derivative $\delta''(x - x_j)$ of the Dirac delta function. The expressions for virtual pressures $e_1(x)$ and $e_2(x)$ are detailed in this section.

2.1 Derivative equations of Dirac delta function

The derivative equations of the Dirac delta function, $\delta'(x - x_j)$ and $\delta''(x - x_j)$, are used to establish the mathematical model. The two expressions can be obtained using the second- and third-order derivatives of the Heaviside step function $H(x - x_j)$. Figure 2 shows a graphical representation of the Heaviside step function $H(x - x_j)$, where x_j is the joint location:

$$H(x - x_j) = \begin{cases} 0, & x < x_j, \\ 0.5, & x = x_j, \\ 1, & x > x_j. \end{cases} \quad (1)$$

The derivative equations for the Dirac delta function are as follows.

$$\begin{aligned} \delta'(x - x_j) &= \lim_{\Delta x \rightarrow 0} \frac{\Delta'' H(x - x_j)}{(\Delta x)''} \\ &= \lim_{h \rightarrow 0} \left\{ \frac{H[x - (x_j - 2h)] - 2H(x - x_j) + H[x - (x_j + 2h)]}{4h^2} \right\} \\ &= \begin{cases} 0, & (x < x_j - 2h) \text{ or } (x > x_j + 2h) \\ 1/(4h^2), & (x_j - 2h < x < x_j) \text{ or } (x_j < x < x_j + 2h) \end{cases} \quad (2) \end{aligned}$$

$$\begin{aligned} \delta''(x - x_j) &= \lim_{\Delta x \rightarrow 0} \frac{\Delta''' H(x - x_j)}{(\Delta x)'''} \\ &= \lim_{h \rightarrow 0} \left\{ \frac{H[x - (x_j - 3h)] - 3H[x - (x_j - h)]}{8h^3} \right. \\ &\quad \left. + \frac{3H[x - (x_j + h)] - H[x - (x_j + 3h)]}{8h^3} \right\} \\ &= \begin{cases} 0, & (x < x_j - 3h) \text{ or } (x > x_j + 3h) \\ 1/(8h^3), & (x_j - 3h < x < x_j - h) \text{ or } (x_j + h < x < x_j + 3h) \\ 1/(4h^3), & (x_j - h < x < x_j + h) \end{cases} \quad (3) \end{aligned}$$

where h denotes an arbitrary infinitesimal value. When $h \rightarrow 0$, function $\delta_h(x - x_j)$ can degenerate to $\delta(x - x_j)$.

Figure 3 shows the mathematical models of the longitudinal joints. Functions $\delta'(x - x_j)$ and $\delta''(x - x_j)$ were utilized to model the shearing dislocation and bending opening of the longitudinal joints, respectively.

2.2 Virtual pressures at longitudinal joints

As shown in Fig. 3, virtual pressures $e_1(x)$ and $e_2(x)$ at the

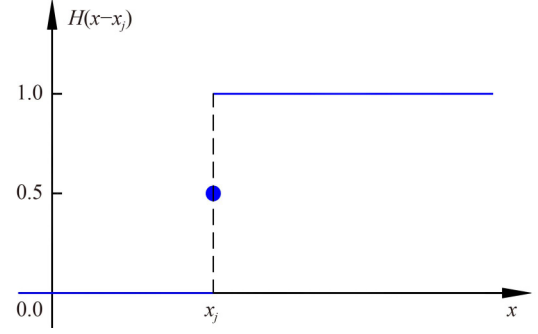


Fig. 2 Graphical representations of Heaviside step function $H(x - x_j)$.

joints are applied and expressed by functions $\delta'(x - x_j)$ and $\delta''(x - x_j)$ to model the bending opening and shearing dislocation of the joints, respectively. Virtual pressure $e_1(x)$ can be expressed as $e_1(x) = C_1 \delta'(x - x_j)$, which is analogous to two equal concentrated forces $2F_1$ and one opposite concentrated force F_1 exerting at the joints. Similarly, virtual pressure $e_2(x)$ can be expressed as $e_2(x) = C_2 \delta''(x - x_j)$, which is analogous to a pair of couples M_2 applying at the joints, where C_1 and C_2 are constants.

Concentrated forces F_1 and M_2 can be obtained using Eqs. (4) and (5), respectively.

$$F_1 = Q_1 = \frac{C_1}{4h^2} \cdot 2h, \quad (4)$$

$$F_2 = \frac{C_2}{8h^3} \cdot 2h = \frac{C_2}{4h^2}, \quad (5)$$

$$M_2 = F_2 \cdot 2h = \frac{C_2}{2h}.$$

Based on material mechanics theory, the internal forces induce the relative rotation angle $\Delta\theta$ and relative dislocation $\Delta\delta$, which are expressed as follows.

$$\Delta\delta = \frac{Q_1}{GA} \cdot 2h = \frac{C_1}{GA}, \quad (6)$$

$$\Delta\theta = \frac{M_2}{EI} \cdot 2h = \frac{C_2}{EI}, \quad (7)$$

where GA is the shear stiffness, and EI is the bending stiffness.

The virtual pressures at the longitudinal joints can be rewritten using the relative rotation angle, relative dislocation, and derivative equations of the Dirac delta function.

$$e_1(x) = GA(\Delta\delta)\delta'(x - x_j), \quad (8)$$

$$e_2(x) = EI\Delta\theta\delta''(x - x_j). \quad (9)$$

3 Tunnel–soil interaction model with longitudinal joints

The existing shield tunnel comprises precast reinforced concrete segments connected by longitudinal and circumferential joints. Each ring of the existing shield tunnel resembles a short and thick beam, which can be modeled as a Timoshenko beam. Meanwhile, an Euler–Bernoulli beam is suitable for modeling a slender beam that reflects only flexure deformation under a bending moment. Joint dislocations under shear forces, which are widely observed, cannot be investigated. Therefore, some scholars [32,43] have indicated that using the Timoshenko beam for solving TSI problems in shield tunnels can provide accurate results.

3.1 Timoshenko beam with Dirac delta functions on Winkler foundation

The existing shield tunnel with longitudinal joints was modeled as a TDW model to precisely calculate the

discontinuous deformation. Figure 4 shows the TDW model. The results were obtained using the EI and GA of the rings, as well as the reduced stiffness of the longitudinal joints instead of the equivalent stiffness of the Timoshenko beam.

According to previous studies [23,32], the governing equation for the TDW model is expressed as follows.

$$\frac{d^4 w(x)}{dx^4} - \frac{k}{GA} \frac{d^2 w(x)}{dx^2} + \frac{k}{EI} w(x) = \frac{q(x)}{EI} - \frac{1}{GA} \frac{d^2 q(x)}{dx^2}, \quad (10)$$

where k is the coefficient of the subgrade reaction, $q(x)$ the additional pressure acting on the beam, and $w(x)$ the beam deflection.

The bending moment and shear force of the rings can be obtained as follows.

$$\begin{aligned} M(x) &= -EI \left(\frac{d^2 w}{dx^2} - \frac{k}{GA} w + \frac{q}{GA} \right), \\ Q(x) &= -EI \left(\frac{d^3 w}{dx^3} - \frac{k}{GA} \frac{dw}{dx} + \frac{1}{GA} \frac{dq}{dx} \right). \end{aligned} \quad (11)$$

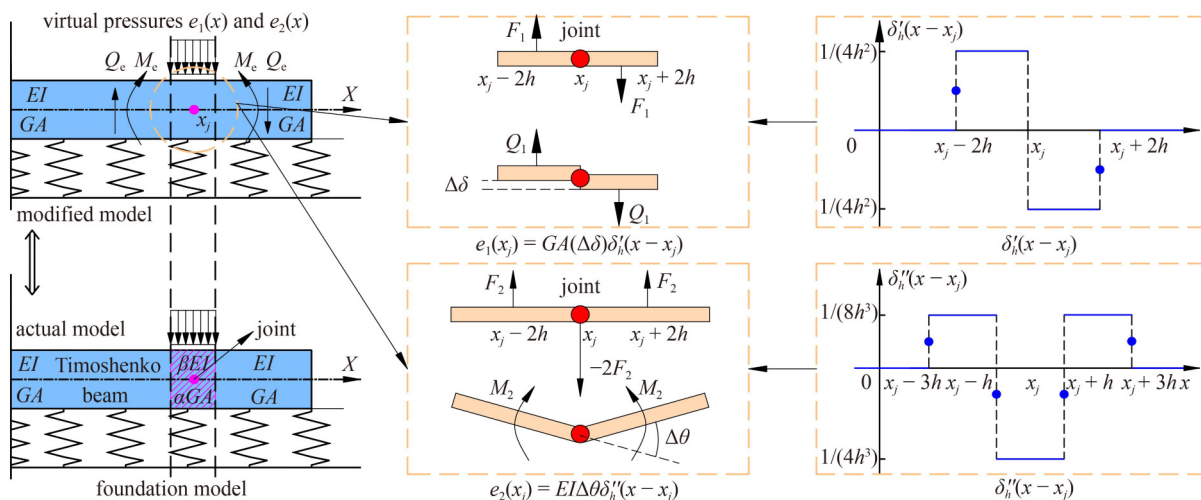


Fig. 3 Mathematical model of longitudinal joints.

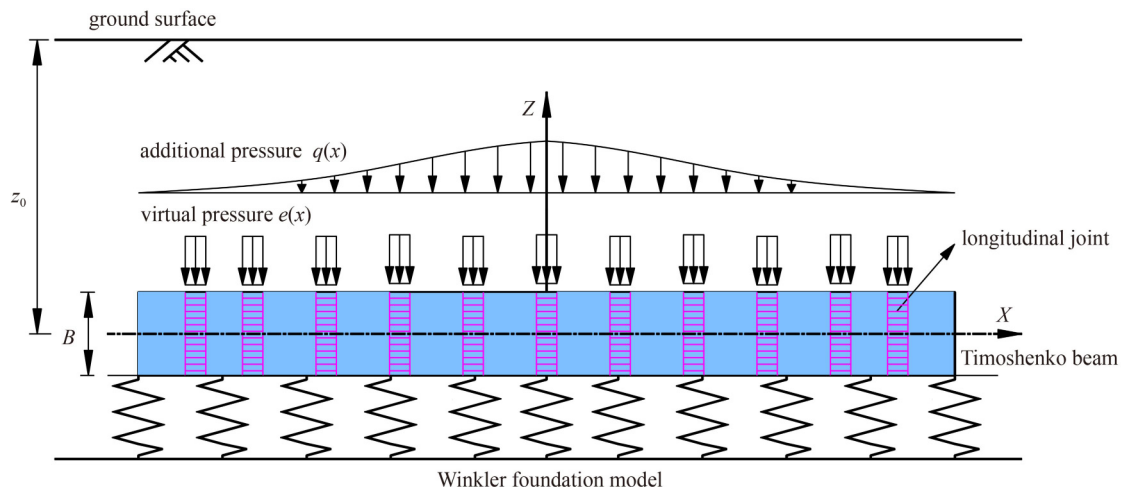


Fig. 4 TDW model.

To represent the reduced stiffness of the longitudinal joints, we assumed that the shearing stiffness and bending stiffness of the longitudinal joints were αGA and βEI , respectively, where α and β are the coefficients of the stiffness reduction. The internal forces of the longitudinal joints are expressed as follows.

$$\begin{aligned} M_j(x_j) &= -\beta EI \left(\frac{d^2 w}{dx^2} - \frac{k}{GA} w + \frac{q}{GA} \right), \\ Q_j(x_j) &= -\alpha EI \left(\frac{d^3 w}{dx^3} - \frac{k}{GA} \frac{dw}{dx} + \frac{1}{GA} \frac{dq}{dx} \right). \end{aligned} \quad (12)$$

If $\alpha = \beta = 1$, then the existing tunnels are without joints. Without considering the longitudinal joints, the stiffness along the existing tunnels was homogeneous. Thus, $M_j(x_j) = M(x_j)$, and $Q_j(x_j) = Q(x_j)$. In particular, the longitudinal joints represent hinge joints if $\alpha = \beta = 0$.

As shown in Fig. 3, virtual pressures $e_1(x)$ and $e_2(x)$ proposed in Section 2 were applied at the longitudinal joints to obtain the virtual internal forces ($M_e(x_j)$ and $Q_e(x_j)$). The internal forces of the rings, i.e., $M(x_j)$ and $Q(x_j)$ at $x = x_j$, must reduce $M_e(x_j)$ and $Q_e(x_j)$ such that the internal forces of the longitudinal joints, i.e., $M_j(x_j)$ and $Q_j(x_j)$, can be obtained.

$$\begin{aligned} M_j(x_j) &= M(x_j) - M_e(x_j), \\ Q_j(x_j) &= Q(x_j) - Q_e(x_j). \end{aligned} \quad (13)$$

Substituting Eqs. (11) and (12) into Eq. (13), $M_e(x_j)$ and $Q_e(x_j)$ can be obtained as follows.

$$\begin{aligned} M_e(x_j) &= -(1 - \beta) EI \left(\frac{d^2 w}{dx^2} - \frac{k}{GA} w + \frac{q}{GA} \right), \\ Q_e(x_j) &= -(1 - \alpha) EI \left(\frac{d^3 w}{dx^3} - \frac{k}{GA} \frac{dw}{dx} + \frac{1}{GA} \frac{dq}{dx} \right). \end{aligned} \quad (14)$$

Based on Eqs. (6) and (7), the reduced internal forces can be rewritten as follows.

$$\begin{aligned} M_e(x) &= \frac{EI \Delta \theta}{2h}, \\ Q_e(x) &= \frac{GA \Delta \delta}{2h}. \end{aligned} \quad (15)$$

By referring to Eqs. (14) and (15), the relative rotation angle $\Delta \theta_j$ and relative dislocation $\Delta \delta_j$ can be expressed as

$$\begin{aligned} \Delta \theta_j &= -2h(1 - \beta) \left(\frac{d^2 w}{dx^2} - \frac{k}{GA} w + \frac{q}{GA} \right), \\ \Delta \delta_j &= -2h(1 - \alpha) \frac{EI}{GA} \left(\frac{d^3 w}{dx^3} - \frac{k}{GA} \frac{dw}{dx} + \frac{1}{GA} \frac{dq}{dx} \right). \end{aligned} \quad (16)$$

Subsequently, virtual pressures $e_1(x)$ and $e_2(x)$ can be written as

$$\begin{aligned} e_1(x_j) &= 2h(1 - \alpha) EI \delta'(x - x_j) \\ &\quad \cdot \left(\frac{d^3 w(x_j)}{dx^3} - \frac{k}{GA} \frac{dw(x_j)}{dx} + \frac{1}{GA} \frac{dq(x_j)}{dx} \right), \\ e_2(x_j) &= 2h(1 - \beta) EI \delta''(x - x_j) \\ &\quad \cdot \left(\frac{d^2 w(x_j)}{dx^2} - \frac{k}{GA} w(x_j) + \frac{q(x_j)}{GA} \right). \end{aligned} \quad (17)$$

3.2 Solution to calculation model

Substituting virtual pressures $e_1(x)$ and $e_2(x)$ into the governing differential equation (Eq. (10)) yields

$$\begin{aligned} \frac{d^4 w(x)}{dx^4} - \frac{k}{GA} \frac{d^2 w(x)}{dx^2} + \frac{k}{EI} w(x) &= \frac{1}{EI} (q(x) + e_1(x_j) + e_2(x_j)) \\ &\quad - \frac{1}{GA} \left(\frac{d^2 q(x)}{dx^2} + \frac{d^2 e_1(x_j)}{dx^2} + \frac{d^2 e_2(x_j)}{dx^2} \right), \end{aligned} \quad (18)$$

where $e_1(x_j)$ and $e_2(x_j)$ are constants at $x = x_j$. The second derivatives of $e_1(x_j)$ and $e_2(x_j)$ are both zero. Therefore, Eq. (18) can be rewritten as follows.

$$\begin{aligned} \frac{d^4 w(x)}{dx^4} - \frac{k}{GA} \frac{d^2 w(x)}{dx^2} + \frac{k}{EI} w(x) \\ = \frac{1}{EI} (q(x) + e_1(x_j) + e_2(x_j)) - \frac{1}{GA} \frac{d^2 q(x)}{dx^2}. \end{aligned} \quad (19)$$

Although the closed solution to Eq. (19) is difficult to obtain, the results can be calculated using the finite-difference method. The shield tunnel contains $n + 5$ elements, each of which exhibits a length l of $2h$. Using Eqs. (2) and (3), the finite differential form of the governing equation can be obtained. Subsequently, we can rewrite the governing equation in the matrix-vector form as follows.

$$\begin{aligned} ([M_1] - [M_2] + [M_3] - [E_{11}] - [E_{21}] + [E_{12}] + [E_{22}])\{w\} \\ = ([P_1] + [QE_2])\{Q_1\} + [QE_1]\{Q_2\} - [P_2]\{Q_3\} - \{Q_4\}. \end{aligned} \quad (20)$$

Matrices $[M_1]$, $[M_2]$, $[M_3]$, $[E_{11}]$, $[E_{12}]$, $[E_{21}]$, and $[E_{22}]$ are displacement stiffness matrices; $\{w\}$ is the displacement vector; $\{Q_1\}$, $\{Q_2\}$, and $\{Q_3\}$ are the modified additional stress vectors; $\{Q_4\}$ is the supplement vector; $[P_1]$ and $[P_2]$ are the stress matrices due to $\{Q_1\}$ and $\{Q_3\}$, respectively; $[QE_1]$ and $[QE_2]$ are defined as the modified additional stress stiffness matrices owing to virtual pressure $e_1(x)$ and $e_2(x)$, respectively. The detailed equations, matrices, and vectors are listed in Appendix B.

The discontinuous deflection of the existing shield tunnel $\{w\}$ can be solved using Eq. (18). Based on Eqs. (11) and (12), the discontinuous internal forces of the existing shield tunnel can be obtained as follows.

$$M = \begin{cases} -EI \left[\frac{d^2 w(x)}{dx^2} - \frac{k}{GA} w(x) + \frac{1}{GA} q(x) \right], & (x \neq x_j) \\ -\beta EI \left[\frac{d^2 w(x)}{dx^2} - \frac{k}{GA} w(x) + \frac{1}{GA} q(x) \right], & (x = x_j) \end{cases} \quad (21)$$

$$Q = \begin{cases} -EI \left[\frac{d^3 w(x)}{dx^3} - \frac{k}{GA} \frac{dw(x)}{dx} + \frac{1}{GA} \frac{dq(x)}{dx} \right], & (x \neq x_j) \\ -\alpha EI \left[\frac{d^3 w(x)}{dx^3} - \frac{k}{GA} \frac{dw(x)}{dx} + \frac{1}{GA} \frac{dq(x)}{dx} \right], & (x = x_j) \end{cases} \quad (22)$$

Subsequently, based on Eq. (16), the relative rotation angle and dislocation at the longitudinal joints can be obtained as follows.

$$\begin{aligned} \Delta\theta_j &= -l(1-\beta) \left(\frac{d^2 w(x_j)}{dx^2} - \frac{k}{GA} w(x_j) + \frac{q(x_j)}{GA} \right), \\ \Delta\delta_j &= -l(1-\alpha) \frac{EI}{GA} \left(\frac{d^3 w(x_j)}{dx^3} - \frac{k}{GA} \frac{dw(x_j)}{dx} + \frac{1}{GA} \frac{dq(x_j)}{dx} \right). \end{aligned} \quad (23)$$

Wu et al. [32] established a geometric relationship between the relative rotation angle and opening of longitudinal joints, as follows.

$$\Delta = \Delta\theta_j \left(\frac{D}{2} + \frac{D}{2} \tan \psi \right), \quad (24)$$

where D is the diameter of the existing shield tunnel, and ψ is the neutral-axis angle.

3.3 Calculation parameters

3.3.1 Tunneling-induced subsurface settlements

The additional pressure exerting on the existing shield tunnel $q(x)$ was obtained based on the tunneling-induced subsurface settlement $s(x)$. The subsurface settlement trough can be described using a Gaussian distribution curve [44].

$$\begin{aligned} s(x) &= s_{\max} \exp \left(-\frac{x^2}{2i^2} \right), \\ s_{\max} &= \frac{\pi r^2 V_l}{\sqrt{2\pi} i}, \\ i &= K(z_0 - z), \end{aligned} \quad (25)$$

where $s_{\max}$ is the maximum subsurface settlement, which primarily depends on the new tunneling-induced volume loss V_l and new tunnel radius r ; i represents the settlement trough width, which is related to the trough width parameter K ; z_0 and z are the buried depths of the new tunnel axis and subsurface, respectively.

Several studies [21,45] have considered the trough width parameter K at different buried depths. Mair et al. [46] proposed the following solution for K , which is widely used.

$$K = \frac{0.325 + 0.175(1 - z/z_0)}{1 - z/z_0}. \quad (26)$$

3.3.2 Coefficient of subgrade reaction

Many scholars [47] have attempted to obtain an accurate coefficient of the subgrade reaction using different solutions. The parameters in the solutions primarily include the soil elastic modulus E_s , soil Poisson's ratio μ , and the diameter of the existing shield D . The following solution proposed by Yu et al. [48] is typically used in studies pertaining to TSIs.

$$\begin{aligned} k &= \frac{3.08}{\eta} \frac{E_s}{1 - \mu^2} \sqrt{\frac{E_s D^4}{EI}}, \\ \eta &= \begin{cases} 2.18, & \text{when } z_0/D \leq 0.5, \\ 1 + \frac{1}{1.7z_0/D}, & \text{when } z_0/D > 0.5. \end{cases} \end{aligned} \quad (27)$$

4 Verification and comparison

Data comparable to those pertaining to the discontinuous deflection of an existing shield tunnel above a new tunnel are rare. Therefore, we verified our proposed method using the experimental data of a jointed pipeline obtained from centrifuge tests conducted by Vorster [49]. A model pipeline constructed using aluminum alloy with an outer diameter of 15.875 mm and a thickness of 1.22 mm was used under a 75g acceleration in the centrifuge tests. The prototype pipeline with nine joints featured an outer diameter of 1.19 m and a thickness of 0.09 m, and was buried at a depth of 4.165 m. A prototype new tunnel with a diameter of 4.5 m and an overburden depth of 11.25 m was excavated directly beneath the middle joint. The 0.3% and 2.0% volume loss ratios of the new tunnel were controlled in Tests 1 and 2, respectively. The elastic modulus and Poisson's ratio of the soil were assumed to be 10 MPa and 0.3, respectively, since Leighton Buzzard Fraction E silica sand was used in the tests. The relevant parameters can be calculated using Eqs. (25), (26), and (27). Based on the parameters published by Vorster [49] and Lin and Huang [50], the geometrical dimensions and physical parameters of the prototype are listed in Table 1. Notably, the coefficients of stiffness reduction, α and β , were set to zero to perform a comparison with the results.

Figures 5 and 6 show the experimental data and corresponding results, respectively, which were used to

Table 1 Geometrical dimensions and physical parameters for verification and comparison [49,50]

pipeline							tunnel			soil		
D (m)	L (m)	E (GPa)	I (m ⁴)	A (m ²)	G (10 ¹⁰ N·m ²)	z_p (m)	R_0 (m)	Z_0 (m)	V_1 (%)	E_s (MPa)	μ	i
									Test 1	Test 1		
1.19	48.06	70	0.0474	0.3	2.9944	4.165	2.25	11.25	0.3	2	10	0.3 4.3

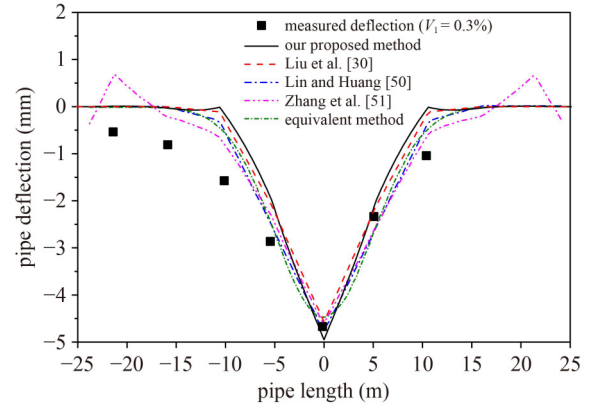
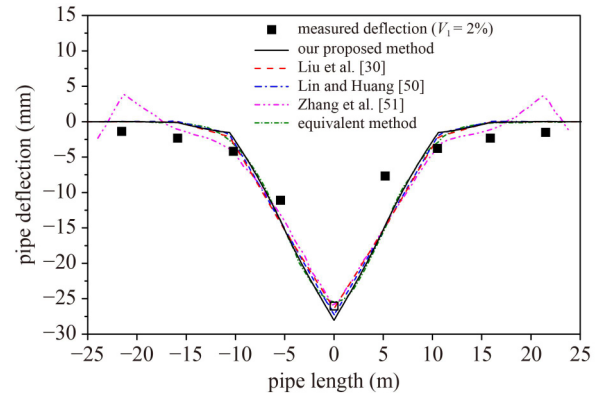
validate and compare our method with the previous methods. The equivalent method does not consider the discontinuity of the joints; thus, the pipe deflection curve is smooth in the longitudinal direction. The elastic continuous method [51] provides acceptable predictions within a specific region. The methods proposed by Lin and Huang [50] and Liu et al. [30] can be used to estimate the joints' rotation and dislocation. The results obtained by Liu et al. [30] are more similar to the experimental data than those obtained by Lin and Huang [50]. The method proposed by Liu et al. [30] uses different stiffnesses of the standard segment and joint instead of the equivalent stiffness. A specific case in which the joints are hinged ($\alpha = \beta = 0$) can be analyzed as well.

The analytical results yielded by our method were consistent with the centrifuge data. Discontinuity was evident at the joints. One of the most important advantages of our method is that it allows us to use the stiffness of the ring and longitudinal joints separately, and not the equivalent stiffness. Differential governing equations with Dirac delta functions that model longitudinal joints have definite physical meanings, and the results can be obtained easily without complicated calculations.

Compared with the method proposed by Liu et al. [30], our method yielded a slightly larger deflection. The reasons for the error are as follows: (1) the calculated deformation was underestimated when the Euler–Bernoulli beam was used; (2) the jointed pipeline in the centrifuge tests is suitable for slender beams such as the Euler–Bernoulli beam, whereas the Timoshenko beam applied in our method is suitable for modeling the shield tunnel [32]; (3) the stiffness at the joints was assumed to be zero in our method. However, some stiffness may remain at the joint in the centrifuge tests, thus affecting the results. In general, our proposed method is applicable for predicting discontinuous deformations.

5 Parametric analyses

In our method, the deflection, rotation, and dislocation results primarily relied on the stiffness reduction of the longitudinal joints. The GA of the Timoshenko beam is an important parameter that differs from that of the Euler–Bernoulli beam. Different additional pressure profiles generated different deformation modes in the shield tunnel. Thus, we considered the effects of the

**Fig. 5** Verification and comparison of our proposed method based on Test 1.**Fig. 6** Verification and comparison of our proposed method based on Test 2.

aforementioned parameters on the discontinuous mechanical behavior of an existing shield tunnel. Table 2 lists the calculation parameters used to obtain the discontinuous deformation and internal forces. The overall length of the shield tunnel L was set to 100 m, and the width of each segmental ring l_s was 1 m.

5.1 Coefficient of shearing stiffness reduction α

The same parameters listed in Table 2, except for α and β , were used to investigate the effects of the coefficient of shearing stiffness reduction α . We assumed α to be 0, 0.01, 0.1, 0.5, and 1. Considering only the effect of α , zero relative rotations were generated at the longitudinal joints because β was set to 1. Our approach incorporates

the general method when α and β are set to 1, where the Timoshenko beam and the Winkler model are used without considering the longitudinal joints.

Figure 7 shows the deflection of an existing shield tunnel and the dislocation of the longitudinal joints. As α increased, the deflection and dislocation decreased. The deflection curve was smooth, and the width of the curve expanded gradually. The dislocation curve represents discontinuity when α is sufficiently small. The discontinuity of the dislocation was not evident as α increased. When $\alpha = 1$, zero dislocations were generated at the longitudinal joints.

Table 2 Input parameters for factor analyses

parameter	unit	value
α	—	0.01
β	—	0.01
D	m	6
L	m	100
l	m	0.1
l_s	m	1
EI	GPa·m ⁴	30
GA	GPa·m ⁴	30
k	MPa/m	30

Figure 8 shows the internal forces of an existing shield tunnel. Both the bending moment and shear force indicated significant levels of discontinuity. The increase in α decreased the internal forces because the discontinuous internal forces at the longitudinal joints decreased gradually. Meanwhile, the shear forces of the rings increased. When α increased to 1, the bending moment and shear force were continuous and smooth, which were similar to the results yielded by the conventional method.

5.2 Coefficient of bending stiffness reduction β

The effects of β on the deflection and relative rotation of the longitudinal joints are shown in Fig. 9. The dislocations of the joints were zero because α was set to 1. The deflection and rotation were slightly discontinuous when β was zero. The deflection and rotation decreased as β increased. The resultant curves became increasingly wider. When $\beta = 1$, the relative rotations at the longitudinal joints were zero.

Figure 10 shows the internal forces of an existing shield tunnel. The bending moment and shear force increased as β increased (and $\alpha = 1$). Although the bending moment curve was smooth, the bending moment was zero at the longitudinal joints when $\beta = 0$. Owing to the shear force discontinuity at the longitudinal joints, the shear force curve was serrated in some regions. When $\beta = 1$, the

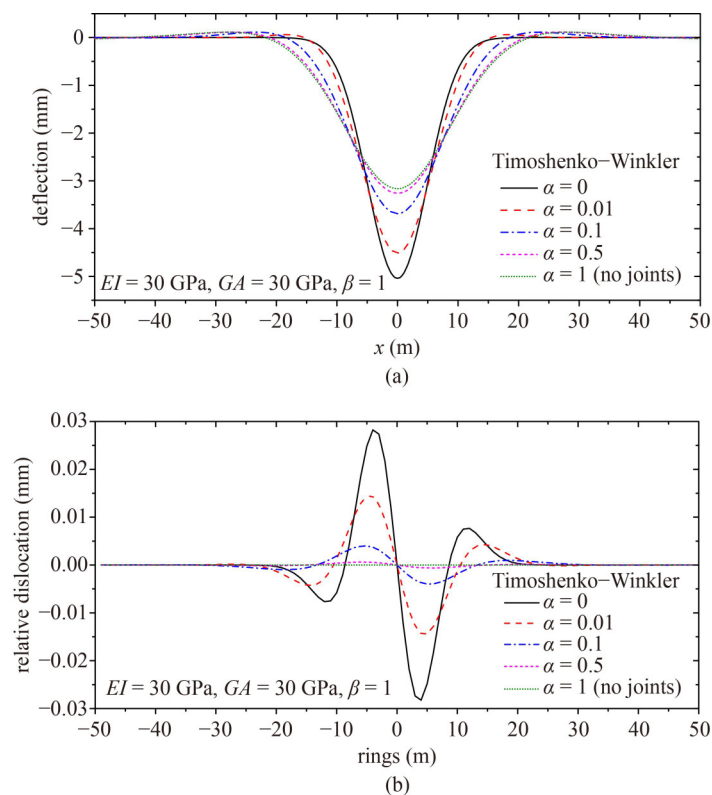


Fig. 7 Discontinuous deformation considering shearing stiffness reduction at longitudinal joints: (a) deflection of an existing shield tunnel; (b) relative dislocation of the longitudinal joints.

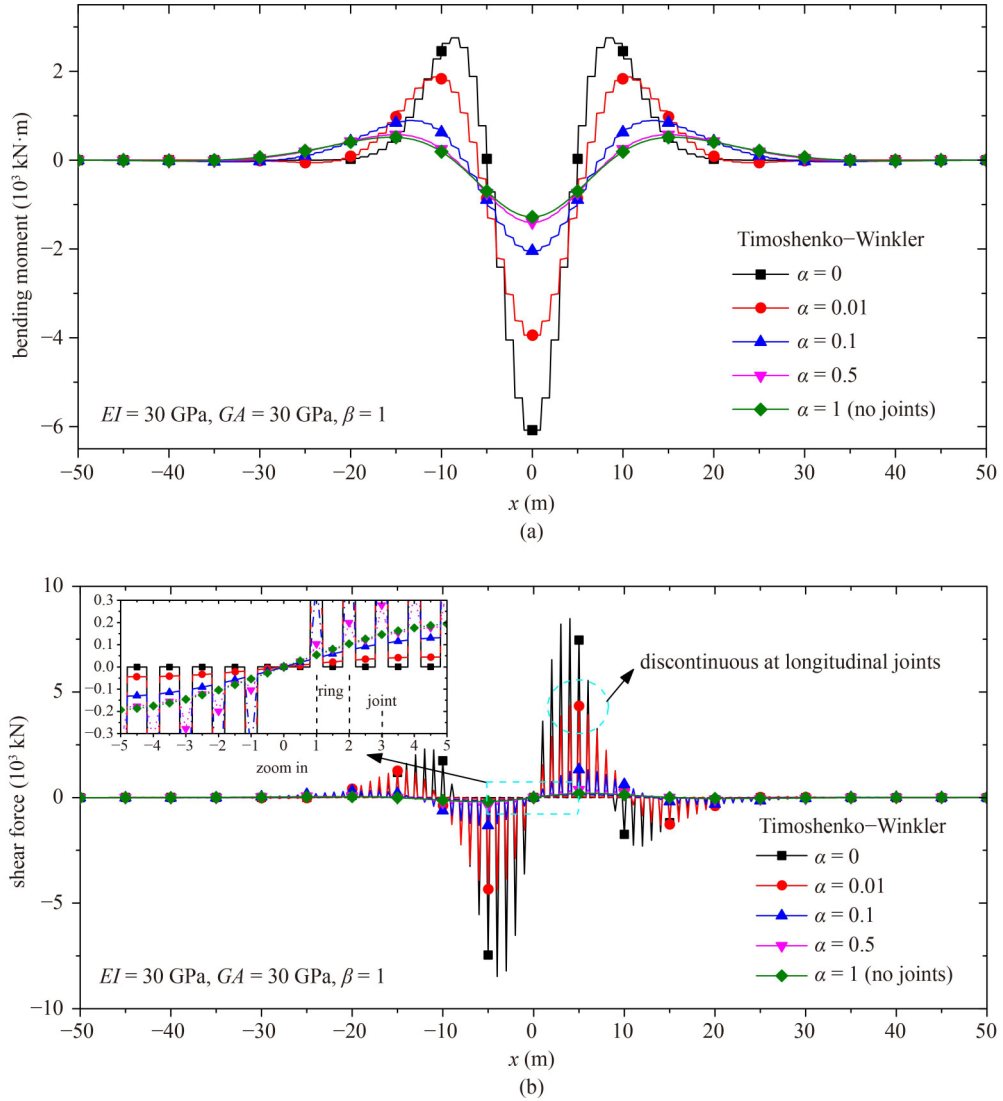


Fig. 8 Discontinuous internal forces considering shearing stiffness reduction at longitudinal joints: (a) bending moment; (b) shear force.

bending moment and shear force remained constant, i.e., without considering the longitudinal joints.

5.3 GA of Timoshenko beam

GA is an essential parameter that defines Timoshenko beams. To investigate the effect of GA on the deformation and internal force, we assumed that $\alpha = \beta = 0.01$, and set GA to 5, 10, and 30 GPa, and infinite. Figure 11 shows the results of deflections, rotations, and dislocations. As GA increased, the deformation and discontinuity decreased. When GA approached infinity, the Timoshenko beam degenerated to the Euler-Bernoulli beam. In this case, the deflection, rotation, and dislocation were continuous and indicated the minimum values. Shearing dislocations did not occur at the longitudinal joints.

Figure 12 shows the bending moment and shear force of an existing shield tunnel. The internal forces of the existing shield tunnel showed a significant level of

discontinuity. As GA increased, the internal forces decreased gradually.

5.4 Additional pressure profile

The additional pressure profile, which is typically assumed to be a Gaussian function in analytical methods, was obtained based on the tunneling-induced subsurface settlement. However, different pressure distributions can result in different deformation mechanisms. In this section, we assume that the subsurface settlement can be regarded as a rectangle, an isosceles trapezoid, or a right triangle. For the rectangular settlement, the subsurface settlement measures 5 mm and is located between -10.5 to 10.5 m. For the isosceles trapezoidal settlement, the maximum settlement measures 5 mm and is located between -10.5 to 10.5 m; furthermore, it reduces linearly to zero between -20.5 and 20.5 m. A right triangle, which

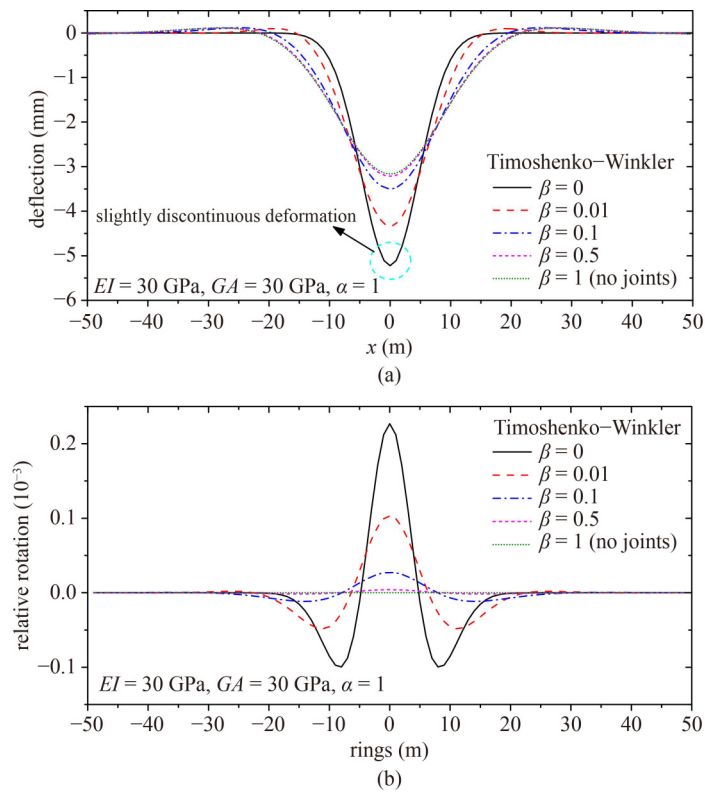


Fig. 9 Discontinuous deformation considering bending stiffness reduction at longitudinal joints: (a) deflection of an existing shield tunnel; (b) relative rotation of the longitudinal joints.

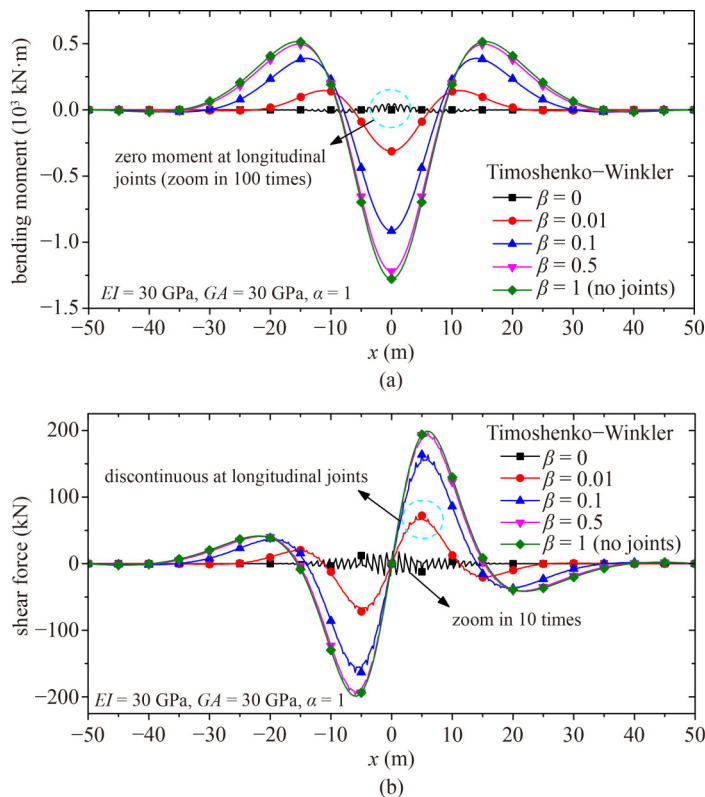


Fig. 10 Discontinuous internal forces considering bending stiffness reduction at longitudinal joints: (a) bending moment; (b) shear force.

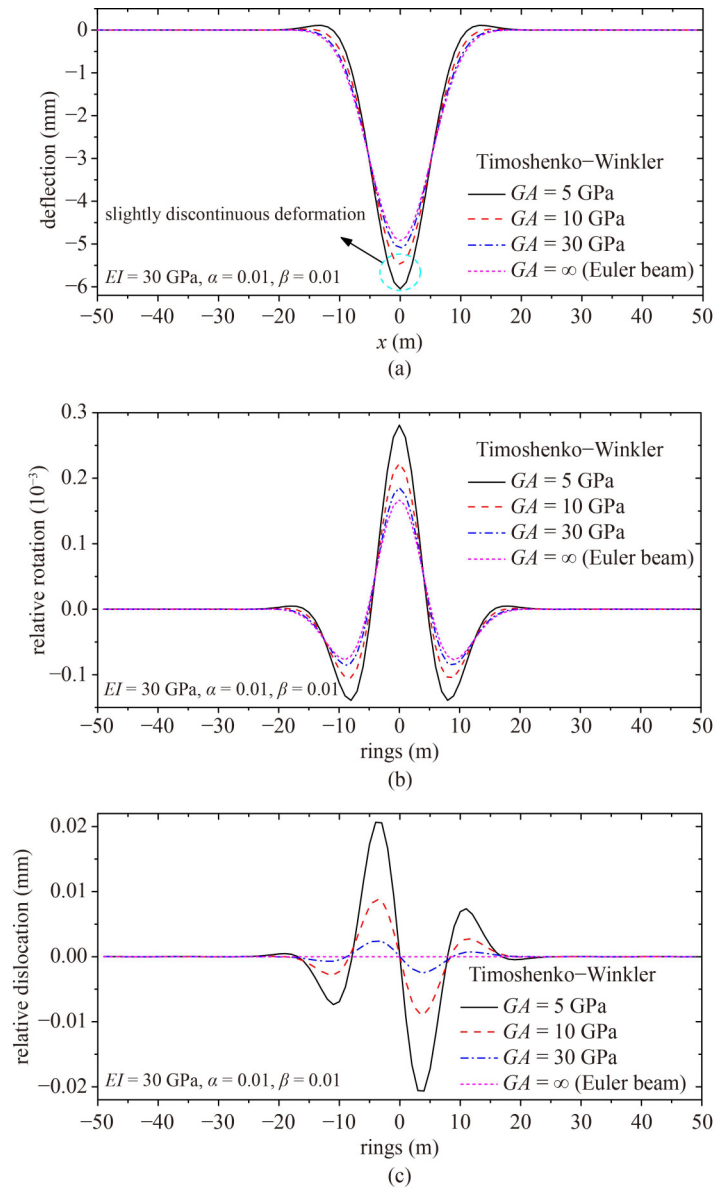


Fig. 11 Discontinuous deformation considering shearing stiffness GA : (a) deflection; (b) relative rotations; (c) relative dislocations.

measures 0 mm at -20.5 m and 5 mm at 20.5 m, is applied to investigate the effects of asymmetric pressures.

Figure 13 shows the deformation of an existing shield tunnel based on different pressure profiles. The deflection curves were similar to those of the additional pressure profile. Extreme rotation and dislocation of the longitudinal joints occurred on both sides of the rectangle and on the right-angle side of the right triangle. Similar to the case at both sides of the isosceles trapezoid, the smoothed pressures resulted in slight rotation and dislocation. Figure 14 shows the bending moment and shear force. At the location where mutation pressures occurred, the internal forces changed rapidly, which was similar to the characteristics of rotation and dislocation. The discontinuities in the bending moment and shear force were evident.

6 Discussion

Figure 15 illustrates the Timoshenko beam for the shield tunnel vs. the Euler-Bernoulli beam for the composite-lining tunnel. The Euler-Bernoulli beam primarily considers flexural deformation under a bending moment and is widely used to model slender beams. The Timoshenko beam is improved based on the Euler-Bernoulli beam by accounting for shearing deformation under shear force. A short and thick beam is typically regarded as a Timoshenko beam. However, these two beams are used indiscriminately to solve the TSI problem.

Euler-Bernoulli beams are suitable for simulating composite-lining tunnels, which are typically several meters wide and several hundred meters long (slender

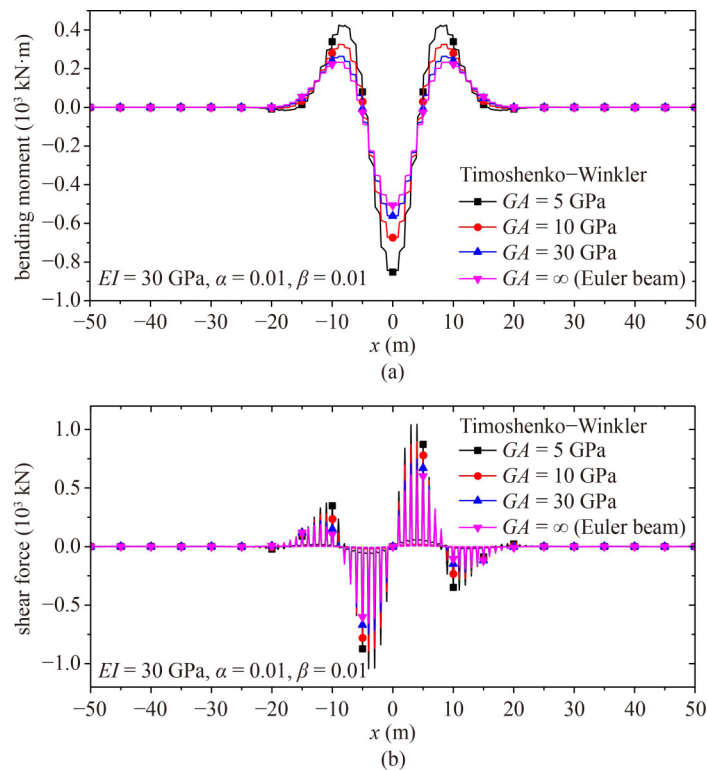


Fig. 12 Discontinuous internal forces considering shearing stiffness GA : (a) bending moment; (b) shear force.

beams). Meanwhile, Timoshenko beams are suitable for modelling shield tunnels because they are connected with several rings. Each ring typically measures 6 m high and 2 m wide (short thick beam). When considering the movement and longitudinal joints, an unsuitable beam model may cause the existing tunnel to deform unsatisfactorily, thus resulting in undesirable internal forces.

The findings of this study and those reported in the literature [26] show that the deformation of composite-lining tunnels exhibit significant discontinuities owing to the presence of movement joints. The number and relative location of movement joints determine the deformation mechanism. Meanwhile, the deformation of the shield tunnel was slightly discontinuous owing to the longitudinal joints, whereas the internal forces were extremely discontinuous. Interestingly, the flexible structures (slender beams) exhibited locally rigid characteristics owing to the movement joints. Several rigid rings (short thick beams) connected by longitudinal joints exhibited flexible characteristics.

This indicates that the structural deformation and discontinuity in vulnerable sections of undercrossing composite-lining tunnels should be monitored more closely. Whereas the internal forces and their discontinuity for undercrossing shield tunnels should be paid more attention to. This enables an effective monitoring of potential abrupt changes in the mechanical response of an

existing tunnel and ensure its structural and operational safety.

In addition, our approach predicts the responses of an existing shield tunnel in the longitudinal direction by considering the longitudinal joints but not the circumferential joints. In the future, we plan to investigate the mechanical behavior of the cross-section of an existing shield tunnel with circumferential joints.

7 Conclusions

An analytical method abbreviated TDW was developed in this study to predict the discontinuous mechanical behaviors of an existing shield tunnel with longitudinal joints. Centrifuge tests were performed to validate the proposed method. The findings obtained were as follows.

1) A mathematical model based on the Dirac delta function was proposed in this study. Compared with conventional methods, our method can provide a more accurate estimation of opening and dislocation at longitudinal joints and the discontinuous internal forces of an existing shield tunnel.

2) As α , β , and GA increased, the deflection, rotation, and dislocation decreased and indicated slight discontinuities. Meanwhile, discontinuities were particularly evident in the bending moment and shear force. Our method can incorporate the general method, which uses the Euler–Bernoulli beam ($GA = \infty$) or does not consider

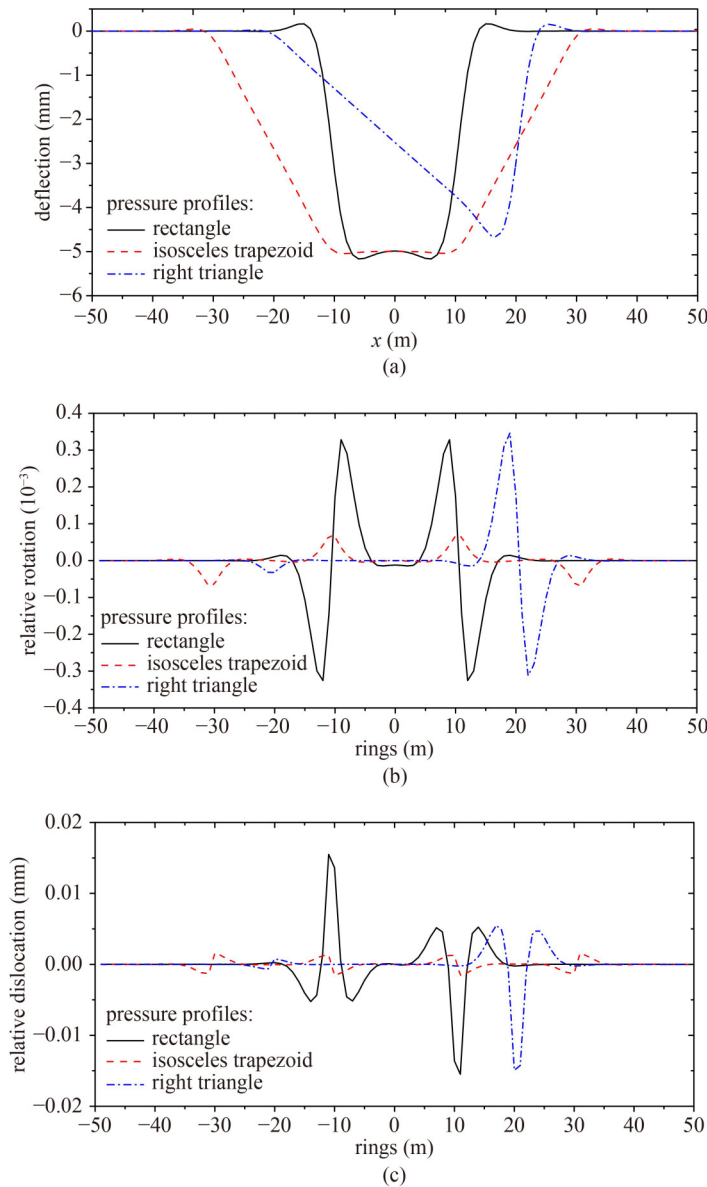


Fig. 13 Discontinuous deformation considering additional pressure profiles: (a) deflection; (b) relative rotations; (c) relative displacements.

longitudinal joints ($\alpha = \beta = 1$).

3) Different additional pressure profiles resulted in different deformation mechanisms. The deflection and pressure profiles of the existing shield tunnel were the same. However, its rotation, dislocation, and internal forces changed significantly at the location of mutation pressure.

4) Because of the movement of longitudinal joints, slender beams used to model flexible structures (composite-lining tunnel) showed locally rigid characteristics. Moreover, short thick beams used to model several rigid rings (shield tunnels) exhibited flexible characteristics.

5) The Timoshenko beam is suitable for simulating existing shield tunnels with longitudinal joints. The deformation of shield tunnels was slightly discontinuous,

whereas the internal forces were extremely discontinuous. During the construction of shield tunnels, the internal forces and abrupt changes in the weaker sections must be prioritized.

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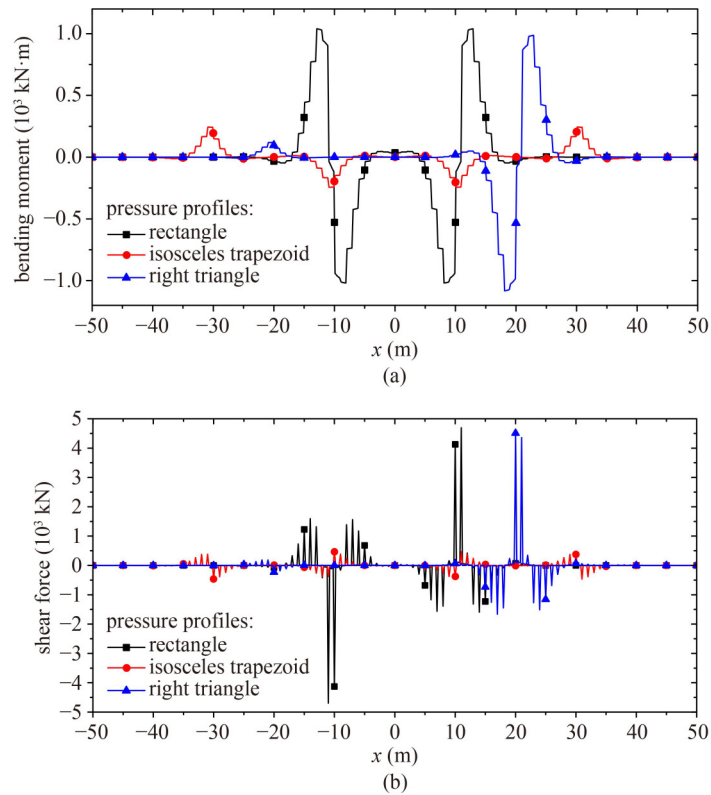


Fig. 14 Discontinuous internal forces considering additional pressure profiles: (a) bending moment; (b) shear force.

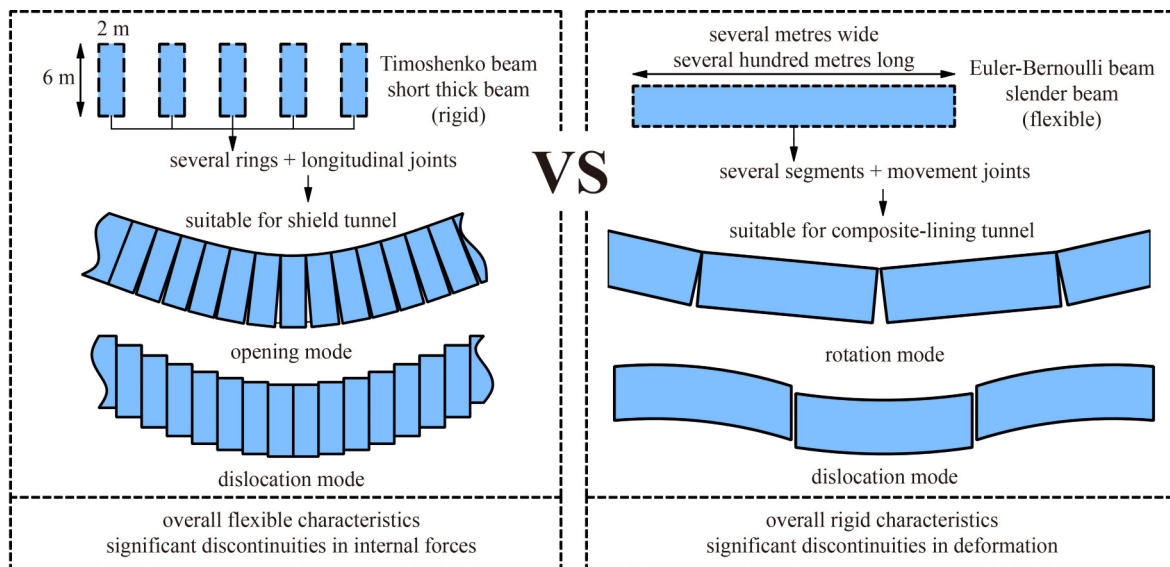


Fig. 15 Timoshenko beam for shield tunnel vs. Euler-Bernoulli beam for composite-lining tunnel.

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