Variant quantifiers in $\mathbf{L_3}$-valued first-order logic

Wei LI, Yuefei SUI

Frontiers of Computer Science, DOI: 10.1007/s11704-020-7399-7

Traditional first-order logic:

The semantics for quantifiers are defined as follows:

$\forall x A(x)$ is satisfied if for any element a, $A(x/a)$ is satisfied

$\exists x A(x)$ is satisfied if for some element a, $A(x/a)$ is satisfied.

B_2-valued first-order logic:

There are four kinds of definition for quantifier $\forall$, where $[\mathtt{t}]A \equiv A$, $[\mathtt{f}]A \equiv \neg A$:

$[\mathtt{t}]\forall^{11}xA(x)$ is satisfied if for any element a, $[\mathtt{t}]A(x/a)$ is satisfied

$[\mathtt{f}]\forall^{11}xA(x)$ is satisfied if for some element a, $[\mathtt{f}]A(x/a)$ is satisfied;

$[\mathtt{t}]\forall^{10}xA(x)$ is satisfied if for any element a, $[\mathtt{t}]A(x/a)$ is satisfied

$[\mathtt{f}]\forall^{10}xA(x)$ is satisfied if for any element a, $[\mathtt{f}]A(x/a)$ is satisfied;

$[\mathtt{t}]\forall^{01}xA(x)$ is satisfied if for some element a, $[\mathtt{t}]A(x/a)$ is satisfied

$[\mathtt{f}]\forall^{01}xA(x)$ is satisfied if for any element a, $[\mathtt{f}]A(x/a)$ is satisfied;

$[\mathtt{t}]\forall^{00}xA(x)$ is satisfied if for some element a, $[\mathtt{t}]A(x/a)$ is satisfied

$[\mathtt{f}]\forall^{00}xA(x)$ is satisfied if for some element a, $[\mathtt{f}]A(x/a)$ is satisfied.

L_3-valued first-order logic:

Assume that there are three modalities $[\mathtt{t}], [\mathtt{m}], [\mathtt{f}]$ to be applied on formulas. There are eight definitions for quantifiers.

- $\mathbf{L}_3^{000}$:

$$[\mathtt{t}]\forall x A(x) \text{ is true if for any element } a, [\mathtt{t}]A(x/a) \text{ is true}$$

$$[\mathtt{m}]\forall x A(x) \text{ is true if for any element } a, [\mathtt{t}]A(x/a) \vee [\mathtt{m}]A(x/a) \text{ is true}$$
$$\text{and for some element } b, [\mathtt{m}]A(x/b) \text{ is true}$$

$$[\mathtt{f}]\forall x A(x) \text{ is true if for some element } a, [\mathtt{f}]A(x/a) \text{ is true;}$$

- $\mathbf{L}_3^{001}$:

$$[\mathtt{t}]\forall x A(x) \text{ is true if for any element } a, [\mathtt{t}]A(x/a) \text{ is true}$$

$$[\mathtt{m}]\forall x A(x) \text{ is true if for any element } a, [\mathtt{t}]A(x/a) \vee [\mathtt{m}]A(x/a) \text{ is true}$$
$$\text{and for some element } b, [\mathtt{m}]A(x/b) \text{ is true}$$

$$[\mathtt{f}]\forall x A(x) \text{ is true if for any element } a, [\mathtt{f}]A(x/a) \text{ is true;}$$

$\mathbf{L}_3$-valued first-order logic:

- $\mathbf{L}_3^{010}$:

$$
\begin{aligned}
&[\mathtt{t}]\forall x A(x) \text{ is true if for any element } a, [\mathtt{t}]A(x/a) \text{ is true}\\
&[\mathtt{m}]\forall x A(x) \text{ is true if for any element } a, [\mathtt{f}]A(x/a) \vee [\mathtt{m}]A(x/a) \text{ is true}\\
&\qquad\qquad\qquad\qquad \text{and for some element } b, [\mathtt{m}]A(x/b) \text{ is true}\\
&[\mathtt{f}]\forall x A(x) \text{ is true if for some element } a, [\mathtt{f}]A(x/a) \text{ is true;}
\end{aligned}
$$

- $\mathbf{L}_3^{011}$:

$$
\begin{aligned}
&[\mathtt{t}]\forall x A(x) \text{ is true if for any element } a, [\mathtt{t}]A(x/a) \text{ is true}\\
&[\mathtt{m}]\forall x A(x) \text{ is true if for any element } a, [\mathtt{f}]A(x/a) \vee [\mathtt{m}]A(x/a) \text{ is true}\\
&\qquad\qquad\qquad\qquad \text{and for some element } b, [\mathtt{m}]A(x/b) \text{ is true}\\
&[\mathtt{f}]\forall x A(x) \text{ is true if for any element } a, [\mathtt{f}]A(x/a) \text{ is true;}
\end{aligned}
$$

$\mathbf{L}_3$-valued first-order logic:

- $\mathbf{L}_3^{100}$:

$$[\mathtt{t}]\forall x A(x) \text{ is true if for some element } a, [\mathtt{t}]A(x/a) \text{ is true}$$

$$[\mathtt{m}]\forall x A(x) \text{ is true if for any element } a, [\mathtt{t}]A(x/a) \vee [\mathtt{m}]A(x/a) \text{ is true}$$
$$\text{and for some element } b, [\mathtt{m}]A(x/b) \text{ is true}$$

$$[\mathtt{f}]\forall x A(x) \text{ is true if for some element } a, [\mathtt{f}]A(x/a) \text{ is true;}$$

- $\mathbf{L}_3^{101}$:

$$[\mathtt{t}]\forall x A(x) \text{ is true if for some element } a, [\mathtt{t}]A(x/a) \text{ is true}$$

$$[\mathtt{m}]\forall x A(x) \text{ is true if for any element } a, [\mathtt{t}]A(x/a) \vee [\mathtt{m}]A(x/a) \text{ is true}$$
$$\text{and for some element } b, [\mathtt{m}]A(x/b) \text{ is true}$$

$$[\mathtt{f}]\forall x A(x) \text{ is true if for any element } a, [\mathtt{f}]A(x/a) \text{ is true;}$$

L_3-valued first-order logic:

- $\mathbf{L}_3^{110}$:

$$[\mathtt{t}]\forall x A(x) \text{ is true if for some element } a, [\mathtt{t}]A(x/a) \text{ is true}$$

$$[\mathtt{m}]\forall x A(x) \text{ is true if for any element } a, [\mathtt{f}]A(x/a) \vee [\mathtt{m}]A(x/a) \text{ is true}$$
$$\text{and for some element } b, [\mathtt{m}]A(x/b) \text{ is true}$$

$$[\mathtt{f}]\forall x A(x) \text{ is true if for some element } a, [\mathtt{f}]A(x/a) \text{ is true;}$$

- $\mathbf{L}_3^{111}$:

$$[\mathtt{t}]\forall x A(x) \text{ is true if for some element } a, [\mathtt{t}]A(x/a) \text{ is true}$$

$$[\mathtt{m}]\forall x A(x) \text{ is true if for any element } a, [\mathtt{f}]A(x/a) \vee [\mathtt{m}]A(x/a) \text{ is true}$$
$$\text{and for some element } b, [\mathtt{m}]A(x/b) \text{ is true}$$

$$[\mathtt{f}]\forall x A(x) \text{ is true if for any element } a, [\mathtt{f}]A(x/a) \text{ is true.}$$

L_3-valued first-order logic:

We will give a sound and complete Gentzen deduction system for each choice of quantifiers.

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