

Shear strength model of the reinforced concrete beams with embedded through-section strengthening bars

Linh Van Hong BUI, Phuoc Trong NGUYEN*

Faculty of Civil Engineering, Ho Chi Minh City Open University, Ho Chi Minh City 700000, Vietnam

**Corresponding author. E-mail: phuoc.nguyen@ou.edu.vn*

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ABSTRACT In this study, finite element (FE) analysis is utilized to investigate the shear capacity of reinforced concrete (RC) beams strengthened with embedded through-section (ETS) bars. Effects of critical variables on the beam shear strength, including the compressive strength of concrete, stiffness ratio between ETS bars and steel stirrups, and use of ETS strengthening system alone, are parametrically investigated. A promising method based on the bond mechanism between ETS strengthening and concrete is then proposed for predicting the shear resistance forces of the strengthened beams. An expression for the maximum bond stress of the ETS bars to concrete is developed. This new expression eliminates the difficulty in the search and selection of appropriate bond parameters from adhesion tests. The results obtained from the FE models and analytical models are validated by comparison with those measured from the experiments. Consequently, the model proposed in this study demonstrates better performance and more accuracy for prediction of the beam shear-carrying capacity than those of existing models. The results obtained from this study can also serve researchers and engineers in selection of the proper shear strength models for design of ETS-strengthened RC beams.

KEYWORDS embedded through-section, strengthening, fiber-reinforced polymer, finite element, shear strength model, bond mechanism

1 Introduction

The fiber-reinforced polymer (FRP) composites for shear strengthening of reinforced concrete (RC) members has become increasingly popular. The techniques that have been proven to be effective for strengthening RC members are externally bonding (EB) and near-surface mounting (NSM) [1–8]. However, these strengthening methods retain the negative points that reduce the member performance, in which the premature debonding of FRP to concrete is a main issue. Despite a number of studies having proposed the techniques for prevention of early loss of adherence, for example, the use of anchorage system or the treatment of adhesive [9], the results remain unconvincing due to the complication of extra elements or the expensive cost of materials and labors.

Recently, a new shear strengthening method named embedded through-section (ETS) has been introduced by

researchers [3,10–16]. The ETS method uses steel or FRP bars embedding into the predrilled holes in the shear zone of RC beams and bonds them to concrete with an adhesive resin. Most previous works have indicated that the shear performance of the beams strengthened by ETS system can be highly enhanced in comparison with those strengthened by the other techniques [3,16]. The main benefits of the ETS method for beam strengthening are the prevention of the early debonding, protection from fire exposure, and the great composite action of the whole member.

However, in general, experimental studies are still few to justify the practical application of the ETS method for strengthening of RC beams. Many numerical studies [17–22] have investigated the fracture behavior of the RC members. In the works [17–20], the authors were concerned with the meshfree methods for modeling discrete cracks in the RC structures. Meanwhile, studies [21,22] proposed an explicit phase field model for the fracture of RC members. However, numerical research on

the RC beams strengthened in shear with the ETS technique seems to be inadequate for understanding the shear behavior of ETS-strengthened beams. Some numerical programs regarding beams with ETS strengthening systems were studied in the literature in Refs. [15,23–26]. These past works primarily aimed at assessing the capability of the finite element method (FEM) in simulating the structural responses of the ETS-strengthened beams by comparison with the experimental results. Godat et al. [24] utilized ADIANA [27] for the simulation program, while Bui et al. [25,26] used ANSYS 15.0 [28] for the FEM simulation. In addition, Breveglieri et al. [15] employed a complicated package to perform numerical analyses.

Conversely, the design models for ETS-strengthened beams seem to be insufficient. Indeed, only the works by Mofidi et al. [12], Bui et al. [8,16], and Breveglieri et al. [29] developed shear strength models, in which the determination of effective strain for ETS strengthening systems was the major goal of their studies. Bui et al. [8], Mofidi et al. [12], and Breveglieri et al. [29] considered the bond behavior between ETS bars and concrete to assess the ETS effective strain. Otherwise, Bui et al. [16] adopted a regression analysis to propose an empirical formula for the effective strain of ETS systems. The verification of those models was made with limited experimental data, which seem not incorporate the diversity of the variables affecting member performance. Additionally, the models of Bui et al. [8], Mofidi et al. [12] required information from the bond tests, and little such information was available. No general and simple application could be generated from their works. Meanwhile, the empirical models by Bui et al. [16] and Breveglieri et al. [29] depended highly on the experimental data; thereby, they seem not to be widely applicable.

The present study focuses on a very specific physical problem, and aims to develop a universal model for prediction of shear strength of the RC beams strengthened with ETS-FRP bars. To this end, datasets in terms of shear strengths of ETS-strengthened beams under a number of design variables are needed. When the reliability of the FEM simulation is confirmed, then FEM simulation becomes suitable to complement the data for beams with ETS-FRP strengthening system. This paper is therefore organized as follows: 1) experimental results with respect to the ETS strengthening method are gathered into a dataset; 2) FEM simulation is validated by comparing and evaluating its results to experimental data; 3) parametric studies on various design factors, such as the ratios of the ETS strengthening stiffness ($E_f \rho_f$) to the existing stirrup stiffness ($E_{sw} \rho_{sw}$), compressive strengths of concrete (f'_c), and single use of ETS strengthening system, are carried out by means of reliable FE models; 4) an analytical shear strength model for the ETS-strengthened beams is developed on the basis of both the

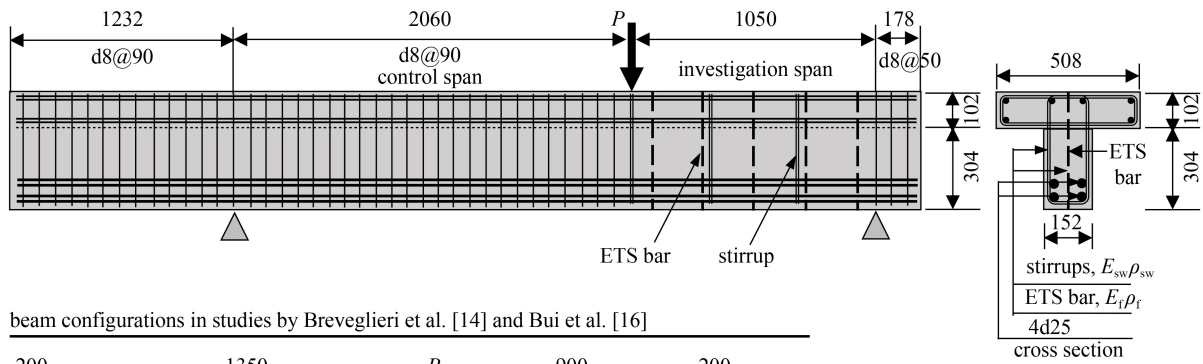
empirical formula and the bond mechanism between ETS-FRP bars and concrete; 5) validation and verification of the proposed analytical model to both the experimental data and the numerical results are carefully assessed; 6) comparison between the proposed model with the existing shear models is considered.

2 Experimental data and introduction to parametric studies

The details and configurations of the ETS-strengthened RC beams tested in the experimental studies of Mofidi et al. [12], Breveglieri et al. [14], and Bui et al. [16] are shown in Fig. 1 and Table 1. In this study, such beams are used to validate the reliability of the FE method, while experimental results of all beams tested in the works of Mofidi et al. [12], Breveglieri et al. [14], and Bui et al. [16] are utilized to validate and develop the available and new shear resisting models. The beams in previous studies had T-shaped section, as in the works by Breveglieri et al. [14] and Bui et al. [16] had the shear span length: 900 mm, web: 180 mm $\times$ 300 mm, and flange: 450 mm $\times$ 100 mm. The beams in the work by Mofidi et al. [12] had the shear span length: 1050 mm, web: 152 mm $\times$ 304 mm, and flange: 508 mm $\times$ 102 mm. Engineering information for the material properties are quantified in Table 1. All beams in those studies were designed having shear failure in the span strengthened with ETS bars; thereby, the amount of tensile steel bars and the number of steel stirrups in the control span had over-reinforced designs.

The specifications of the transverse steels and longitudinal reinforcement are illustrated in Fig. 1. The sensitive analysis quantifying the influence of all uncertain input parameters, done in the studies [17,30], was made to assess the accuracy of the numerical models. In fact, the experimental data base for the ETS-strengthened RC beams is still limited to the different experiments using the same material properties. Therefore, sensitive analysis is not considered in this study. The primary objective of the present study is to use the FE models to implement parametric analyses, and the study aims to propose a shear strength model based on ETS bonding mechanism. To this end, the validation of the FEM against accessible experiments is first made via comparisons in shear capacity, ETS shear contribution, load-deflection curves, and failure patterns. Thereafter, a reliable FE model is extended to conduct parametric studies considering various design parameters. The results obtained from the experiments and the parametric studies with respect to the shear strength of beams and the shear contribution of ETS strengthening bars are used to validate and verify the proposed analytical shear strength model. The accuracy of the proposed shear strength

beam configurations in study by Mofidi et al. [12]



beam configurations in studies by Breveglieri et al. [14] and Bui et al. [16]

Fig. 1 Specifications of the beams tested in previous studies [12,14,16] (dimensions in mm). (Reprinted from Journal of Composites for Construction, 16(5), Mofidi A, Chaallal O, Benmokrane B, Neale K W, Experimental tests and design model for RC beams strengthened in shear using the embedded through-section FRP method, 540–550, Copyright 2012, with permission from American Society of Civil Engineers.) (Reprinted from Composite Structures, 126, Breveglieri M, Aprile A, Barros J A O, Embedded through-section shear strengthening technique using steel and CFRP bars in RC beams of different percentage of existing stirrups, 101–113, Copyright 2015, with permission from Elsevier.)

Table 1 Beam configurations for experiments in previous studies

group	beams	ρ_{sw} (%)	E_{sw} (GPa)	d_{sw} (mm)	d_b (mm)	ρ_f (%)	E_f (GPa)	f'_c (MPa)	f'_t (MPa)	$E_f \rho_f / E_{sw} \rho_{sw}$	$E_f \rho_f + E_{sw} \rho_{sw}$ (MPa)
Mofidi et al. [12]	S0-12d130s	–			12.7	0.64	148	29.6	1885	–	947.2
	S1-9d260s	0.25	200	8	9.5	0.18	148	29.6	1885	0.533	766.4
	S1-12d260s	0.25	200	8	12.7	0.32	148	29.6	1885	0.947	973.6
	S1-12d130s	0.38	200	8	12.7	0.64	148	29.6	1885	1.246	1707
	S1-9d260p	0.25	200	8	9.5	0.18	155	29.6	2800	0.558	779
	S3-12d130s	0.38	200	8	12.7	0.64	148	29.6	1885	1.246	1707
Breveglieri et al. [14]	2S-C180-90 (C1)	0.10	200	6	8	0.16	160	29.7	1920	1.280	456
	2S-C180-45 (C2)	0.10	200	6	8	0.22	160	29.7	1920	1.760	552
	4S-C180-90 (C3)	0.17	200	6	8	0.16	160	32.3	1920	0.753	596
	4S-C180-45 (C4)	0.17	200	6	8	0.22	160	32.3	1920	1.035	692
Bui et al. [16]	B1	0.11	200	6	10	0.24	50	38	1076	0.551	341.2
	B2	0.11	200	6	10	0.34	50	38	1076	0.779	391.4
	B3	0.24	200	9	10	0.24	50	38	1076	0.253	601.2
	B4	0.24	200	9	10	0.34	50	38	1076	0.357	651.4

Notes: E_{sw} is the elastic modulus of transverse steel (GPa); ρ_{sw} is the ratio of steel stirrups (%); d_{sw} is the stirrup bar diameter (mm); E_f is the elastic modulus of ETS bars (MPa); ρ_f is the ratio of ETS bars (%); d_b is the ETS bar diameter (mm); and f'_c is the concrete compressive strength (MPa).

model is assessed via the variance and mean value for both experiments and simulations.

The extensive parametric studies are conducted using a

reliable FE model of the ETS-strengthened RC beam as presented in the study of Bui et al. [16]. The primary

objective of the parametric investigation is to provide the

datasets for the shear capacity of the ETS-strengthened beams. Following the main purpose, the data from parametric studies are then used to propose a new shear strength model. The design variables for the parametric studies are the ratios of the ETS strengthening stiffness ($E_f \rho_f$) to the existing stirrup stiffness ($E_{sw} \rho_{sw}$) for different compressive strengths of concrete (f'_c). In addition, the beams with no shear steel reinforcement and strengthened with ETS bars are investigated. The parametric studies are categorized into four groups; the details and specifications of those four groups are given in Table 2. The yielding strengths of steel reinforcement with 6-, 9-, and 25-mm diameters are respectively identified as 235, 235, and 395 MPa according to the standards of Thai Industrial Standards Institute (TIS20-2543 [31] and TIS24-2548 [32]). Further descriptions on the parametric studies are presented in the relevant sections.

3 Finite element method

3.1 Constitutive models for materials and elements

A commercial software ANSYS 15.0 [28], which is popular in the structural engineering community, is adopted to prepare the 3D finite element (FE) models of the ETS-strengthened RC beams. Several numerical works have presented the element and material models for constitution of FE models of the beams [24,26,33]. The 3D elements LINK180, SOLID65, and SOLID45 are used to represent the behaviors of the steel or FRP bars, concrete, and supporting or loading, respectively. The properties of the elements have been depicted in detail in the literature of Bui et al. [25,26] and Hawileh [33]. The elements SOLID65 and SOLID45 are the eight-node elements with three directions of degree of freedom at each node. The element LINK180 includes the uniaxial

Table 2 Beam configurations for parametric studies using FEM

group	beams	ρ_{sw} (%)	E_{sw} (GPa)	d_{sw} (mm)	d_b (mm)	ρ_f (%)	E_f (GPa)	f'_c (MPa)	f_f (MPa)	$E_f \rho_f / E_{sw} \rho_{sw}$	$E_f \rho_f + E_{sw} \rho_{sw}$ (MPa)
Group 1 (for $f'_c = 20$ MPa varying stiffness ratio)	G1_Original	0.24	200	9	10	0.24	50	20	1076	0.257	592.4
	G1_B1	0.14	200	6	10	0.29	40	20	755	0.417	395.6
	G1_B2	0.10	200	6	10	0.15	75	20	1366	0.521	318.5
	G1_B3	0.31	200	9	9.5	0.13	100	20	1500	0.209	759.6
	G1_B4	0.10	200	6	10	0.10	120	20	1700	0.556	325.8
	G1_B5	0.39	200	9	9	0.08	160	20	2400	0.160	911.1
	G1_B6	0.19	200	10	8	0.06	200	20	–	0.320	512.0
Group 2 (for $f'_c = 38$ MPa varying stiffness ratio)	G2_Original	0.24	200	9	10	0.24	50	38	1076	0.257	592.4
	G2_B1	0.14	200	6	10	0.29	40	38	755	0.417	395.6
	G2_B2	0.10	200	6	10	0.15	75	38	1366	0.521	318.5
	G2_B3	0.31	200	9	9.5	0.13	100	38	1500	0.209	759.6
	G2_B4	0.10	200	6	10	0.10	120	38	1700	0.556	325.8
	G2_B5	0.39	200	9	9	0.08	160	38	2400	0.160	911.1
	G2_B6	0.19	200	10	8	0.06	200	38	–	0.320	512.0
Group 3 (for $f'_c = 70$ MPa varying stiffness ratio)	G3_Original	0.24	200	9	10	0.24	50	70	1076	0.257	592.4
	G3_B1	0.14	200	6	10	0.29	40	70	755	0.417	395.6
	G3_B2	0.10	200	6	10	0.15	75	70	1366	0.521	318.5
	G3_B3	0.31	200	9	9.5	0.13	100	70	1500	0.209	759.6
	G3_B4	0.10	200	6	10	0.10	120	70	1700	0.556	325.8
	G3_B5	0.39	200	9	9	0.08	160	70	2400	0.160	911.1
	G3_B6	0.19	200	10	8	0.06	200	70	–	0.320	512.0
Group 4 (for ETS only)	G4_Original	–	–	–	10	0.24	50	38	1076	–	121.2
	G4_B1	–	–	–	10	0.29	40	38	755	–	116.4
	G4_B2	–	–	–	9.5	0.13	100	38	1366	–	131.3
	G4_B3	–	–	–	10	0.10	120	38	1500	–	116.4
	G4_B4	–	–	–	9	0.08	160	38	2400	–	125.7
	G4_B5	–	–	–	8	0.06	200	38	–	–	124.1

tension-compression behavior. The following descriptions present the constitutive models for the materials, which can be applied for the 3D elements in the FE models.

A parabolic equation (Eq. (1)) proposed by Hognestad et al. [34] is used to simulate concrete behavior in tension. The curve of the model of Hognestad et al. [34] is shown in Fig. 2(a).

$$f_c = f'_c \left[2 \left(\frac{\varepsilon}{\varepsilon_0} \right) - \left(\frac{\varepsilon}{\varepsilon_0} \right)^2 \right], \quad (1)$$

where f_c is the compressive stress of concrete (MPa) corresponding to the specified strain ε ; f'_c is the concrete compressive strength (MPa); $\varepsilon_0 = 2f'_c/E_c$, where $E_c = 3300\sqrt{f'_c} + 6900$ is the concrete elastic modulus (MPa) [24], and $\varepsilon_{cu} = 0.0038$ is the concrete crushing strain shown in Fig. 2(a).

The smeared crack approach is used in the FEM simulation. The tensile properties of concrete are employed according to the model proposed by Willam and Warnke [35], as illustrated in Fig. 2(b). The tensile concrete strength is taken as $0.6\sqrt{f'_c}$ (MPa) [24]. After the peak, the tensile stress relaxation makes a steep drop to the tensile strength by $0.6f'_{ct}$. Then, the stress-strain relationship continues until stress is zero, where the strain

attains a value six times higher than that at the peak stress. After the steep drop, the post-cracking stiffness of the concrete is reduced to R' , which depends on the shear crack transfer. The fracture energy (G_F) can be determined from the stress-strain curve, as indicated in Fig. 2(b). The concrete crack model is important for the numerical simulation of the RC beams [17,36–38]. The shear transfer coefficients, which are ranged from 0 to 1.0, are indispensable for assessment of the concrete crack features. The higher the shear transfer coefficient, the larger the shear transfer at crack plane. Very low shear transfer results in smooth cracks, while full shear transfer results in rough cracks. No unified values of the shear transfer coefficients for the crack modeling of the RC beams have been recommended in past works. In this study, the open and closed shear transfer coefficients (β_v , β_c) = (0.5, 1.0) are used for the input data for the FE models. The values of the shear transfer coefficients are selected by considering the accuracy and convergency of the FE models. The failure modes of concrete using the aforementioned constitutive models are assumed to be the crushing and tensile cracking. The constitutive laws for the steel and FRP materials are demonstrated in Fig. 2(c). The elastoplastic behavior with the yielding phase is assumed for the steel reinforcement, while the linear up to rupture relationship is utilized for the FRP bars. The

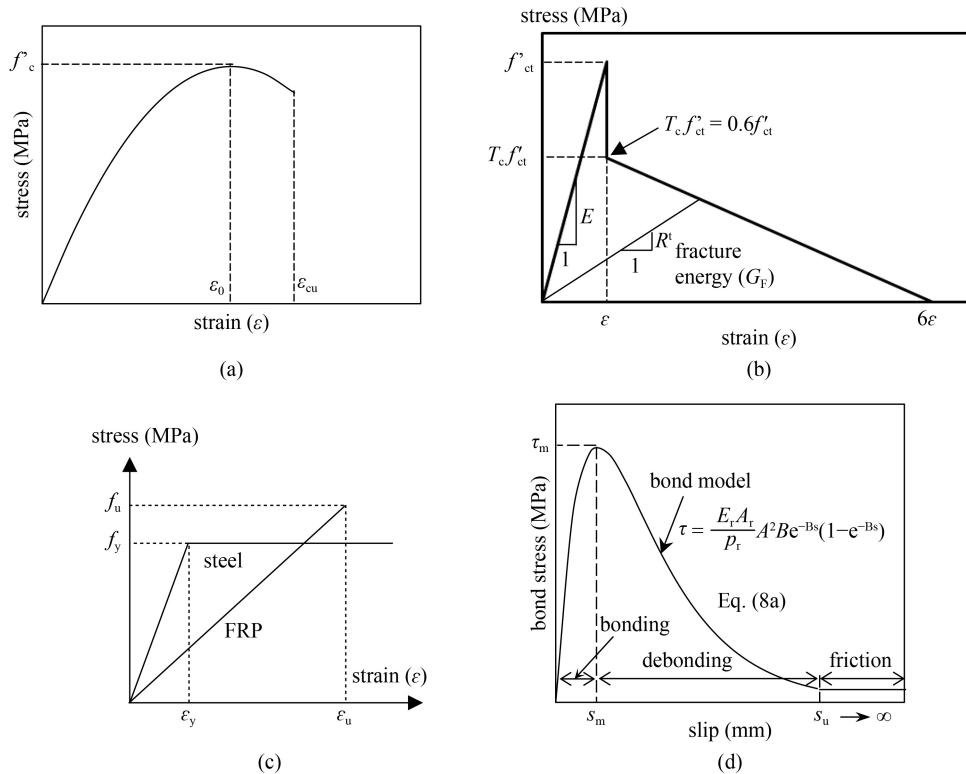


Fig. 2 Modelings for FEM simulation: (a) concrete model in compression [25,26]; (b) concrete model in tension [25,26]; (c) steel and FRP material models [25,26]; (d) bond model [25,26]. (Reprinted from Journal of Building Engineering, 29, Bui L V H, Stitmannaitum B, Jongvivatsakul P, Comprehensive investigation on bond mechanism of embedded throughsection fiber-reinforced polymer bars to concrete for structural analysis, 101180, Copyright 2020, with permission from Elsevier.)

Poisson's ratios of the concrete, steel, and FRP materials are 0.2, 0.3, and 0.22, respectively.

The bond link between steel or FRP bars and concrete is simulated by the element COMBIN39 in ANSYS 15.0 [28], which is represented by a non-linear spring model. A bond model proposed by Bui et al. [26] is used, and the shape of their bond law is shown in Fig. 2(d). The bond model suggested by Bui et al. [26] requires the interfacial parameters A and B , which are derived from a series of pullout tests. The element COMBIN39 requires the bond force–displacement (F_n – D_n) curve, which is determined from the bond model. The bond force can be calculated as $F_n = \tau_m \times \pi \times d_f \times 25$, in which, τ_m is the bond stress, d_f is the bar diameter, and the value of 25 is the mesh size. Meanwhile, D_n is the slip between ETS bars and concrete obtained from the bond model. Because the bond model proposed by Bui et al. [26] considers the failure type of the bar pullout, the limitations of the bond model to the failure modes of the bar rupture and concrete splitting remain. However, in actual tests of the ETS-strengthened beams, the bar rupture and the concrete splitting surrounded ETS bars are not observed. Therefore, the bond model proposed by Bui et al. [26] with pullout failure of the ETS bars from concrete can be applied in the FEM simulation of the ETS-strengthened RC beams.

A half FE model of a beam strengthened in shear with ETS-FRP bars is shown in Fig. 3. Although the failure is commonly asymmetric due to imperfections in the concrete [17,18,37], the symmetry boundary condition is assumed in this study to decrease the complication and computation time. The mesh size of elements is selected to obtain a high accuracy and convergency of the FE model, and it needs to be suitable for the capacity of the computer. Many FE models with different element sizes are trained to choose the proper mesh size. The suitable mesh size is 25 mm × 25 mm × 25 mm. The failure mode

of the FE models is the steel stirrup yielding following the large principal strain occurring in the concrete. The failure involving the ETS-FRP bar rupture and debonding is analyzed based on the strain response in the ETS-FRP bars.

3.2 FEM validation

Several numerical works, such as Godat et al. [24], Bui et al. [26], and Breveglieri et al. [29], have investigated the reliability of FE models for prediction of the shear behaviors of concrete beams strengthened with ETS-FRP bars. This section briefly summarizes the verification of the FEM simulation in terms of the shear capacity of ETS-strengthened beams, the shear contribution of ETS-FRP bars, and the failure mechanism. The FE models for the ETS-strengthened beams, which were prepared in the study of Bui et al. [26], are reproduced to extend analyses in this study. Some beams in the experimental works of Breveglieri et al. [14] and Bui et al. [16], specifically 2S-C180-90 (C1), 2S-C180-45 (C2), B1, B2, B3, and B4, are selected for the FEM verification. The details of those beams are presented in Table 1, and the FE models for those beams are provided in above sections.

Figures 4(a)–4(c) illustrate the appraisal of the FEM results relative to the experimental data in terms of the load–deflection curves of the representative specimens, the beam shear capacity, and the ETS shear contribution. The FE models can predict the stiffness of the ETS-strengthened beams well. Clearly, the maximum difference between test and simulation in the shear capacity of the beams is 15% approximately, for specimen C2, while the remaining specimens provide the discrepancies in shear capacities between tests and simulations of less than 5%. The beam C2 might offer a longest embedment length of ETS bars due to its diagonal arrangement

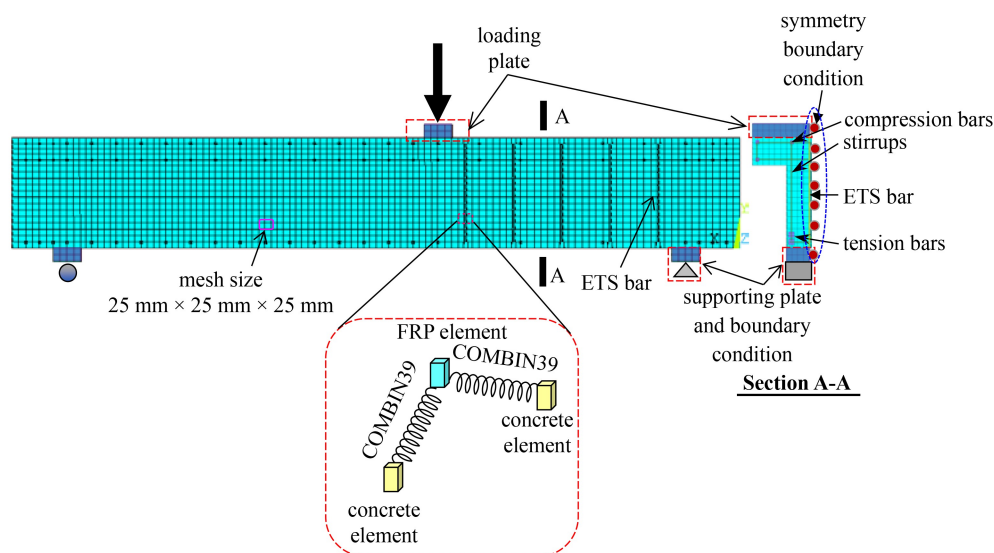


Fig. 3 A half FE model.

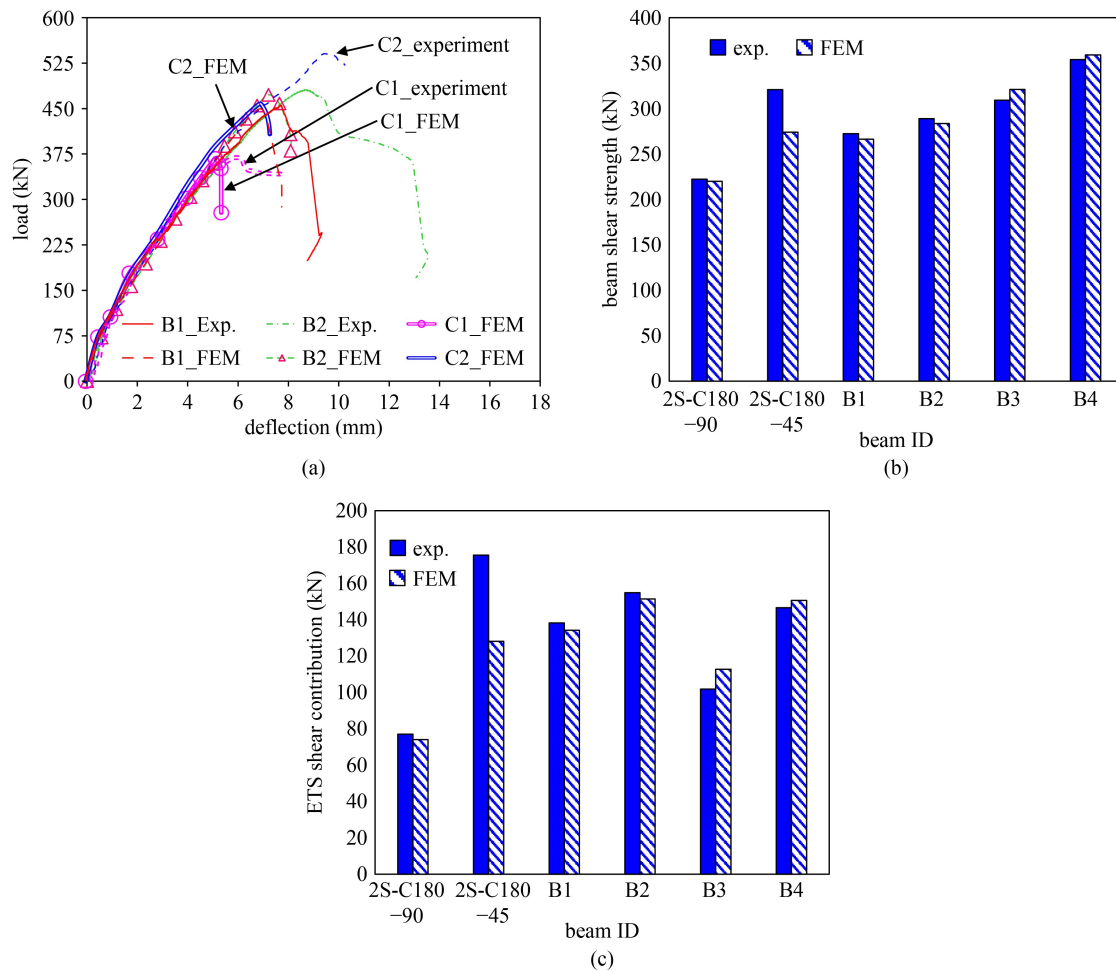


Fig. 4 Comparison between experiment and FEM simulation: (a) load–deflection curves for representative specimens B1, B2, C1, and C2; (b) beam shear capacity; (c) ETS shear contribution.

ensuring that the interfacial performance of the ETS-FRP-concrete joints can be fully developed. Therefore, the bond variables assumed for the interfacial element of ETS-FRP-concrete joints in the FE model of the specimen C2, which were collected from the pullout tests with no long embedment length, are not suitable to reflect the actual bond behavior in the test beam. These lead to the larger shear strengths of the beam and ETS bars in the test beam than those in the FE model.

Conversely, the beams with the ETS bars inclined at 45° give higher beam shear strength and ETS shear resisting forces than those of the specimens with vertical ETS bars. The beam C1 using carbon FRP (CFRP) bars for its ETS system furnishes the smallest shear capacity. This may be mainly due to the fact that the local debonding of the vertical ETS-CFRP bars to concrete occurred in the specimen C1. The phenomenon by local debonding that occurred along inadequate bond length led to the reduction in the shear bond strength of the ETS-CFRP-concrete interfaces following the decrease of the shear resisting forces of the ETS-CFRP bars. However, the beam B1 used the vertical ETS-glass FRP (GFRP) bars displays an impressive shear strength since the

anchorage system was used to prevent the debonding of the ETS bars to concrete, triggering the effectiveness of the ETS strengthening. Considering the beams B1, B2, B3, and B4, the increase in percentage of the transverse steels enhances the beam shear capacities but decreases the shear contribution of the ETS bars. These findings achieved from the experiments are in good agreement with the results obtained from the FEM simulations.

Figure 5 presents the verification of FEM simulation results in the crack patterns of the ETS-strengthened beams. Two specimens C1 and C2 are considered. In FEM simulations, the maximum principal strain is examined to describe the failure of the beams. Apparently, the actual failure cracks that occurred in the tests are in the shear zones where the maximum principal strain is concentrated. Further, it can be observed in Fig. 5 that the cracks (i.e., the principal strain contour in the FE models) formed in the beam with diagonal ETS bars passed the whole system, while in the beam with vertical ETS bars, the cracks did not cross the entire strengthening. This implies that the diagonal arrangement of the strengthening is more effective than the vertical one. Additionally, the flexural cracks at the beam web and compressive

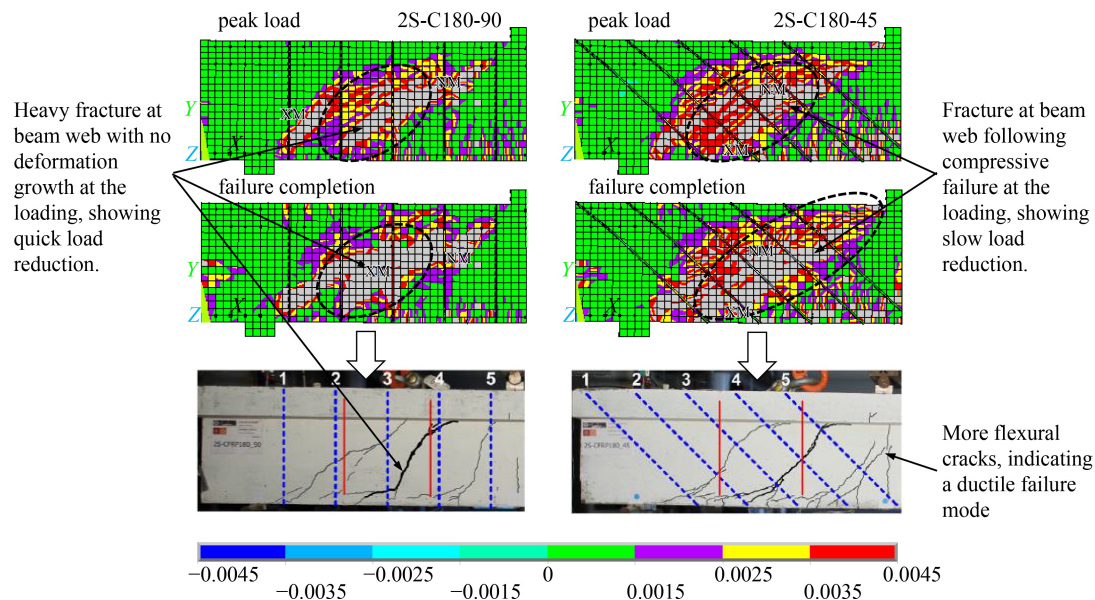


Fig. 5 Failure patterns. Note: The gray color displayed in the strain contour demonstrates the exceeding ultimate strain occurred in concrete. (photos of the experimental results of the beams are reproduced from Breveglieri et al. [14] with permission). (Reprinted from Composite Structures, 126, Breveglieri M, Aprile A, Barros J A O, Embedded through-section shear strengthening technique using steel and CFRP bars in RC beams of different percentage of existing stirrups, 101–113, Copyright 2015, with permission from Elsevier.)

cracks at the loading area of the beam C1 are fewer than those in the specimen C2. This phenomenon also agrees with the deformation produced by the FE models. The aforementioned findings demonstrate that the failure mode of beam C2 might change from brittle shear fracture to a more ductile situation through bending and compressive failures. On the other hand, the growth of the principal strain from the peak load stage to the failure completion is depicted in Fig. 5. Obviously, the beam with vertical ETS system furnishes a quick load reduction because no clear development in the principal strain is observed. Meanwhile, the beam with diagonal ETS bars provides a slow load reduction (i.e., a ductile post peak behavior) due to the presence of strain development after the peak.

Figure 6 presents the stress response in the reinforcement and strengthening systems of beam C2 recorded in the FE model. At the last step of the FEM simulation, the steel stirrups yield, while the stress in ETS-CFRP bars yields to the high stress but without rupturing. The stress in the tensile steel bar close to the beam soffit nearly reaches its yielding strength, leading to the ductility of the ETS-strengthened beam (C2). These observations indicate that the beam C2 fails with the yielding of the transverse steels and the quasi-yielding of the longitudinal reinforcement followed by the concrete fracture in shear and the compressive cracks of concrete at loading. No early loss of adhesion of the ETS-CFRP-concrete interfaces and no rupturing of CFRP bars are detected. The foregoing findings of the FEM simulations agree with the experimental results measured in the three-point loading test of the beam C2.

4 Analyses of shear strength of the ETS-strengthened RC beams

4.1 Parametric investigation

The reliability of the FE tool for simulation of the ETS-strengthened RC beams has been verified in Subsection 3.2. To comprehensively assess the shear resisting models, more variables and factors on the RC beams with ETS shear strengthening should be studied. Therefore, this section shows the parametric studies on the shear capacity of the ETS-strengthened beams. A FE model with dimensions similar to those of the beam B1 simulated in Section 3.2 is used to develop the parametric studies. The ratios of the ETS stiffness to transverse steel stiffness ($E_f \rho_f / E_{sw} \rho_{sw}$), which range between 0.257 and 0.556, are investigated to ascertain effects of hybrid usage of steel transverse stirrups and ETS-FRP bars. Effects of the compressive strength of concrete (f'_c), of which three values of $f'_c = 20, 38$, and 70 MPa are examined, on the beam shear capacity are also assessed. Additionally, the beams strengthened in shear with only ETS-FRP bars ($E_f \rho_f$ range between 116.4 and 131.3 MPa) without steel stirrups are considered. The details of parametric studies for deriving the beam shear strength are clearly shown in Table 2. In the present study, the results in the total shear capacity of the beams in the FEM simulations for the parametric analyses are used for verifying and developing the shear strength models. Structural behaviors of the beams in the parametric studies will be analyzed in a separate work prepared by the authors.

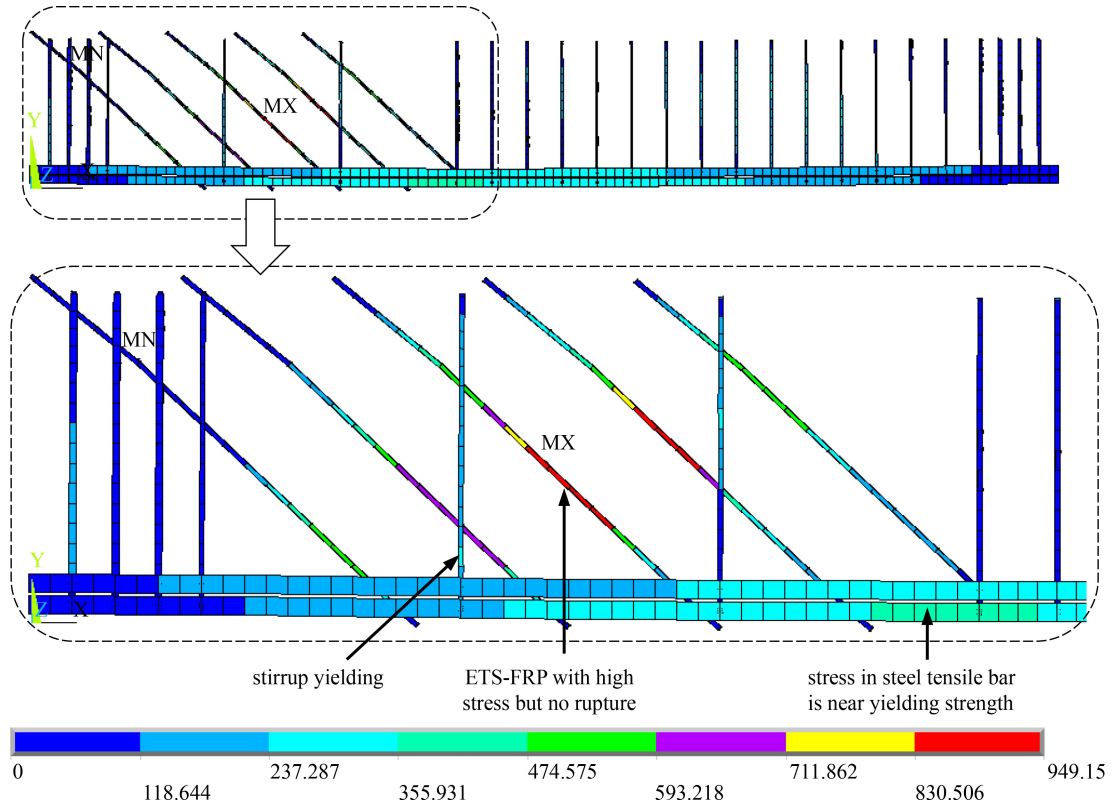


Fig. 6 Stress in steel reinforcement and ETS-FRP bars in the specimen 2S-C180-45 (C2) (stress unit in MPa).

4.2 Calculation of total shear strength

The total shear strength of a RC beam strengthened by ETS bars can be expressed as below.

$$V_n = V_c + V_s + V_f, \quad (2)$$

where V_n is the total shear strength (kN); V_c is shear resistance by concrete (kN); V_s is the shear resisting force of stirrups (kN); and V_f is the shear contribution of ETS bars (kN).

The strength in shear of concrete (V_c) can be determined by an equation in ACI 318 [39] modified by Breveglieri et al. [15], as the following expression.

$$V_c = 0.23 \sqrt{f'_c} b_w d, \quad (3)$$

where f'_c is the concrete compressive strength (MPa); b_w is the width of the beam web (mm); d is the effective depth in the beam section (mm).

The shear resisting force of steel shear reinforcement can be estimated using truss analogy using the following equation:

$$V_s = A_v f_y \frac{d(\cot \theta + \cot \alpha)}{s} \sin \alpha, \quad (4)$$

where f_y is the yielding strength of steel transverse reinforcement (MPa); s is the center-to-center of reinforcements measured parallel to longitudinal bars (mm); α is

the angle between inclined stirrups or spirals and longitudinal axis ($^\circ$); θ is the failure crack angle in the beams ($^\circ$). The ACI guideline [39] assumes the failure crack angle in the beams to be 45° .

Employing truss theory, the shear resistance of ETS strengthening bars matches the following equations:

$$V_f = A_f E_f \epsilon_{fe} \frac{d(\cot \theta + \cot \alpha)}{s} \sin \alpha. \quad (5)$$

The key feature of the above equation is the determination of the effective strain of the ETS strengthening system (ϵ_{fe}). The next section presents the expressions of the ETS effective strain.

4.3 Expressions of the effective strain for ETS strengthening system

In the available literature, Bui et al. [8,16] and Mofidi et al. [12] proposed approaches to estimate the effective strain in the ETS strengthening system. Bui et al. [8] and Mofidi et al. [12] furnished the formulations based on the bond mechanism between ETS bars and concrete, in which the interfacial bond variables with respect to the bond stress and slip were required. Meanwhile, Bui et al. [16] built the effective strain equation using the regression analysis of the experimental results associated with truss theory. They indicated that the proposed equations for effective strain of the ETS strengthening

bars performed a reasonable agreement with the experimentally accessible database in comparison with the other FRP shear contribution models proposed for FRP-RC beams.

The model of Mofidi et al. [12] stipulates the following expression for the effective strain of ETS strengthening:

$$\varepsilon_{fe} = \sqrt{\frac{8}{d_b E_f} \left(\frac{\tau_m s_m}{1+p} \right)} \leq 0.004, \quad (6)$$

where d_b is the ETS bar diameter (mm); τ_m and s_m are the maximum bond stress (MPa) and slip at peak bond stress (mm); p is the bond exponential value for fitting the bond-slip curves.

The model of Bui et al. [16] stipulates the below formulation for the effective strain of ETS bars:

$$\varepsilon_{fe} = -0.00127 + 0.0162 \times \frac{\sqrt{f'_c}}{\sqrt{a/d} + 1} e^{\frac{-1000}{\rho_s E_s} - 0.05 \sqrt{\rho_{sw} E_{sw} + \rho_t E_t}}, \quad (7)$$

where ε_{fe} is the effective strain of shear strengthening system; a/d is the shear span-to-effective depth ratio of the beam; E_f is the elastic modulus of ETS bars (MPa); ρ_f is the ratio of ETS bars (%); E_s is the elastic modulus of tensile steel reinforcement (MPa); ρ_s is the ratio of tensile steel reinforcement (%); E_{sw} is the elastic modulus of transverse steel (MPa); and ρ_{sw} is the ratio of steel stirrups (%).

4.4 Bonding-based approach for shear contribution of ETS-FRP bars

4.4.1 Bond model between anchored ETS bars and concrete

There are many bond models to describe for the behavior of the FRP composites bonded to concrete interfaces. However, bond models representing ETS-FRP bars–concrete interfaces are still limited. Bui et al. [26] developed the bond stress–slip (τ – s) model on the foundations of the model proposed by Dai et al. [40], which were originally established for the analysis of the EB-FRP sheet–concrete interfaces. The bond model of Bui et al. [26] has been sufficiently validated by comparison with the pullout test results regarding the ETS strengthening method. The shape of the bond model of Bui et al. [26] is illustrated in Fig. 2(d); therein three stages of bonding, debonding, and friction representing the interfacial behaviors are identified. According to the concept of Bui et al. [26], the equations representing the τ – s relationship, maximum bond stress (τ_m), and slip at peak bond stress (s_m) are briefly presented as below.

$$\tau = \frac{E_r A_r}{p_r} A^2 B e^{-Bs} (1 - e^{-Bs}), \quad (8a)$$

$$\tau_m = 0.5 B G_f, \quad (8b)$$

$$s_m = \ln(2) / B, \quad (8c)$$

where τ is the bond stress (MPa); s is the slip (mm); s_m is the slip at peak bond stress (mm); τ_m is the maximum bond stress (MPa); G_f is the interfacial fracture energy defined by the area underneath the τ – s curve (N/mm); E_r , A_r , and p_r are the elastic modulus (GPa), cross-sectional area (mm²), and perimeter (mm) of the ETS strengthening bars, respectively; and A and B are the calibration parameters for fitting the bond–slip curve.

4.4.2 Outline of proposed model

As initially developed by Bui et al. [8], the proposal of bonding-based approach considers the following concepts. The bars that are intersected the critical crack plane are the influenced ETS bars (red lines), as demonstrated in Fig. 7(a). Then, the number of influenced ETS bars (N_f) and the length of each influenced ETS bar (L_{fi}) are calculated as equations below.

$$N_f = \text{round off} \left[h_w \frac{\cot \theta + \cot \alpha}{s_{fw}} \right], \quad (9)$$

$$L_{fi} = \begin{cases} i s_{fw} \frac{\sin \theta}{\sin(\theta + \alpha)}, & \text{for } x_{fi} < \frac{h_w}{2} (\cot \theta + \cot \alpha), \\ L_f - i s_{fw} \frac{\sin \theta}{\sin(\theta + \alpha)}, & \text{for } x_{fi} \geq \frac{h_w}{2} (\cot \theta + \cot \alpha), \end{cases} \quad (10)$$

where $x_{fi} = i s_{fw}$ is the distance from the end of main crack plane to the end of the i th FRP single bar passed the critical crack plane (mm); s_{fw} is the ETS spacing (mm); h_w is the beam height for beams with the ETS bars (mm); N_f is the number of influenced ETS bars; L_{fi} is the length of each influenced ETS bar (mm); and θ and α are the crack angle and the inclination of strengthening systems ($^\circ$), respectively.

The location of critical crack plane and crack angle (θ) is determined by the inclined smeared cracks exported from the FEM simulation. An example for deriving the critical crack plane and crack angle (θ) of the beam G4_B1 from the FEM simulation is shown in Fig. 7(b). Thus, the crack angle (θ) values for all simulated beams are in the range 35°–52°.

All influenced ETS bars in an ETS-strengthened beam are converted to be a concrete block embedded by a single ETS bar with the equivalent configuration and size of the conceptual block demonstrated in Fig. 7(a). Hereinafter, that concept is called the equivalent block.

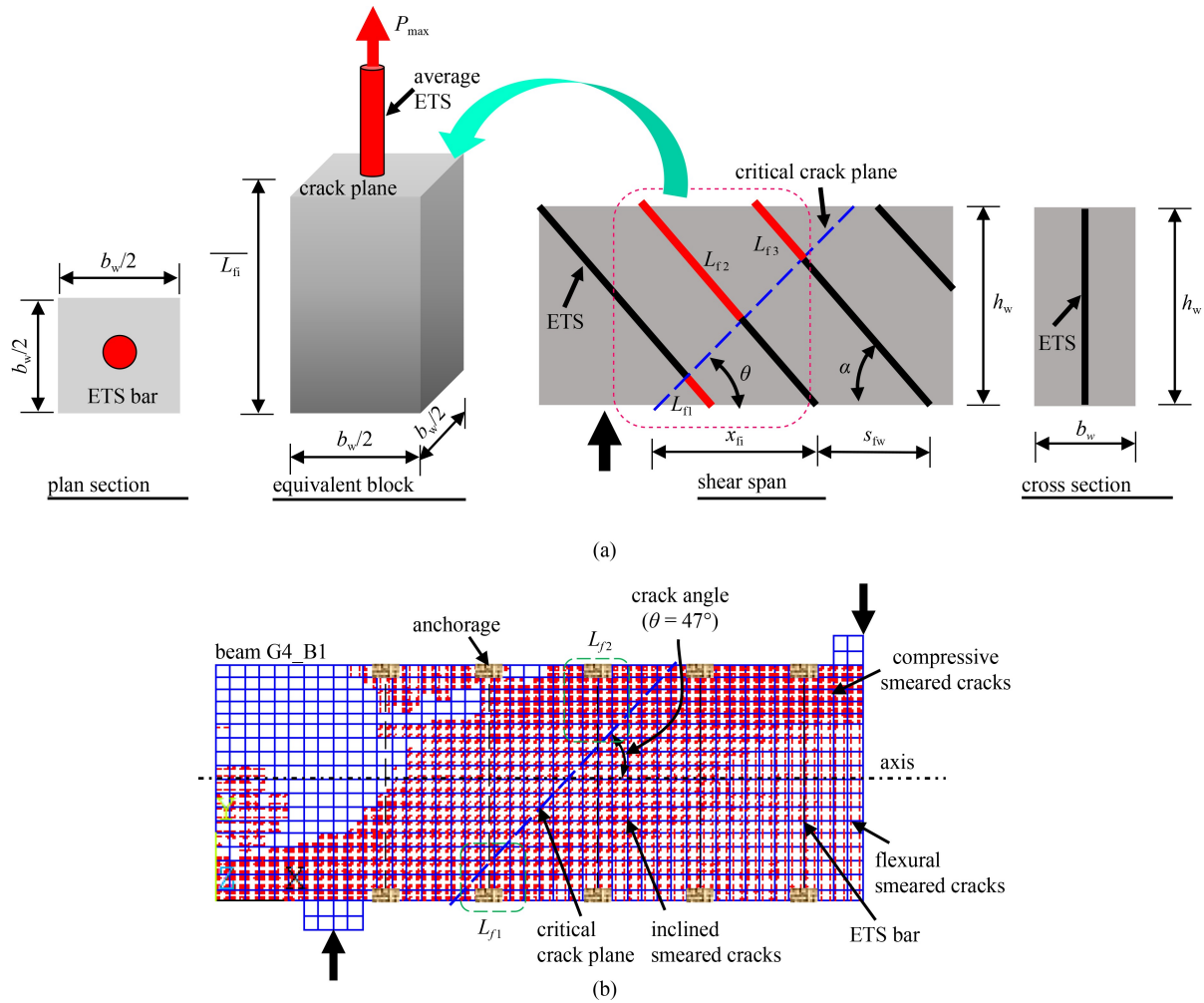


Fig. 7 Scheme to determine equivalent bond mechanism: (a) equivalent bond block [25] (Reprinted from Journal of Building Engineering, 29, Bui L V H, Stitmannathum B, Jongvivatsakul P, Comprehensive investigation on bond mechanism of embedded throughsection fiber-reinforced polymer bars to concrete for structural analysis, 101180, Copyright 2020, with permission from Elsevier.); (b) example for determination of the critical crack plane and crack angle via smeared cracks from the FEM simulation.

The bond length of the equivalent block, which is defined by the average bond length of the influenced ETS bars ($\bar{L}_{fi}$) in the shear zone of a beam, is calculated using Eq. (11).

$$\bar{L}_{fi} = \frac{1}{N_f} \sum_{i=1}^{N_f} L_{fi}, \quad (11)$$

where $\bar{L}_{fi}$ is the average bond length of the influenced ETS bars (mm). After that, the configurations and dimensions of the equivalent concrete block subjected to pullout specimen drawn in Fig. 7(a) can be fully identified. The knowledge of the bond model proposed in a study by the authors [20] for the ETS-concrete interfaces through pullout process is used to determine the bond force of ETS bar to concrete.

The bond parameter A , interfacial fracture energy (G_f), and theoretical maximum tensile force in FRP (P_{max}) of the equivalent pullout specimens are mathematically

defined by Eqs. (12)–(14), respectively. s_u is the maximum slip as indicated in Fig. 2(d). Note that Eq. (12) is defined by the equilibrium force between interfacial response and bar tension at the loaded end of the conceptual equivalent block. Obviously, via Eqs. (12)–(14), when the bond parameters, such as $\bar{L}_{fi}$, τ_m , s_m , and s_u , and the information of ETS strengthening are known, the bond force can be easily determined by Eq. (14). Bui et al. [8] and Mofidi et al. [12] revealed that the maximum bond stress (τ_m) is the most important factor, and it significantly impacts the shear capacity of the ETS bars defined via the interfacial bond mechanism. Based on the pullout test results attained by Bui et al. [26], for simplicity, in this study, s_u is assumed to be 0.7 mm, while s_m is taken to be 0.15 mm for all specimens, and $\bar{L}_{fi}$ is derived by Eq. (11). The next section attempts to propose a simplified model for the maximum bond stress (τ_m) between ETS bars and concrete in the ETS-strengthened RC beams.

$$A = \frac{p_r \tau_m \bar{L}_{fi}}{A_r E_r (1 - e^{-B s_u})}, \quad (12)$$

$$G_f = \int_0^{s_u} \tau ds = \frac{E_r A_r}{p_r} \times A^2 \times \left(\frac{1}{2} e^{-2 s_u B} - e^{-s_u B} + \frac{1}{2} \right), \quad (13)$$

$$P_{\max} = E_r A_r \varepsilon_{\max} = E_r A_r A = E_r A_r \sqrt{2 G_f \frac{p_r}{E_r A_r}}. \quad (14)$$

In the FEM simulation, premature failure at the anchorage was not observed; therefore, G_f in Eq. (13) can be assumed as the interfacial fracture energy, which is obtained from bond stress distribution along bonded length. This means that the total bond force at the failure is mainly carried by bond stresses. Depending on the frictional mechanism between ETS bars and concrete after debonding, the maximum slip (s_u) can be assumed to be infinite (∞).

Then, the bond forces are converted into the shear resisting forces of ETS bars in the strengthened beams as Eq. (15). Afterwards, the total shear strength of beam can be calculated according to the equations presented in Section 4.2.

$$V_{n, \text{ETS}}^b = N_r P_{\max} = N_r E_r A_r \sqrt{2 G_f \frac{p_r}{E_r A_r}}. \quad (15)$$

4.5 Validation of the shear strength models

Figure 8 presents the verification of the shear resisting models for prediction of the shear strength of ETS-strengthened RC beams by comparison with the experimental and reliable numerical data. The models proposed by Mofidi et al. [12] and built into the present study are based on the concept of bond mechanism between ETS bars and concrete. The key bond parameters for these models are τ_m , s_m , s_u , and p , which can be obtained from the calibration of the bond models to the pullout test results. As mentioned in Subsubsection 4.4.2, this study respectively uses s_m , s_u , and p by 0.15, 0.7, and 0.1 for all specimens and for both Mofidi et al. [12]'s model and proposed model. However, the current calculation employs the maximum bond stress with rigorous calibration for achieving the absolute fits to the shear contribution of ETS bars derived from the tests and simulations. Conversely, Bui et al. [16]'s model, which was established from a regression analysis, is used to compare with the remaining models in the calculation of the total shear strength of ETS-strengthened beams. In the computation made using all models, the shear strengths provided by stirrups and concrete are determined by the expressions in the ACI guideline [39], as shown in Eqs. (3) and (4).

Obviously, Fig. 8 indicates that the model of Mofidi et al. [12] underestimates the experimental and numerical results in terms of the total shear capacity of ETS-strengthened beams. As presented in Table 3, the mean of the ratios of the shear strength produced by the empirical model to the shear strength obtained from the tests and simulations is 0.72, and the coefficient of variation (*Cov*) of the mean is 18%. This could be due to the limitation of the effective strain in ETS-FRP system to 4000 $\mu\text{m}/\text{m}$ defined by the model of Mofidi et al. [12], while the fact is that the strain of ETS-FRP strengthening can be greater than that value depending on the elastic modulus of FRP. The foregoing limitation in the FRP strain was originally proposed for the FRP-RC beams in the past literature, and requires the prevention of the crack opening for ensuring the aggregate interlocking of concrete. However, for ETS strengthening technique, the FRP system interacts with the concrete via an adhesive layer, which could provide the stress transfer to affect the concrete aggregate interlocking. Therefore, the condition limiting the effective strain in FRP bars to 4000 $\mu\text{m}/\text{m}$ seems to be not suitable for the ETS strengthening method. The

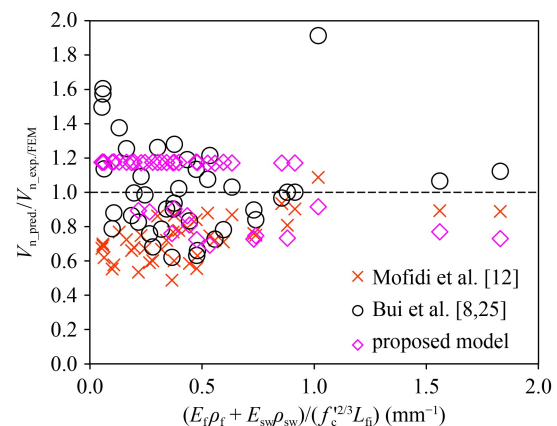


Fig. 8 Verification of shear resisting models. Note: τ_m in the model of Mofidi et al. [12] and the proposed model is selected to have best fit with ETS shear contribution.

Table 3 Accuracy of the models

items	Mofidi et al.'s model [12]	Bui et al.'s model [16]	proposed model
Calibration of τ_m to have best fit with V_f	—	—	—
average ($V_{n,pred}/V_{n,exp}/FEM$)	0.73	1.03	1.05
<i>Cov</i> (average)	0.18	0.27	0.18
τ_m expression (Eq. (16)) with $k = 1$	—	—	—
average ($V_{n,pred}/V_{n,exp}/FEM$)	0.73	1.03	1.04
<i>Cov</i> (average)	0.21	0.27	0.26
τ_m expression (Eq. (16)) with $k = 0.8$	—	—	—
average ($V_{n,pred}/V_{n,exp}/FEM$)	0.71	1.03	0.98
<i>Cov</i> (average)	0.20	0.27	0.25

results indicate that the values of the bond factors obtained from the absolute fit to the ETS shear contribution and lead to the conservativeness of the shear capacity model of Mofidi et al. [12].

In contrast, the proposed model furnishes a better prediction in the total shear strength than that offered by the model of Mofidi et al. [12] via the mean of 1.05 and the *Cov* of the mean of 18%, as shown in Table 3. The possible reason is attributed to the best fit of the bond variables for obtaining the best fit of the ETS shear contribution. The bond model used for establishing the shear resisting model allows the strain in the ETS strengthening to be developed until beam failure occurs, and can be included in the local debonding of ETS bars to concrete followed by the yielding of stirrups and the heavy fracture of concrete. Conversely, verification of the model of Bui et al. [16] reveals that their model can estimate the total shear strength of the ETS-strengthened beams with the average of 1.03 and the mean *Cov* of 27%. The model of Bui et al. [16] takes into account the effective strain of ETS strengthening based on a regression analysis of the experimental values associated with the truss theory. The more reliable test data are, the better the regression model is. The properties and geometries of the beams designed in this study are in range of the empirical model developed by Bui et al. [16].

Many pullout tests of the FRP bars–concrete joints in the previous studies, for example, Bui et al. [8], Godat et al. [24], and Caro et al. [41], have indicated that the bond performance substantially depends on the properties of FRP, concrete, and bond length. In addition, as reported in the studies by Breveglieri et al. [15], Bui et al. [16], and Triantafillou et al. [42], the strain of FRP strengthening as shear reinforcement correlates the term $(E_f \rho_f + E_{sw} \rho_{sw}) / (f_c'^{2/3} \bar{L}_{fi})$. It is noted that the bond mechanism between FRP and concrete is mainly analyzed via the strain of FRP. Further, the maximum bond stress is obviously affected by the bond length of FRP-concrete interface, which has been evident in the relevant past studies [26,40]. Thereby, this study introduces the term $(E_f \rho_f + E_{sw} \rho_{sw}) / (f_c'^{2/3} \bar{L}_{fi})$ to reflect its relationship to the maximum bond stress (τ_m). The values of τ_m used for calibration to have the absolute fit with the ETS shear contribution between the model and the experiment or simulation (see Fig. 8) are adopted for conducting the correlational analysis. The result of the correlational analysis is then shown in Fig. 9.

Figure 9 indicates that the increase of the term $(E_f \rho_f + E_{sw} \rho_{sw}) / (f_c'^{2/3} \bar{L}_{fi})$ results in the decrease of the maximum bond stress (τ_m). This means that higher concrete strength and bond length provide larger maximum bond stress. This finding is consistent with that found in a previous study by Bui et al. [26]. The increase of f_c' and/or $\bar{L}_{fi}$ offers great interfacial performance due to the properties of the concrete matrix and/or the characteristics of ‘strain’

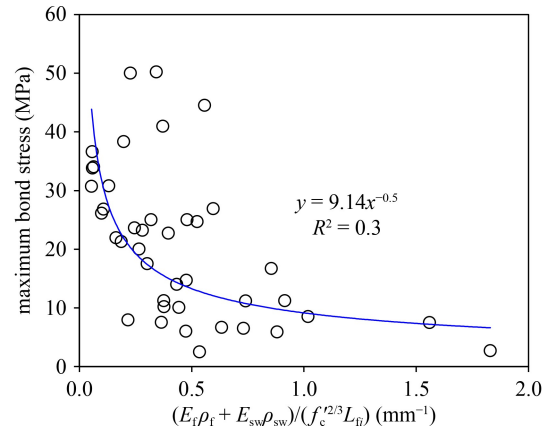


Fig. 9 Relationship between material function and maximum bond stress.

length. However, the relationship in Fig. 9 reveals that the enhancement of total shear reinforcement and ETS strengthening stiffness ($E_f \rho_f + E_{sw} \rho_{sw}$) decreases the maximum bond stress between ETS-FRP bars and concrete. This is attributable to the presence of steel stirrups that reduces the capacity of the ETS-FRP bars, which is shown in previous literature by Breveglieri et al. [14] and Bui et al. [16]. This result leads to the reduction of the bond performance including the maximum bond stress of the ETS-FRP bars to concrete. Based on the relationship obtained from Fig. 9, the fitting line can be expressed through the following equation:

$$\tau_m = 9.14 \sqrt{\frac{f_c'^{2/3} \times (k \bar{L}_{fi})}{E_f \rho_f + E_{sw} \rho_{sw}}} \quad (16)$$

A parameter k is introduced to modify the bond length in Eq. (16). This is because in some cases the bond length carries the bond stress transfer; thereby, the effective bond length, which is defined by the adequate bond area bearing the interfacial bond stress at the peak load, is examined. The effects of k are investigated in the following section. Eq. (16) shows that the maximum bond stress between ETS-FRP bars and concrete in the strengthened beams can be determined when the details of the beams are known with no necessity to search and select the bond parameters from the corresponding pullout tests, which are difficult and inadequate.

The expression of the maximum bond stress presented in Eq. (16) is used to recalculate the total shear strength of ETS-strengthened beams using the proposed model and Mofidi et al.'s model [12]. The total shear strengths of the beams investigated in previous sections from both tests and simulations are utilized to verify the shear strength models.

Figure 10 and Table 3 demonstrate the comparison between the experimental or numerical results and the model results in the total shear capacity of all ETS-strengthened beams in this study. By applying the

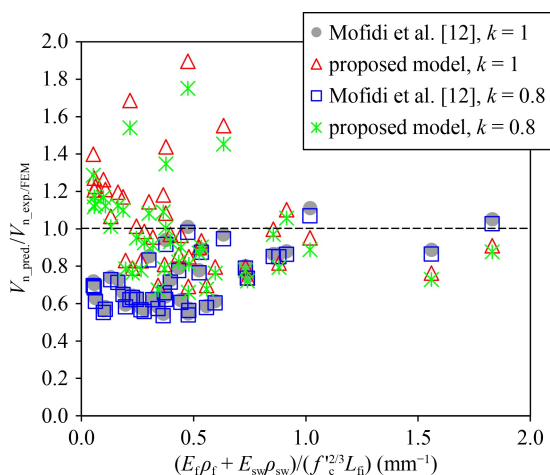


Fig. 10 Verification of the shear strength models with proposed expression for maximum bond stress.

proposed expression for the maximum bond stress, the model of Mofidi et al. [12] remains conservative. Meanwhile, the model proposed in this study, incorporating the new formula for maximum bond stress, can predict accurately and safely the total shear capacity of the ETS-strengthened beams. Despite the fact that the proposed model for design with $k = 1$ is good enough, the reduction factor of $k = 0.8$ on the average bond length between ETS-FRP bars and concrete should be suggested to have a safer design.

5 Conclusions

The following main conclusions can be drawn.

1) The FEM is a convenient tool for modeling the structural behaviors of ETS-strengthened beams with high accuracy.

2) A bonding-based model was successfully developed in this study based on the interfacial mechanism between ETS strengthening and concrete. A simplified formula for maximum bond stress of ETS bars to concrete in the strengthened beams was proposed.

3) Assemblage of the shear resisting models for concrete, steel stirrups, and ETS strengthening system proposed in this paper was a comprehensive and rational technique for estimation of the shear strength of the ETS-strengthened RC beams considering number of critical parameters.

4) Shear resisting model of Mofidi et al. [12] provided conservative results in the calculation of the shear capacity of the ETS-strengthened beams. Meanwhile, the proposed model and Bui et al. [16]'s model could predict accurately the total shear strength of ETS-strengthened RC beams. These foregoing models satisfied the requirements of safety and simplicity.

5) To achieve a safer design of the shear strength of ETS-strengthened RC beams using the proposed model,

the reduction factor on the average bond length by $k = 0.8$ was suggested.

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